

Discrete Mathematics in Computer Science

B1. Tuples & Cartesian Product

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Tuples and the Cartesian Product

Motivation

- A **set** is an **unordered collection** of **distinct** objects.
- We often need a more structured way of representation.
 - A person is associated with a name, address, phone number.
 - A set of persons makes sense in many contexts.
 - Representing the associated data as a set rather not.
- We could for example want to
 - directly access the name of a person, or
 - have a separate billing and delivery address for some order, but in general, these can be the same.
- **Tuples** are mathematical building blocks that support this.

Sets vs. Tuples

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Sets vs. Tuples

- A **set** is an **unordered collection** of **distinct** objects.
- A **tuple** is an **ordered sequence** of objects.

Tuples

- **k -tuple**: ordered sequence of k objects ($k \in \mathbb{N}_0$)
- written $(o_1, \dots, o_k)$ or $\langle o_1, \dots, o_k \rangle$
- unlike sets, **order matters** ($\langle 1, 2 \rangle \neq \langle 2, 1 \rangle$)
- objects may occur multiple times in a tuple

German: k -Tupel, Komponente, (geordnetes) Paar, Tripel, Quadrupel

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- objects contained in tuples are called **components**
- terminology:
 - $k = 2$: (ordered) pair
 - $k = 3$: triple
 - more rarely: quadruple, quintuple, sextuple, septuple, ...
- if k is clear from context (or does not matter),
often just called **tuple**

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Equality of Tuples

Definition (Equality of Tuples)

Two n -tuples $t = \langle o_1, \dots, o_n \rangle$ and $t' = \langle o'_1, \dots, o'_n \rangle$ are **equal** ($t = t'$) if for $i \in \{1, \dots, n\}$ it holds that $o_i = o'_i$.

Cartesian Product

Definition (Cartesian Product and Cartesian Power)

Let $S_1, \dots, S_n$ be sets. The **Cartesian product** $S_1 \times \dots \times S_n$ is the following set of n -tuples:

$$S_1 \times \dots \times S_n = \{ \langle x_1, \dots, x_n \rangle \mid x_1 \in S_1, x_2 \in S_2, \dots, x_n \in S_n \}.$$

René Descartes: French mathematician and philosopher (1596–1650)

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Example: $A = \{a, b\}$, $B = \{1, 2, 3\}$

$A \times B =$

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The k -ary **Cartesian power** of a set S (with $k \in \mathbb{N}_1$) is the set $S^k = \{ \langle o_1, \dots, o_k \rangle \mid o_i \in S \text{ for all } i \in \{1, \dots, k\} \} = \underbrace{S \times \dots \times S}_{k \text{ times}}.$

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Example: $A = \{a, b\}$, $B = \{1, 2, 3\}$

$A^2 =$

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(Non-)properties of the Cartesian Product

The Cartesian product is

- **not commutative**, in most cases $A \times B \neq B \times A$.
- **not associative**, in most cases $(A \times B) \times C \neq A \times (B \times C)$

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Why? Exceptions?

Questions



Questions?

Summary

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- A k -tuple is an ordered sequence of k objects, called the components of the tuple.
- 2-tuples are also called pairs and 3-tuples triples.
- The Cartesian Product $S_1 \times \cdots \times S_n$ of set $S_1, \dots, S_n$ is the set of all tuples $\langle o_1, \dots, o_n \rangle$, where for all $i \in \{1, \dots, n\}$ component o_i is an element of S_i .