

Planning and Optimization

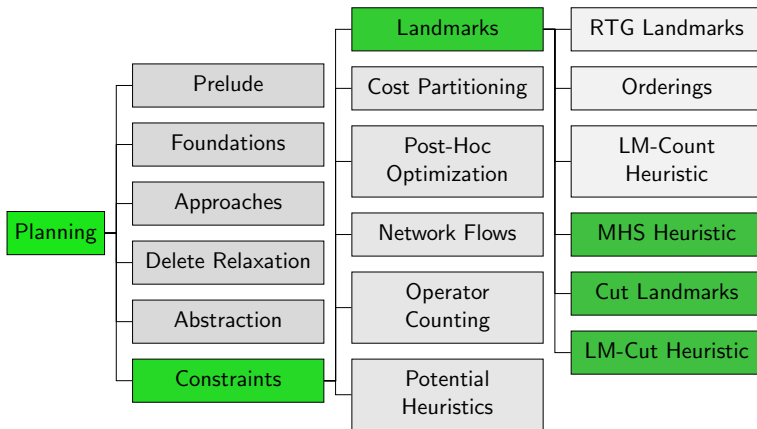
F4. Landmarks: Minimum Hitting Set Heuristic

Malte Helmert and Gabriele Röger

Universität Basel

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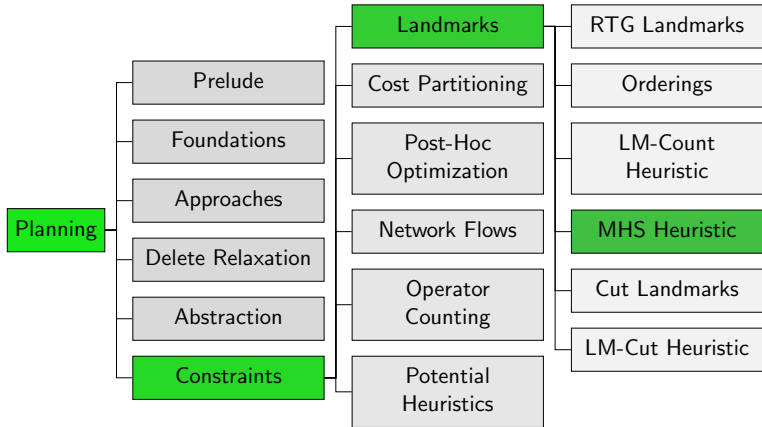
Content of the Course



The remaining landmark topics focus on disjunctive action landmarks.

Minimum Hitting Set Heuristic

Content of the Course



Exploiting Disjunctive Action Landmarks

- The cost $cost(L)$ of a disjunctive action landmark L is an admissible heuristic, but it is usually not very informative.
- Landmark heuristics typically aim to combine multiple disjunctive action landmarks.

How can we exploit a given set \mathcal{L} of disjunctive action landmarks?

- Sum of costs $\sum_{L \in \mathcal{L}} cost(L)$?
 \rightsquigarrow **not admissible!**
- Maximize costs $\max_{L \in \mathcal{L}} cost(L)$?
 \rightsquigarrow **usually very weak heuristic**
- **better:** Hitting sets

Hitting Sets

Definition (Hitting Set)

Let X be a set, $\mathcal{F} = \{F_1, \dots, F_n\} \subseteq 2^X$ be a family of subsets of X and $c : X \rightarrow \mathbb{R}_0^+$ be a cost function for X .

A **hitting set** is a subset $H \subseteq X$ that “hits” all subsets in \mathcal{F} , i.e., $H \cap F \neq \emptyset$ for all $F \in \mathcal{F}$. The **cost** of H is $\sum_{x \in H} c(x)$.

A **minimum hitting set (MHS)** is a hitting set with minimal cost.

MHS is a “classical” NP-complete problem (Karp, 1972)

Example: Hitting Sets

Example

$$X = \{o_1, o_2, o_3, o_4\}$$

$$\mathcal{F} = \{\{o_4\}, \{o_1, o_2\}, \{o_1, o_3\}, \{o_2, o_3\}\}$$

$$c(o_1) = 3, \quad c(o_2) = 4, \quad c(o_3) = 5, \quad c(o_4) = 0$$

Specify a minimum hitting set.

Example: Hitting Sets

Example

$$X = \{o_1, o_2, o_3, o_4\}$$

$$\mathcal{F} = \{\{o_4\}, \{o_1, o_2\}, \{o_1, o_3\}, \{o_2, o_3\}\}$$

$$c(o_1) = 3, \quad c(o_2) = 4, \quad c(o_3) = 5, \quad c(o_4) = 0$$

Specify a minimum hitting set.

Solution: $\{o_1, o_2, o_4\}$ with cost $3 + 4 + 0 = 7$

Hitting Sets for Disjunctive Action Landmarks

Idea: **disjunctive action landmarks** are interpreted as instance of **minimum hitting set**

Definition (Hitting Set Heuristic)

Let \mathcal{L} be a set of disjunctive action landmarks. The **hitting set heuristic** $h^{MHS}(\mathcal{L})$ is defined as the cost of a minimum hitting set for \mathcal{L} with $c(o) = cost(o)$.

Proposition (Hitting Set Heuristic is Admissible)

Let \mathcal{L} be a set of disjunctive action landmarks for state s . Then $h^{MHS}(\mathcal{L})$ is an admissible estimate for s .

Hitting Set Heuristic: Discussion

- The hitting set heuristic is the **best possible** heuristic that only uses the given information...
- ...but is NP-hard to compute.
- \rightsquigarrow Use approximations that can be efficiently computed.
 \Rightarrow LP-relaxation, cost partitioning (both discussed later)

Summary

Summary

- **Hitting sets** yield the most accurate heuristic for a given set of disjunctive action landmarks.
- The computation of a **minimal hitting set** is NP-hard.