Planning and Optimization

E10. Merge-and-Shrink: Algorithm

Malte Helmert and Gabriele Röger

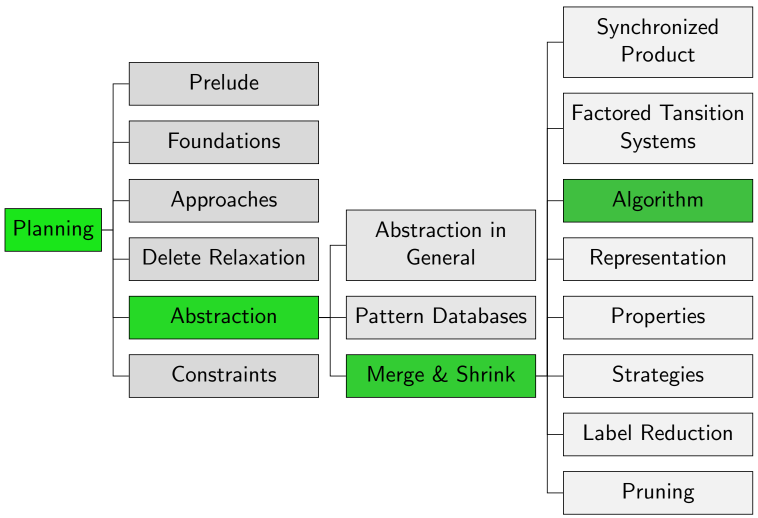
Universität Basel

November 18, 2024

Planning and Optimization

November 18, 2024 — E10. Merge-and-Shrink: Algorithm

Content of the Course



E10.1 Generic Algorithm

Generic Merge-and-shrink Abstractions: Outline

Using the results of the previous chapter, we can develop a generic abstraction computation procedure that takes all state variables into account.

- Initialization: Compute the FTS consisting of all atomic projections.
- Loop: Repeatedly apply a transformation to the FTS.
  - Merging: Combine two factors by replacing them with their synchronized product.
  - Shrinking: If the factors are too large, make one of them smaller by abstracting it further (applying an arbitrary abstraction to it).
- Termination: Stop when only one factor is left.

The final factor is then used for an abstraction heuristic.

Generic Algorithm Template

Generic Merge & Shrink Algorithm for planning task Π

$F := F(\Pi)$
while $|F| > 1$:
  select $type \in \{\text{merge}, \text{shrink}\}$
  if $type = \text{merge}$:
    select $\mathcal{T}_1, \mathcal{T}_2 \in F$
    $F := (F \setminus \{\mathcal{T}_1, \mathcal{T}_2\}) \cup \{\mathcal{T}_1 \otimes \mathcal{T}_2\}$
  if $type = \text{shrink}$:
    select $\mathcal{T} \in F$
    choose an abstraction mapping β on $\mathcal{T}$
    $F := (F \setminus \{\mathcal{T}\}) \cup \{\mathcal{T}^\beta\}$
return the remaining factor $\mathcal{T}^\alpha$ in F

In Ch. E12 and E13, we will include more transformation types (label reduction and pruning)

Merge-and-Shrink Strategies

Choices to resolve to instantiate the template:

- When to merge, when to shrink?
  $\rightsquigarrow$ general strategy
- Which abstractions to merge?
  $\rightsquigarrow$ merge strategy
- Which abstraction to shrink, and how to shrink it (which β)?
  $\rightsquigarrow$ shrink strategy

merge and shrink strategies $\rightsquigarrow$ Ch. E11/E12

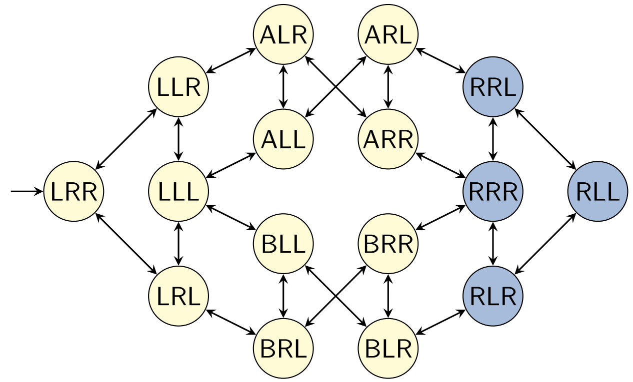
General Strategy

A typical general strategy:

- define a limit N on the number of states allowed in each factor
- in each iteration, select two factors we would like to merge
- merge them if this does not exhaust the state number limit
- otherwise shrink one or both factors just enough to make a subsequent merge possible

E10.2 Example

Back to the Running Example

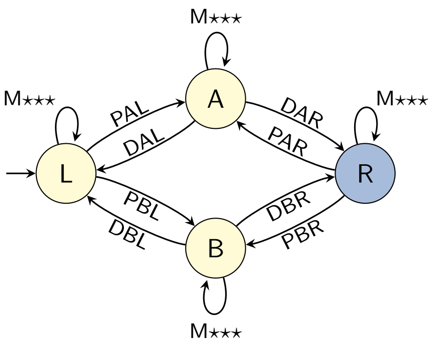


Logistics problem with one package, two trucks, two locations:

- state variable package: $\{L, R, A, B\}$
- state variable truck A: $\{L, R\}$
- state variable truck B: $\{L, R\}$

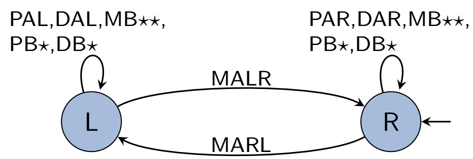
Initialization Step: Atomic Projection for Package

$\mathcal{T}^{\pi_{\{\text{package}\}}}$:



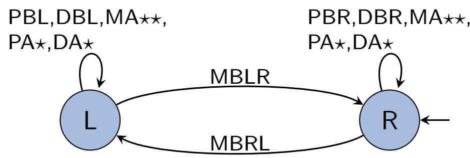
Initialization Step: Atomic Projection for Truck A

$\mathcal{T}^{\pi_{\{\text{truck A}\}}}$:



Initialization Step: Atomic Projection for Truck B

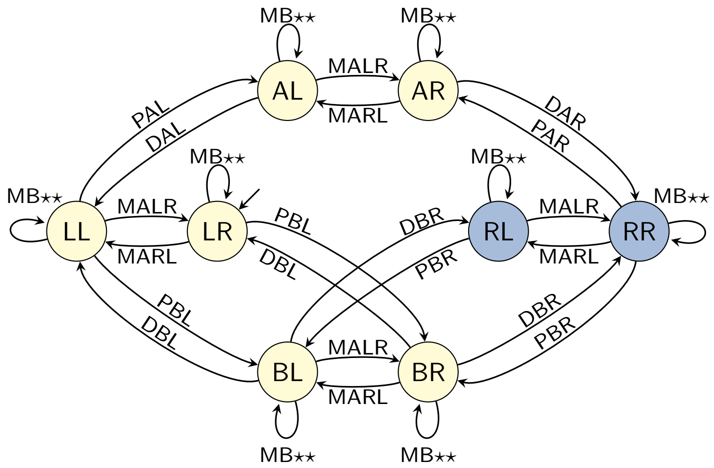
$\mathcal{T}^{\pi_{\{\text{truck B}\}}}$:



current FTS: $\{\mathcal{T}^{\pi_{\{\text{package}\}}}, \mathcal{T}^{\pi_{\{\text{truck A}\}}}, \mathcal{T}^{\pi_{\{\text{truck B}\}}}\}$

First Merge Step

$\mathcal{T}_1 := \mathcal{T}^{\pi_{\{\text{package}\}}} \otimes \mathcal{T}^{\pi_{\{\text{truck A}\}}}$:



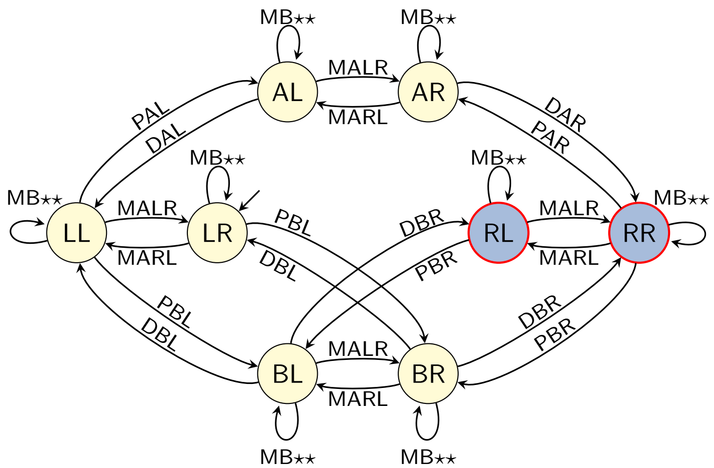
current FTS: $\{\mathcal{T}_1, \mathcal{T}^{\pi_{\{\text{truck B}\}}}\}$

Need to Shrink?

- With sufficient memory, we could now compute $\mathcal{T}_1 \otimes \mathcal{T}^{\pi_{\{\text{truck B}\}}}$ and recover the full transition system of the task.
- However, to illustrate the general idea, we assume that memory is too restricted: we may never create a factor with more than 8 states.
- To make the product fit the bound, we shrink $\mathcal{T}_1$ to 4 states. We can decide freely how exactly to abstract $\mathcal{T}_1$.
- In this example, we manually choose an abstraction that leads to a good result in the end. Making good shrinking decisions algorithmically is the job of the shrink strategy.

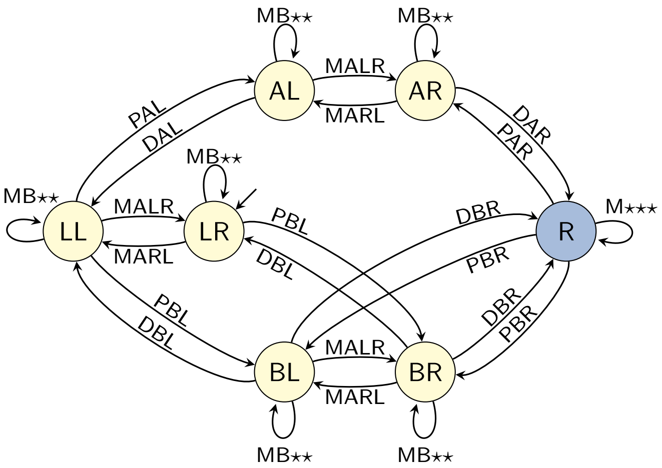
First Shrink Step

$\mathcal{T}_2 :=$ some abstraction of $\mathcal{T}_1$



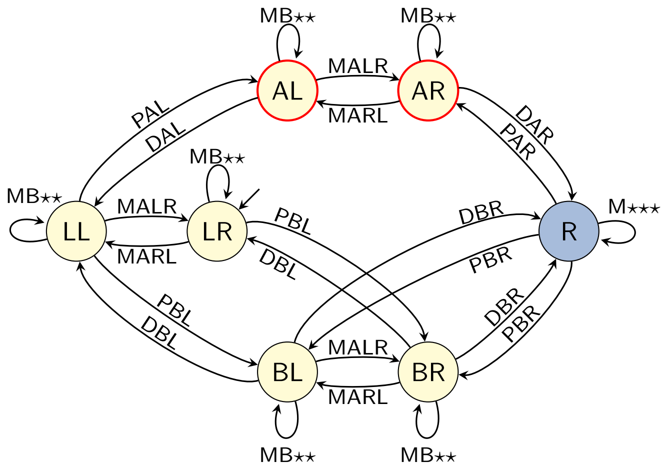
First Shrink Step

$\mathcal{T}_2 :=$ some abstraction of $\mathcal{T}_1$



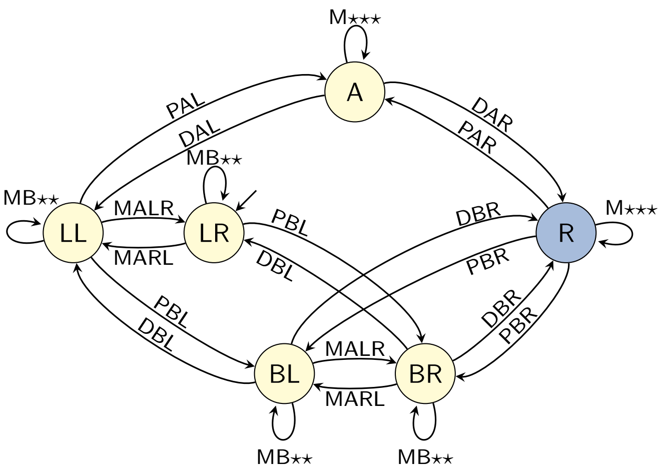
First Shrink Step

$\mathcal{T}_2 :=$ some abstraction of $\mathcal{T}_1$



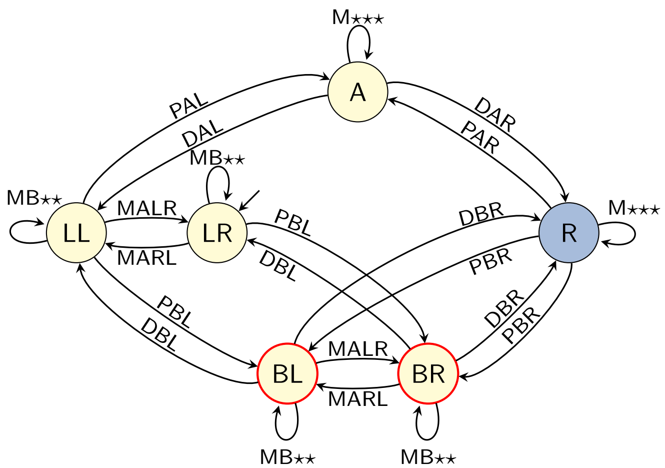
First Shrink Step

$\mathcal{T}_2 :=$ some abstraction of $\mathcal{T}_1$



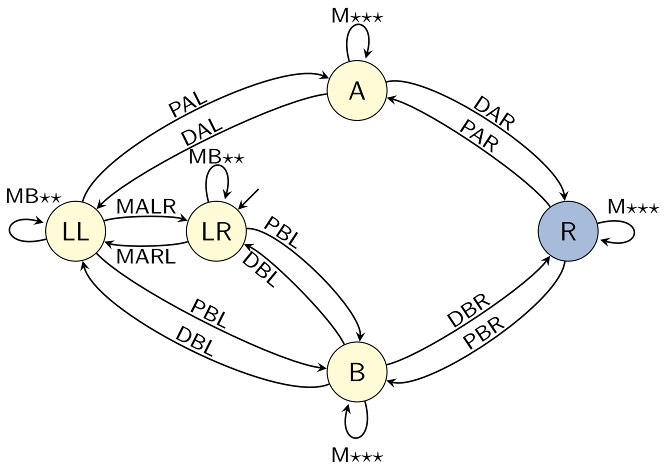
First Shrink Step

$\mathcal{T}_2 :=$ some abstraction of $\mathcal{T}_1$



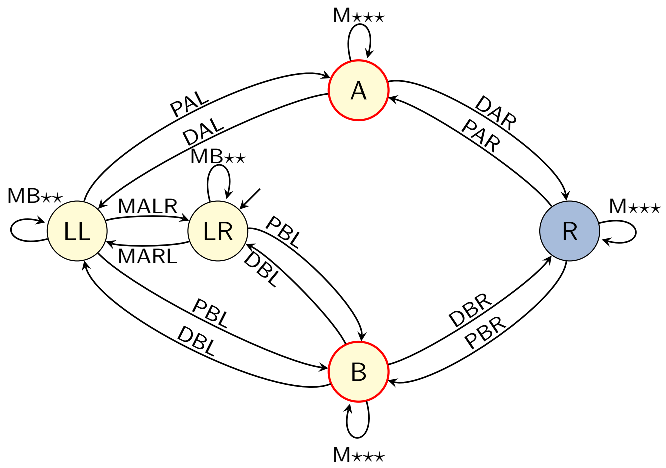
First Shrink Step

$\mathcal{T}_2 :=$ some abstraction of $\mathcal{T}_1$



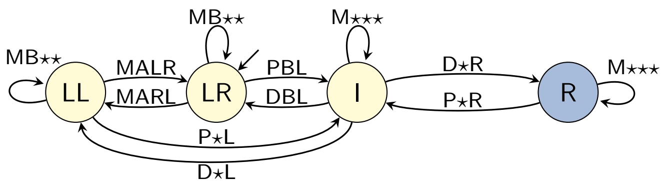
First Shrink Step

$\mathcal{T}_2 :=$ some abstraction of $\mathcal{T}_1$



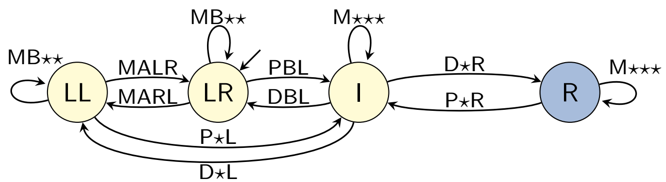
First Shrink Step

$\mathcal{T}_2 :=$ some abstraction of $\mathcal{T}_1$



First Shrink Step

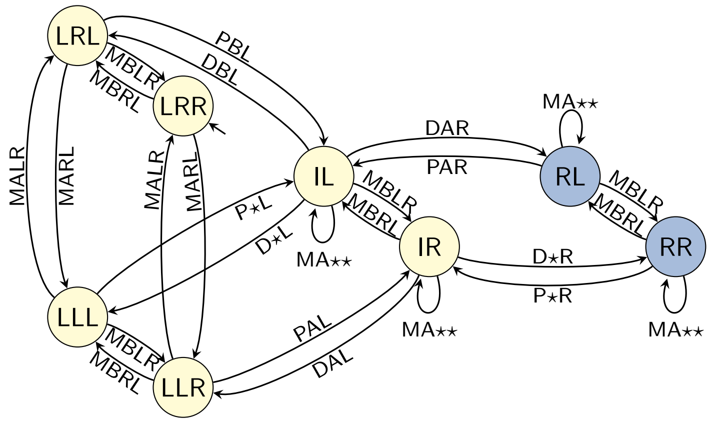
$\mathcal{T}_2 :=$ some abstraction of $\mathcal{T}_1$



current FTS: $\{\mathcal{T}_2, \mathcal{T}^{\pi_{\{\text{truck B}\}}}\}$

Second Merge Step

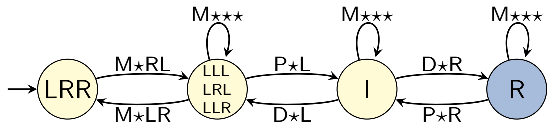
$\mathcal{T}_3 := \mathcal{T}_2 \otimes \mathcal{T}^{\pi_{\{\text{truck B}\}}}$:



current FTS: $\{\mathcal{T}_3\}$

Another Shrink Step?

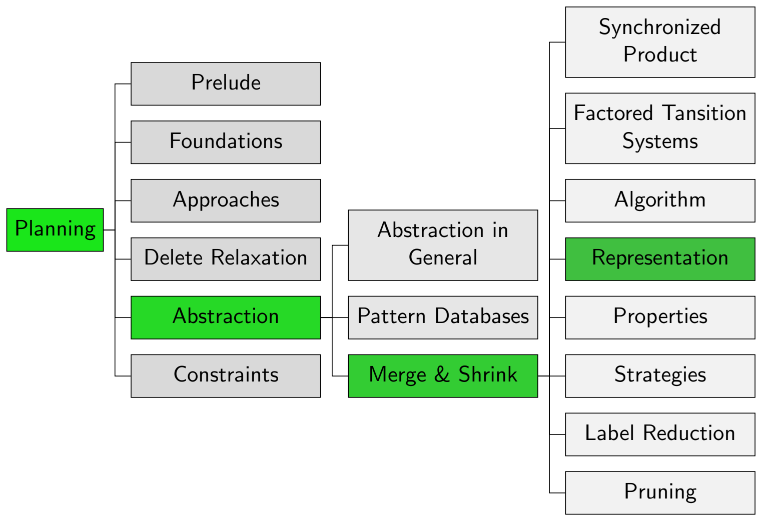
- At this point, merge-and-shrink construction stops. The distances in the final factor define the heuristic function.
- If there were further state variables to integrate, we would shrink again, e.g., leading to the following abstraction (again with four states):



- We get a heuristic value of 3 for the initial state, better than any PDB heuristic that is a proper abstraction.
- The example generalizes to arbitrarily many trucks, even if we stick to the fixed size limit of 8.

E10.3 Maintaining the Abstraction

Content of the Course



Generic Algorithm Template

Generic Merge & Shrink Algorithm for planning task Π

$F := F(\Pi)$
while $|F| > 1$:
  select $type \in \{\text{merge}, \text{shrink}\}$
  if $type = \text{merge}$:
    select $\mathcal{T}_1, \mathcal{T}_2 \in F$
    $F := (F \setminus \{\mathcal{T}_1, \mathcal{T}_2\}) \cup \{\mathcal{T}_1 \otimes \mathcal{T}_2\}$
  if $type = \text{shrink}$:
    select $\mathcal{T} \in F$
    choose an abstraction mapping β on $\mathcal{T}$
    $F := (F \setminus \{\mathcal{T}\}) \cup \{\mathcal{T}^\beta\}$
return the remaining factor $\mathcal{T}^\alpha$ in F

- The algorithm computes an abstract transition system.
- For the heuristic evaluation, we need an abstraction.
- How to maintain and represent the corresponding abstraction?

The Need for Succinct Abstractions

- One major difficulty for non-PDB abstraction heuristics is to succinctly represent the abstraction.
- For pattern databases, this is easy because the abstractions – projections – are very structured.
- For less rigidly structured abstractions, we need another idea.

How to Represent the Abstraction? (1)

Idea: the computation of the abstraction follows the sequence of product computations

- For the atomic abstractions $\pi_{\{v\}}$, we generate a one-dimensional table that denotes which value in $\text{dom}(v)$ corresponds to which abstract state in $\mathcal{T}^{\pi_{\{v\}}}$.
- During the merge (product) step $\mathcal{A} := \mathcal{A}_1 \otimes \mathcal{A}_2$, we generate a two-dimensional table that denotes which pair of states of $\mathcal{A}_1$ and $\mathcal{A}_2$ corresponds to which state of $\mathcal{A}$.
- During the shrink (abstraction) steps, we make sure to keep the table in sync with the abstraction choices.

How to Represent the Abstraction? (2)

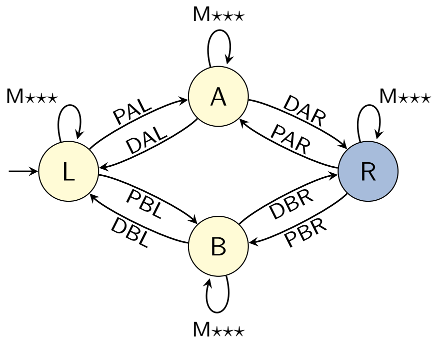
Idea: the computation of the abstraction mapping follows the sequence of product computations

- Once we have computed the final abstract transition system, we compute all abstract goal distances and store them in a one-dimensional table.
- At this point, we can throw away all the abstract transition systems – we just need to keep the tables.
- During search, we do a sequence of table lookups to navigate from the atomic abstraction states to the final abstract state and heuristic value
  $\rightsquigarrow 2|V|$ lookups, $O(|V|)$ time

Again, we illustrate the process with our running example.

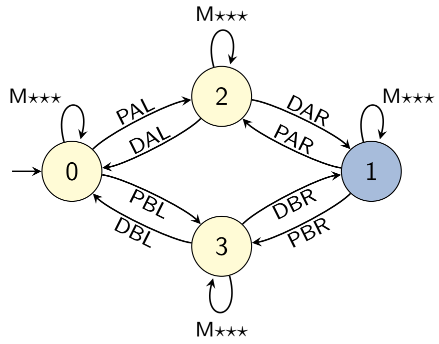
Abstraction Example: Atomic Abstractions

Computing abstractions for the transition systems of atomic abstractions is simple. Just number the states (domain values) consecutively and generate a table of references to the states:



Abstraction Example: Atomic Abstractions

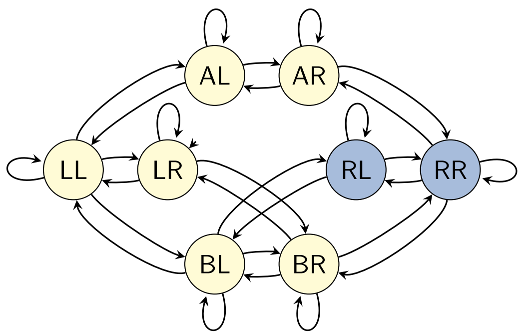
Computing abstractions for the transition systems of atomic abstractions is simple. Just number the states (domain values) consecutively and generate a table of references to the states:



L	R	A	B
0	1	2	3

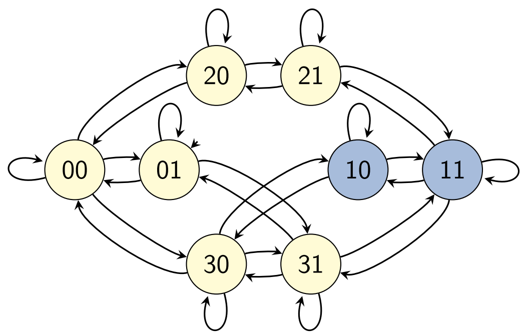
Abstraction Example: Merge Step

For product transition systems $\mathcal{A}_1 \otimes \mathcal{A}_2$, we again number the product states consecutively and generate a table that links state pairs of $\mathcal{A}_1$ and $\mathcal{A}_2$ to states of $\mathcal{A}$:



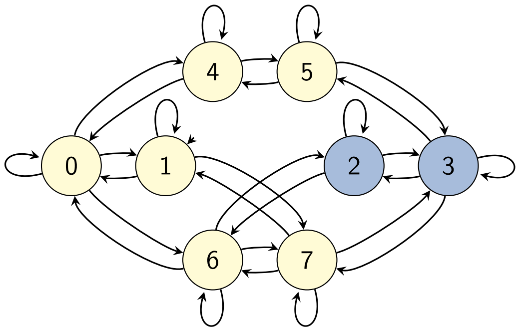
Abstraction Example: Merge Step

For product transition systems $\mathcal{A}_1 \otimes \mathcal{A}_2$, we again number the product states consecutively and generate a table that links state pairs of $\mathcal{A}_1$ and $\mathcal{A}_2$ to states of $\mathcal{A}$:



Abstraction Example: Merge Step

For product transition systems $\mathcal{A}_1 \otimes \mathcal{A}_2$, we again number the product states consecutively and generate a table that links state pairs of $\mathcal{A}_1$ and $\mathcal{A}_2$ to states of $\mathcal{A}$:



	$s_2 = 0$	$s_2 = 1$
$s_1 = 0$	0	1
$s_1 = 1$	2	3
$s_1 = 2$	4	5
$s_1 = 3$	6	7

Maintaining the Abstraction when Shrinking

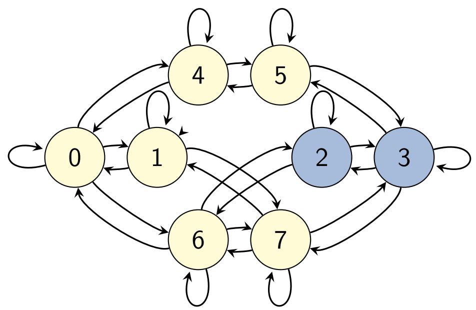
- The hard part in representing the abstraction is to keep it consistent when shrinking.
- In theory, this is easy to do:
  - When combining states i and j, arbitrarily use one of them (say i) as the number of the new state.
  - Find all table entries in the table for this abstraction which map to the other state j and change them to i.
- However, doing a table scan each time two states are combined is very inefficient.
- Fortunately, there also is an efficient implementation which takes constant time per combination.

Maintaining the Abstraction Efficiently

- Associate each abstract state with a linked list, representing all table entries that map to this state.
- Before starting the shrink operation, initialize the lists by scanning through the table, then discard the table.
- While shrinking, when combining i and j, splice the list elements of j into the list elements of i.
  - For linked lists, this is a constant-time operation.
- Once shrinking is completed, renumber all abstract states so that there are no gaps in the numbering.
- Finally, regenerate the mapping table from the linked list information.

Abstraction Example: Shrink Step

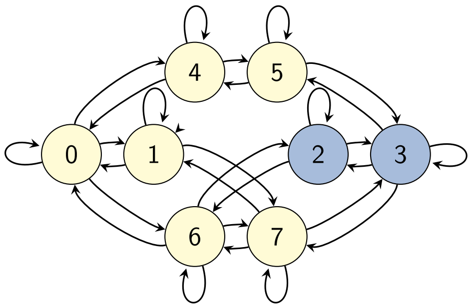
Representation before shrinking:



	$s_2 = 0$	$s_2 = 1$
$s_1 = 0$	0	1
$s_1 = 1$	2	3
$s_1 = 2$	4	5
$s_1 = 3$	6	7

Abstraction Example: Shrink Step

1. Convert table to linked lists and discard it.

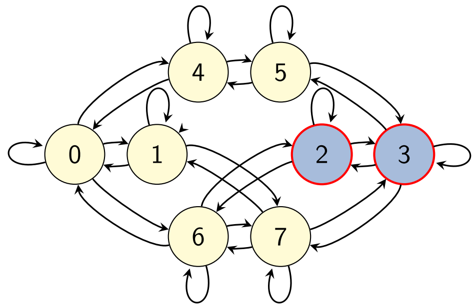


$list_0 = \{(0, 0)\}$
$list_1 = \{(0, 1)\}$
$list_2 = \{(1, 0)\}$
$list_3 = \{(1, 1)\}$
$list_4 = \{(2, 0)\}$
$list_5 = \{(2, 1)\}$
$list_6 = \{(3, 0)\}$
$list_7 = \{(3, 1)\}$

	$s_2 = 0$	$s_2 = 1$
$s_1 = 0$	0	1
$s_1 = 1$	2	3
$s_1 = 2$	4	5
$s_1 = 3$	6	7

Abstraction Example: Shrink Step

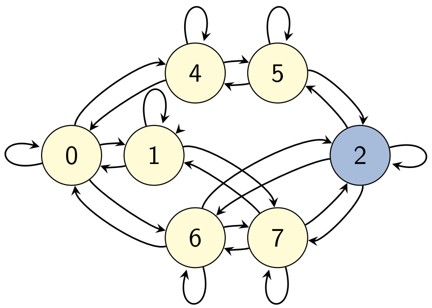
2. When combining i and j, splice $list_j$ into $list_i$.



$list_0 = \{(0, 0)\}$
$list_1 = \{(0, 1)\}$
$list_2 = \{(1, 0)\}$
$list_3 = \{(1, 1)\}$
$list_4 = \{(2, 0)\}$
$list_5 = \{(2, 1)\}$
$list_6 = \{(3, 0)\}$
$list_7 = \{(3, 1)\}$

Abstraction Example: Shrink Step

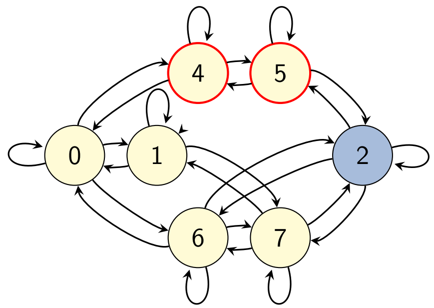
2. When combining i and j, splice $list_j$ into $list_i$.



$list_0 = \{(0, 0)\}$
$list_1 = \{(0, 1)\}$
$list_2 = \{(1, 0), (1, 1)\}$
$list_3 = \emptyset$
$list_4 = \{(2, 0)\}$
$list_5 = \{(2, 1)\}$
$list_6 = \{(3, 0)\}$
$list_7 = \{(3, 1)\}$

Abstraction Example: Shrink Step

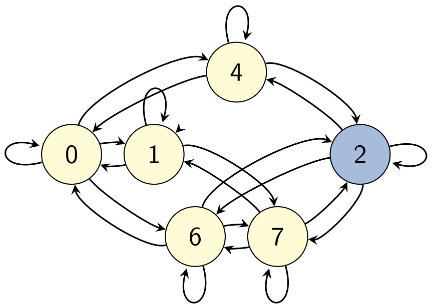
2. When combining i and j, splice $list_j$ into $list_i$.



$list_0 = \{(0, 0)\}$
$list_1 = \{(0, 1)\}$
$list_2 = \{(1, 0), (1, 1)\}$
$list_3 = \emptyset$
$list_4 = \{(2, 0)\}$
$list_5 = \{(2, 1)\}$
$list_6 = \{(3, 0)\}$
$list_7 = \{(3, 1)\}$

Abstraction Example: Shrink Step

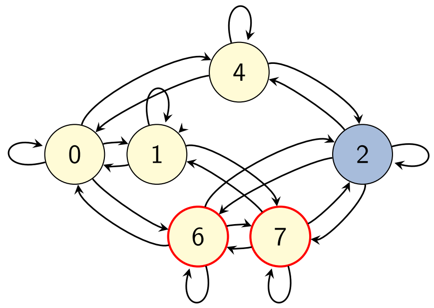
2. When combining i and j, splice $list_j$ into $list_i$.



$list_0 = \{(0,0)\}$
$list_1 = \{(0,1)\}$
$list_2 = \{(1,0),(1,1)\}$
$list_3 = \emptyset$
$list_4 = \{(2,0),(2,1)\}$
$list_5 = \emptyset$
$list_6 = \{(3,0)\}$
$list_7 = \{(3,1)\}$

Abstraction Example: Shrink Step

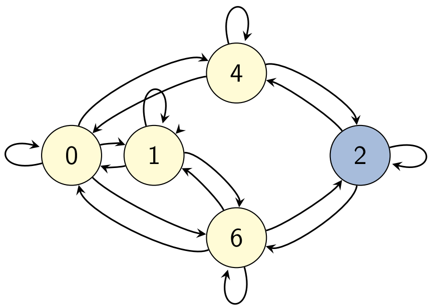
2. When combining i and j, splice $list_j$ into $list_i$.



$list_0 = \{(0,0)\}$
$list_1 = \{(0,1)\}$
$list_2 = \{(1,0),(1,1)\}$
$list_3 = \emptyset$
$list_4 = \{(2,0),(2,1)\}$
$list_5 = \emptyset$
$list_6 = \{(3,0)\}$
$list_7 = \{(3,1)\}$

Abstraction Example: Shrink Step

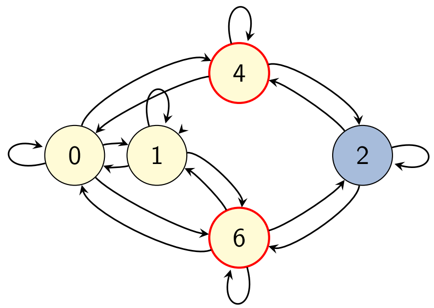
2. When combining i and j, splice $list_j$ into $list_i$.



$list_0 = \{(0,0)\}$
$list_1 = \{(0,1)\}$
$list_2 = \{(1,0),(1,1)\}$
$list_3 = \emptyset$
$list_4 = \{(2,0),(2,1)\}$
$list_5 = \emptyset$
$list_6 = \{(3,0),(3,1)\}$
$list_7 = \emptyset$

Abstraction Example: Shrink Step

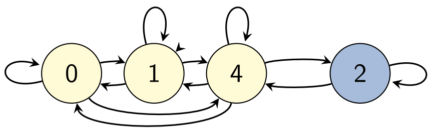
2. When combining i and j, splice $list_j$ into $list_i$.



$list_0 = \{(0,0)\}$
$list_1 = \{(0,1)\}$
$list_2 = \{(1,0),(1,1)\}$
$list_3 = \emptyset$
$list_4 = \{(2,0),(2,1)\}$
$list_5 = \emptyset$
$list_6 = \{(3,0),(3,1)\}$
$list_7 = \emptyset$

Abstraction Example: Shrink Step

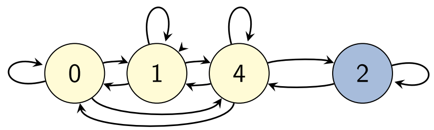
2. When combining i and j, splice $list_j$ into $list_i$.



$list_0 = \{(0,0)\}$
$list_1 = \{(0,1)\}$
$list_2 = \{(1,0),(1,1)\}$
$list_3 = \emptyset$
$list_4 = \{(2,0),(2,1),$
$(3,0),(3,1)\}$
$list_5 = \emptyset$
$list_6 = \emptyset$
$list_7 = \emptyset$

Abstraction Example: Shrink Step

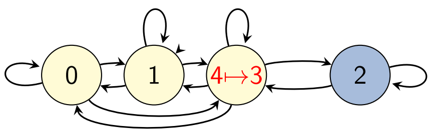
2. When combining i and j, splice $list_j$ into $list_i$.



$list_0 = \{(0,0)\}$
$list_1 = \{(0,1)\}$
$list_2 = \{(1,0),(1,1)\}$
$list_3 = \emptyset$
$list_4 = \{(2,0),(2,1),$
$(3,0),(3,1)\}$
$list_5 = \emptyset$
$list_6 = \emptyset$
$list_7 = \emptyset$

Abstraction Example: Shrink Step

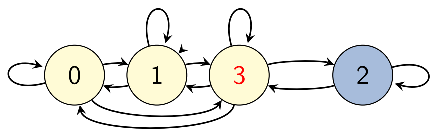
3. Renumber abstract states consecutively.



$list_0 = \{(0,0)\}$
$list_1 = \{(0,1)\}$
$list_2 = \{(1,0),(1,1)\}$
$list_3 = \emptyset$
$list_4 = \{(2,0),(2,1),$
$(3,0),(3,1)\}$
$list_5 = \emptyset$
$list_6 = \emptyset$
$list_7 = \emptyset$

Abstraction Example: Shrink Step

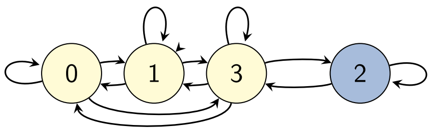
3. Renumber abstract states consecutively.



$list_0 = \{(0,0)\}$
$list_1 = \{(0,1)\}$
$list_2 = \{(1,0),(1,1)\}$
$list_3 = \{(2,0),(2,1),$
$(3,0),(3,1)\}$
$list_4 = \emptyset$
$list_5 = \emptyset$
$list_6 = \emptyset$
$list_7 = \emptyset$

Abstraction Example: Shrink Step

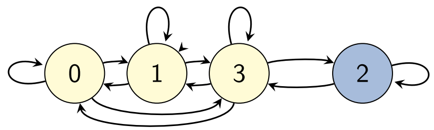
4. Regenerate the mapping table from the linked lists.



$list_0 = \{(0,0)\}$
$list_1 = \{(0,1)\}$
$list_2 = \{(1,0),(1,1)\}$
$list_3 = \{(2,0),(2,1),$
$(3,0),(3,1)\}$
$list_4 = \emptyset$
$list_5 = \emptyset$
$list_6 = \emptyset$
$list_7 = \emptyset$

Abstraction Example: Shrink Step

4. Regenerate the mapping table from the linked lists.



$list_0 = \{(0,0)\}$
$list_1 = \{(0,1)\}$
$list_2 = \{(1,0),(1,1)\}$
$list_3 = \{(2,0),(2,1),$
$(3,0),(3,1)\}$
$list_4 = \emptyset$
$list_5 = \emptyset$
$list_6 = \emptyset$
$list_7 = \emptyset$

	$s_2 = 0$	$s_2 = 1$
$s_1 = 0$	0	1
$s_1 = 1$	2	2
$s_1 = 2$	3	3
$s_1 = 3$	3	3

The Final Heuristic Representation

At the end, our heuristic is represented by six tables:

- three one-dimensional tables for the atomic abstractions:

T_{package}	L	R	A	B
	0	1	2	3

$T_{\text{truck A}}$	L	R
	0	1

$T_{\text{truck B}}$	L	R
	0	1

- two tables for the two merge and subsequent shrink steps:

$T^1_{\text{m\&s}}$	$s_2 = 0$	$s_2 = 1$
$s_1 = 0$	0	1
$s_1 = 1$	2	2
$s_1 = 2$	3	3
$s_1 = 3$	3	3

$T^2_{\text{m\&s}}$	$s_2 = 0$	$s_2 = 1$
$s_1 = 0$	1	1
$s_1 = 1$	1	0
$s_1 = 2$	2	2
$s_1 = 3$	3	3

- one table with goal distances for the final transition system:

T_h	$s = 0$	$s = 1$	$s = 2$	$s = 3$
$h(s)$	3	2	0	1

Given a state $s = \{\text{package} \mapsto L, \text{truck A} \mapsto L, \text{truck B} \mapsto R\}$,
its heuristic value is then looked up as:

- $h(s) = T_h[T^2_{\text{m\&s}}[T^1_{\text{m\&s}}[T_{\text{package}}[L], T_{\text{truck A}}[L]], T_{\text{truck B}}[R]]]$

E10.4 Summary

Summary (1)

- Merge-and-shrink abstractions are constructed by iteratively transforming the factored transition system of a planning task.
- Merge transformations combine two factors into their synchronized product.
- Shrink transformations reduce the size of a factor by abstracting it.
- Merge-and-shrink abstractions are represented by a set of reference tables, one for each atomic abstraction and one for each merge-and-shrink step.
- The heuristic representation uses an additional table for the goal distances in the final abstract transition system.

Summary (2)

- Projections of SAS^+ tasks correspond to merges of atomic factors.
- By also including shrinking, merge-and-shrink abstractions generalize projections: they can reflect all state variables, but in a potentially lossy way.