

Planning and Optimization

E9. Merge-and-Shrink: Factored Transition Systems

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November 18, 2024

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E9.1 Motivation

E9.2 Main Idea

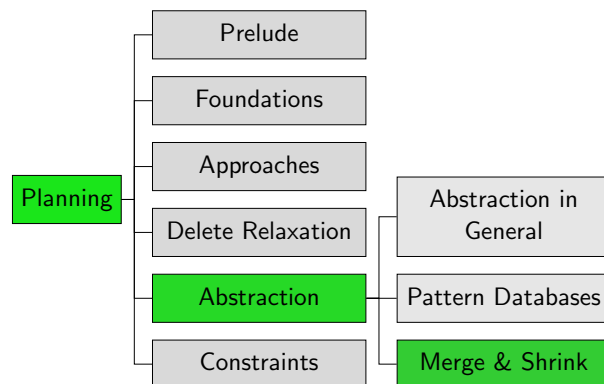
E9.3 Atomic Projections

E9.4 Synchronized Product

E9.5 Factored Transition Systems

E9.6 Summary

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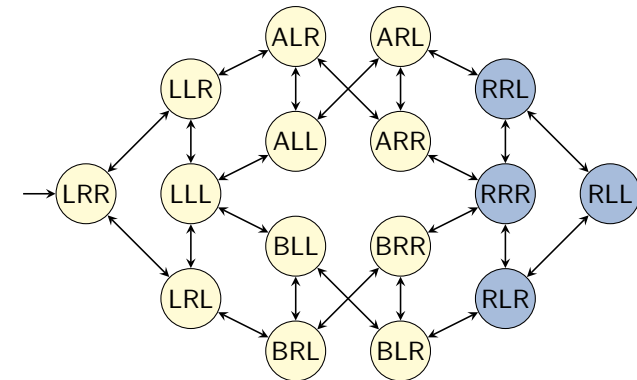


E9.1 Motivation

Beyond Pattern Databases

- ▶ Despite their popularity, pattern databases have some **fundamental limitations** (\rightsquigarrow example on next slides).
- ▶ Today and next time, we study a class of abstractions called **merge-and-shrink abstractions**.
- ▶ Merge-and-shrink abstractions can be seen as a **proper generalization** of pattern databases.
 - ▶ They can do everything that pattern databases can do (modulo polynomial extra effort).
 - ▶ They can do some things that pattern databases cannot.

Back to the Running Example

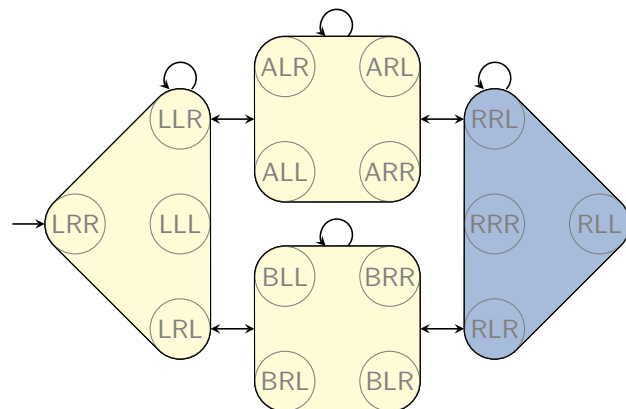


Logistics problem with one package, two trucks, two locations:

- ▶ state variable **package**: $\{L, R, A, B\}$
- ▶ state variable **truck A**: $\{L, R\}$
- ▶ state variable **truck B**: $\{L, R\}$

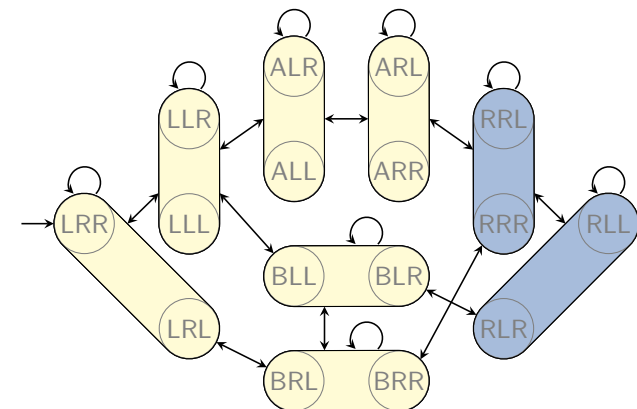
Example: Projection (1)

$\mathcal{T}^\pi_{\{\text{package}\}}$:



Example: Projection (2)

$\mathcal{T}^\pi_{\{\text{package, truck A}\}}$:



Limitations of Projections

How accurate is the PDB heuristic?

- ▶ consider **generalization of the example**:
 N trucks, 1 package
- ▶ consider **any** pattern that is a proper subset of variable set V
- ▶ $h(s_0) \leq 2 \rightsquigarrow$ **no better** than atomic projection to **package**

These values cannot be improved by maximizing over several patterns or using additive patterns.

Merge-and-shrink abstractions can represent heuristics with $h(s_0) \geq 3$ for tasks of this kind of any size.

Time and space requirements are **linear in N** .

(In fact, with time/space $O(N^2)$ we can construct a merge-and-shrink abstraction that gives the **perfect heuristic h^*** for such tasks, but we do not show this here.)

E9.2 Main Idea

Merge-and-Shrink Abstractions: Main Idea

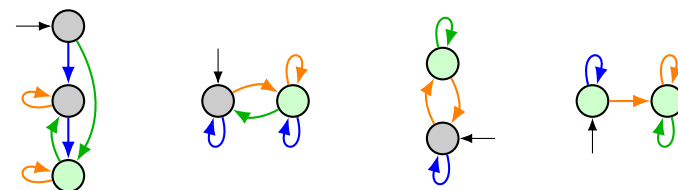
Main Idea of Merge-and-shrink Abstractions
 (due to Dräger, Finkbeiner & Podelski, 2006):

Instead of **perfectly** reflecting **a few** state variables, reflect **all** state variables, but in a **potentially lossy** way.

- ▶ Represent planning task as **factored transition system (FTS)**: a set of (small) abstract transition systems (**factors**) that jointly represent the full transition system of the task.
- ▶ Iteratively **transform** FTS by:
 - ▶ **merging**: combining two factors into one
 - ▶ **shrinking**: reducing the size of a single factor by abstraction
- ▶ When only a single factor is left, its goal distances are the merge-and-shrink heuristic values.

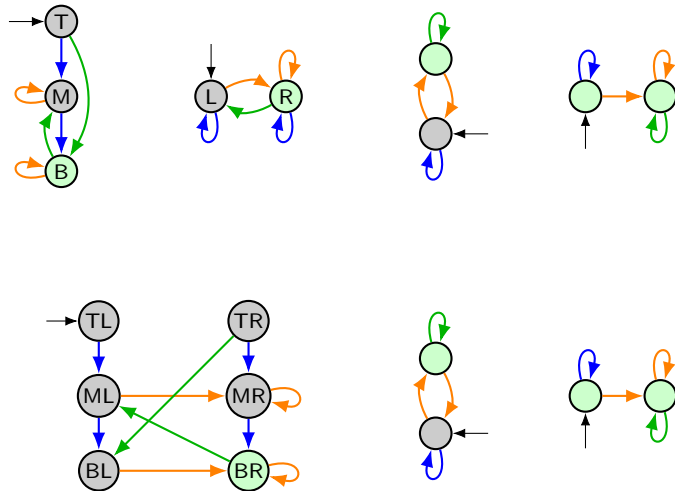
Merge-and-Shrink Abstractions: Idea

Start from atomic factors (projections to single state variables)



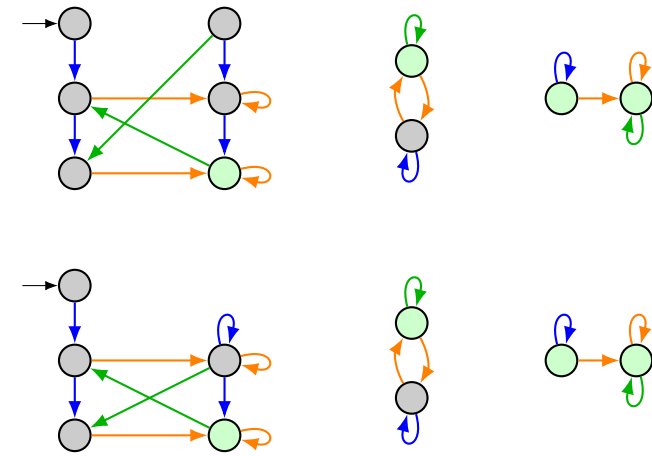
Merge-and-Shrink Abstractions: Idea

Merge: replace two factors with their product

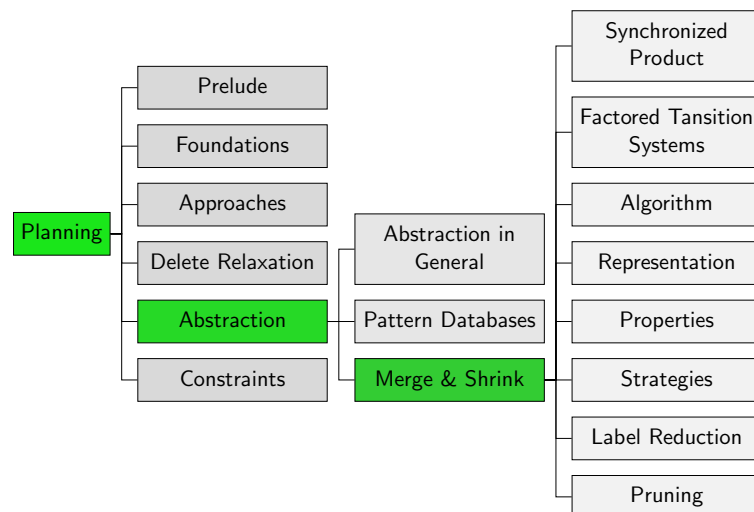


Merge-and-Shrink Abstractions: Idea

Shrink: replace a factor by an abstraction of it



Content of the Course

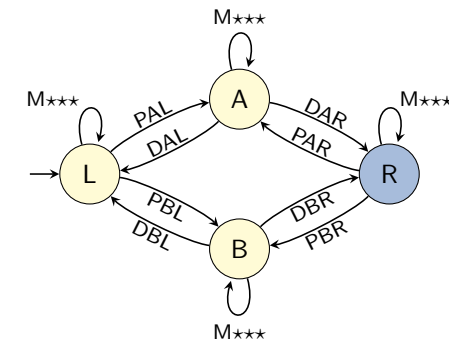


E9.3 Atomic Projections

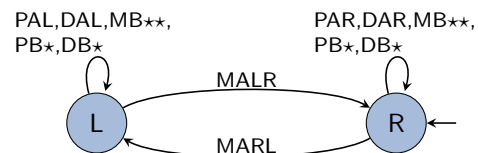
Running Example: Explanations

- ▶ **Atomic projections** (projections to a single state variable) play an important role for merge-and-shrink abstractions.
- ▶ Unlike previous chapters, **transition labels** are critically important for merge-and-shrink.
- ▶ Hence we now look at the transition systems for atomic projections of our example task, including transition labels.
- ▶ We abbreviate labels (operator names) as in these examples:
 - ▶ **MALR**: move truck **A** from **left** to **right**
 - ▶ **DAR**: drop package from truck **A** at **right** location
 - ▶ **PBL**: pick up package with truck **B** at **left** location
- ▶ We abbreviate parallel arcs with **commas** and **wildcards (*)** as in these examples:
 - ▶ **PAL, DAL**: two parallel arcs labeled **PAL** and **DAL**
 - ▶ **MA****: two parallel arcs labeled **MALR** and **MARL**

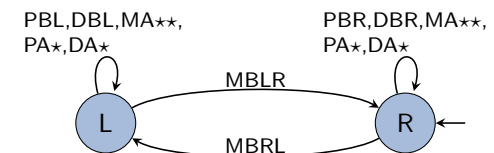
Running Example: Atomic Projection for Package

 $\mathcal{T}^{\pi}\{\text{package}\}:$


Running Example: Atomic Projection for Truck A

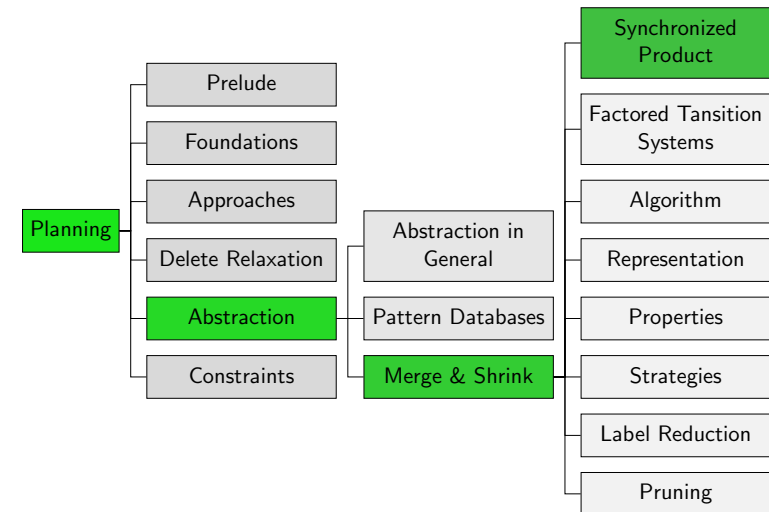
 $\mathcal{T}^{\pi}\{\text{truck A}\}:$


Running Example: Atomic Projection for Truck B

 $\mathcal{T}^{\pi}\{\text{truck B}\}:$


E9.4 Synchronized Product

Content of the Course



Synchronized Product: Idea

- ▶ Given two abstract transition systems with the same labels, we can compute a **product transition system**.
- ▶ The product transition system **captures all information** of both transition systems.
- ▶ A sequence of labels is a solution for the product iff it is a solution for both factors.

Synchronized Product of Transition Systems

Definition (Synchronized Product of Transition Systems)

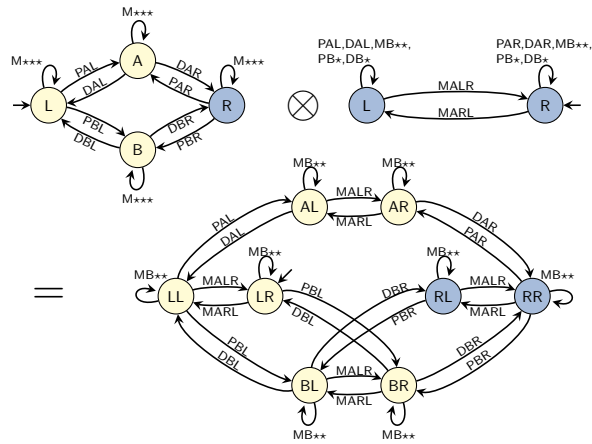
For $i \in \{1, 2\}$, let $\mathcal{T}_i = \langle S_i, L, c, T_i, s_{0_i}, S_{*i} \rangle$ be transition systems with the same labels and cost function.

The **synchronized product** of \mathcal{T}_1 and \mathcal{T}_2 , in symbols $\mathcal{T}_1 \otimes \mathcal{T}_2$, is the transition system $\mathcal{T}_\otimes = \langle S_\otimes, L, c, T_\otimes, s_{0_\otimes}, S_{*\otimes} \rangle$ with

- ▶ $S_\otimes = S_1 \times S_2$
- ▶ $T_\otimes = \{ \langle s_1, s_2 \rangle \xrightarrow{\ell} \langle t_1, t_2 \rangle \mid s_1 \xrightarrow{\ell} t_1 \in T_1 \text{ and } s_2 \xrightarrow{\ell} t_2 \in T_2 \}$
- ▶ $s_{0_\otimes} = \langle s_{0_1}, s_{0_2} \rangle$
- ▶ $S_{*\otimes} = S_{*1} \times S_{*2}$

Example: Synchronized Product

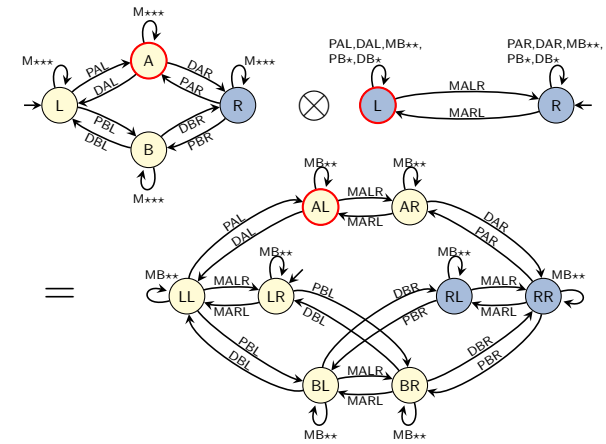
$$\mathcal{T}^\pi\{\text{package}\} \otimes \mathcal{T}^\pi\{\text{truck A}\}:$$



Example: Synchronized Product

$$\mathcal{T}^\pi\{\text{package}\} \otimes \mathcal{T}^\pi\{\text{truck A}\}:$$

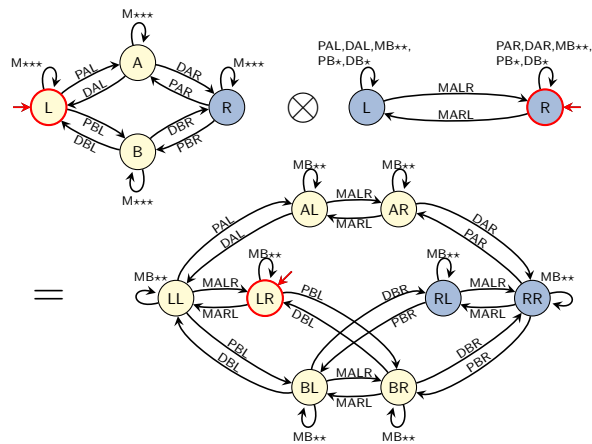
$$S_{\otimes} = S_1 \times S_2$$



Example: Synchronized Product

$$\mathcal{T}^\pi\{\text{package}\} \otimes \mathcal{T}^\pi\{\text{truck A}\}:$$

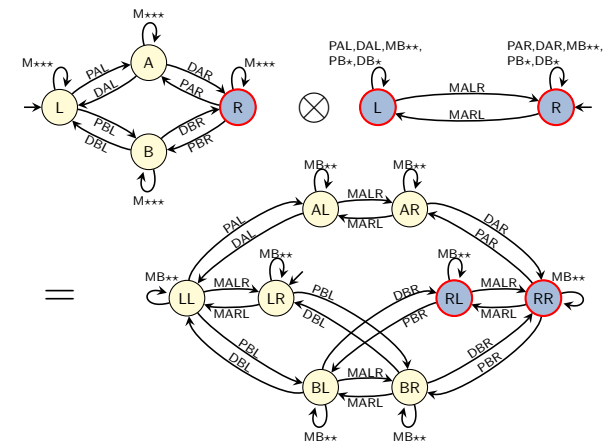
$$S_{\otimes} = \langle s_{01}, s_{02} \rangle$$



Example: Synchronized Product

$$\mathcal{T}^\pi\{\text{package}\} \otimes \mathcal{T}^\pi\{\text{truck A}\}:$$

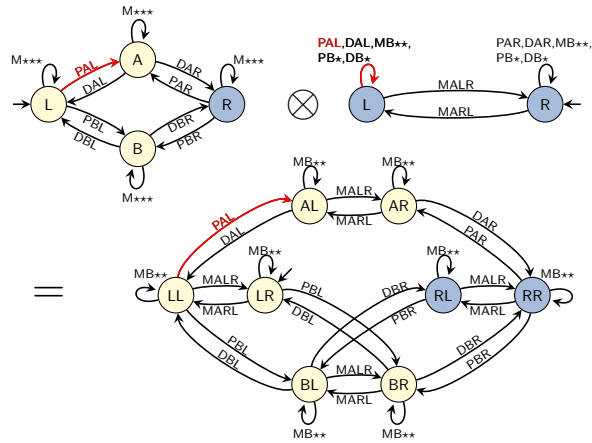
$$S_{*\otimes} = S_{*1} \times S_{*2}$$



Example: Synchronized Product

 $\mathcal{T}^\pi\{\text{package}\} \otimes \mathcal{T}^\pi\{\text{truck A}\}:$

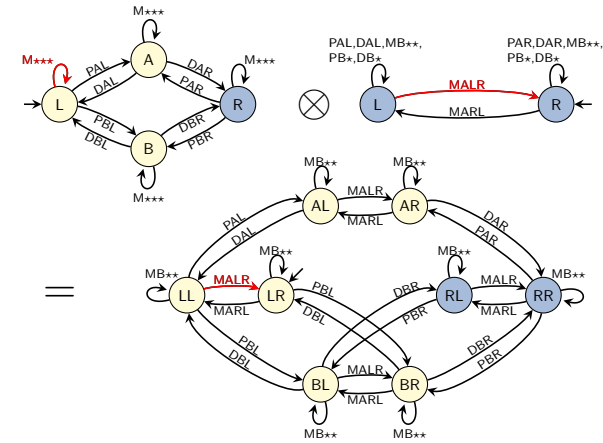
$$T_\otimes = \{\langle s_1, s_2 \rangle \xrightarrow{\ell} \langle t_1, t_2 \rangle \mid s_1 \xrightarrow{\ell} t_1 \in T_1 \text{ and } s_2 \xrightarrow{\ell} t_2 \in T_2\}$$



Example: Synchronized Product

 $\mathcal{T}^\pi\{\text{package}\} \otimes \mathcal{T}^\pi\{\text{truck A}\}:$

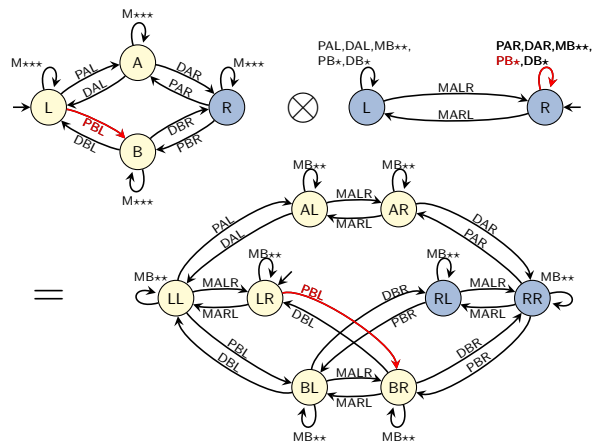
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Example: Synchronized Product

 $\mathcal{T}^\pi\{\text{package}\} \otimes \mathcal{T}^\pi\{\text{truck A}\}:$

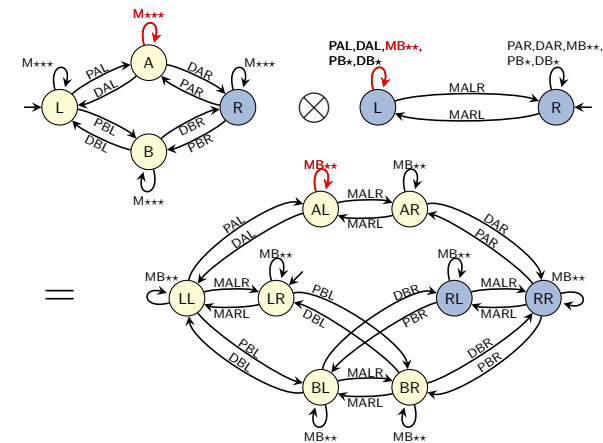
$$T_\otimes = \{\langle s_1, s_2 \rangle \xrightarrow{\ell} \langle t_1, t_2 \rangle \mid s_1 \xrightarrow{\ell} t_1 \in T_1 \text{ and } s_2 \xrightarrow{\ell} t_2 \in T_2\}$$



Example: Synchronized Product

 $\mathcal{T}^\pi\{\text{package}\} \otimes \mathcal{T}^\pi\{\text{truck A}\}:$

$$T_\otimes = \{\langle s_1, s_2 \rangle \xrightarrow{\ell} \langle t_1, t_2 \rangle \mid s_1 \xrightarrow{\ell} t_1 \in T_1 \text{ and } s_2 \xrightarrow{\ell} t_2 \in T_2\}$$

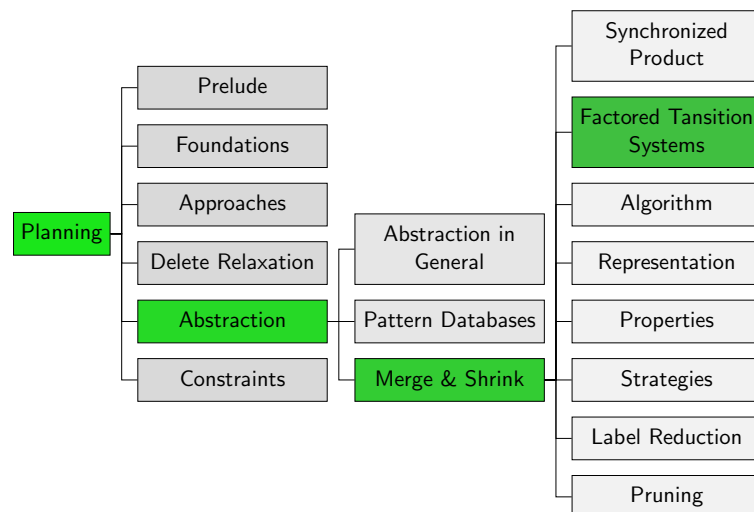


Associativity and Commutativity

- ▶ Up to isomorphism (“names of states”), products are associative and commutative:
 - ▶ $(\mathcal{T} \otimes \mathcal{T}') \otimes \mathcal{T}'' \sim \mathcal{T} \otimes (\mathcal{T}' \otimes \mathcal{T}'')$
 - ▶ $\mathcal{T} \otimes \mathcal{T}' \sim \mathcal{T}' \otimes \mathcal{T}$
- ▶ We do not care about names of states and thus treat products as associative and commutative.
- ▶ We can then define the product of a **set** $F = \{\mathcal{T}_1, \dots, \mathcal{T}_n\}$ of transition systems: $\bigotimes F := \mathcal{T}_1 \otimes \dots \otimes \mathcal{T}_n$

E9.5 Factored Transition Systems

Content of the Course



Factored Transition System

Definition (Factored Transition System)

A finite set $F = \{\mathcal{T}_1, \dots, \mathcal{T}_n\}$ of transition systems with the same labels and cost function is called a **factored transition system (FTS)**.
 F represents the transition system $\bigotimes F$.

A planning task gives rise to an FTS via its atomic projections:

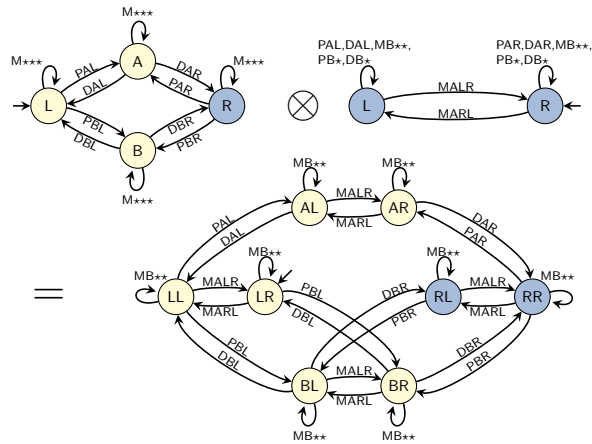
Definition (Factored Transition System Induced by Planning Task)

Let Π be a planning task with state variables V .

The **factored transition system induced by Π** is the FTS $F(\Pi) = \{\mathcal{T}^{\pi\{v\}} \mid v \in V\}$.

Back to the Example Product

$\mathcal{T}^\pi\{\text{package}\} \otimes \mathcal{T}^\pi\{\text{truck A}\} :$



We have $\mathcal{T}^\pi\{\text{package}\} \otimes \mathcal{T}^\pi\{\text{truck A}\} \sim \mathcal{T}^\pi\{\text{package, truck A}\}$. Coincidence?

Products of Projections

Theorem (Products of Projections)

Let Π be a **SAS⁺** planning task with variable set V , and let V_1 and V_2 be **disjoint subsets** of V .

Then $\mathcal{T}^{\pi_{V_1}} \otimes \mathcal{T}^{\pi_{V_2}} \sim \mathcal{T}^{\pi_{V_1 \cup V_2}}$.

(Proof omitted.)

\rightsquigarrow products allow us to build finer projections from coarser ones

Recovering $\mathcal{T}(\Pi)$ from the Factored Transition System

- By repeated application of the theorem, we can recover **all pattern database heuristics** of a SAS⁺ planning task as products of atomic factors.
- Moreover, by computing the product of **all** atomic projections, we can recover the **identity abstraction** $\text{id} = \pi_V$.

This implies:

Corollary (Recovering $\mathcal{T}(\Pi)$ from the Factored Transition System)

Let Π be a **SAS⁺** planning task. Then $\bigotimes F(\Pi) \sim \mathcal{T}(\Pi)$.

This is an important result because it shows that $F(\Pi)$ **represents all important information** about Π .

E9.6 Summary

Summary

- ▶ A **factored transition system** is a set of transition systems that represents a larger transition system by focusing on its individual components (**factors**).
- ▶ For planning tasks, these factors are the **atomic projections** (projections to single state variables).
- ▶ The **synchronized product** $\mathcal{T} \otimes \mathcal{T}'$ of two transition systems with the same labels captures their “joint behaviour”.
- ▶ For SAS⁺ tasks, all **projections** can be obtained as products of atomic projections.
- ▶ In particular, the product of all factors of a SAS⁺ task results in the **full** transition system of the task.