Planning and Optimization

E7. Pattern Databases: Multiple Patterns

Malte Helmert and Gabriele Röger

Universität Basel

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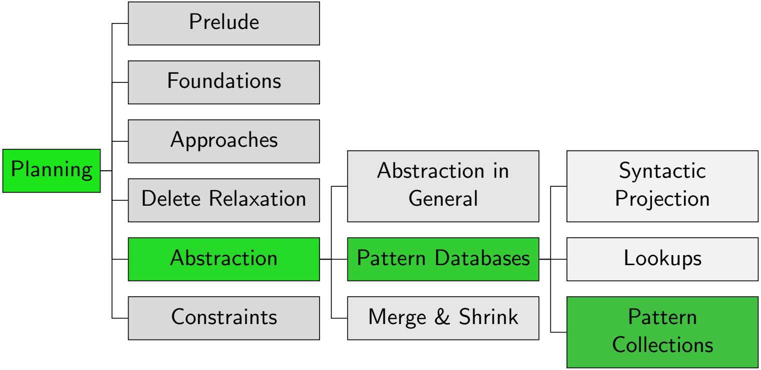
Content of the Course



E7.1 Additivity & the Canonical Heuristic

Pattern Collections

- The space requirements for a pattern database grow exponentially with the number of state variables in the pattern.
- This places severe limits on the usefulness of single PDB heuristics h^P for larger planning task.
- To overcome this limitation, planners using pattern databases work with collections of multiple patterns.
- When using two patterns P_1 and P_2, it is always possible to use the maximum of h^{P_1} and h^{P_2} as an admissible and consistent heuristic estimate.
- However, when possible, it is much preferable to use the sum of h^{P_1} and h^{P_2} as a heuristic estimate, since $h^{P_1} + h^{P_2} \geq \max\{h^{P_1}, h^{P_2}\}$.

Criterion for Additive Patterns

Theorem (Additive Pattern Sets)

Let $P_1, \dots, P_k$ be disjoint patterns for an FDR planning task Π.

If there exists no operator that has an effect on a variable $v_i \in P_i$ and on a variable $v_j \in P_j$ for some $i \neq j$, then $\sum_{i=1}^{k} h^{P_i}$ is an admissible and consistent heuristic for Π.

Proof.

If there exists no such operator, then no label of $\mathcal{T}(\Pi)$ affects both $\mathcal{T}(\Pi)^{\pi_{P_i}}$ and $\mathcal{T}(\Pi)^{\pi_{P_j}}$ for $i \neq j$. By the theorem on affecting transition labels, this means that any two projections π_{P_i} and π_{P_j} are orthogonal. The claim follows with the theorem on additivity for orthogonal abstractions. □

A pattern set $\{P_1, \dots, P_k\}$ which satisfies the criterion of the theorem is called an additive pattern set or additive set.

Finding Additive Pattern Sets

The theorem on additive pattern sets gives us a simple criterion to decide which pattern heuristics can be admissibly added.

Given a pattern collection $\mathcal{C}$ (i.e., a set of patterns), we can use this information as follows:

1. Build the compatibility graph for $\mathcal{C}$.
   - Vertices correspond to patterns $P \in \mathcal{C}$.
   - There is an edge between two vertices iff no operator affects both incident patterns.
2. Compute all maximal cliques of the graph. These correspond to maximal additive subsets of $\mathcal{C}$.
   - Computing large cliques is an NP-hard problem, and a graph can have exponentially many maximal cliques.
   - However, there are output-polynomial algorithms for finding all maximal cliques (Tomita, Tanaka & Takahashi, 2004) which have led to good results in practice.

Finding Additive Pattern Sets: Example

Example

Consider a planning task with state variables $V = \{v_1, \dots, v_5\}$ and the pattern collection $\mathcal{C} = \{P_1, \dots, P_5\}$ with $P_1 = \{v_1, v_2, v_3\}$, $P_2 = \{v_1, v_2\}$, $P_3 = \{v_3\}$, $P_4 = \{v_4\}$ and $P_5 = \{v_5\}$.

There are operators affecting each individual variable, variables v_1 and v_2, variables v_3 and v_4 and variables v_3 and v_5.

What are the maximal cliques in the compatibility graph for $\mathcal{C}$?

Answer: $\{P_1\}$, $\{P_2, P_3\}$, $\{P_2, P_4, P_5\}$

The Canonical Heuristic Function

Definition (Canonical Heuristic Function)

Let $\mathcal{C}$ be a pattern collection for an FDR planning task.

The canonical heuristic $h^{\mathcal{C}}$ for pattern collection $\mathcal{C}$ is defined as

$$h^{\mathcal{C}}(s) = \max_{\mathcal{D} \in \mathit{cliques}(\mathcal{C})} \sum_{P \in \mathcal{D}} h^P(s),$$

where $\mathit{cliques}(\mathcal{C})$ is the set of all maximal cliques in the compatibility graph for $\mathcal{C}$.

For all choices of $\mathcal{C}$, heuristic $h^{\mathcal{C}}$ is admissible and consistent.

Canonical Heuristic Function: Example

Example

Consider a planning task with state variables $V = \{v_1, \dots, v_5\}$ and the pattern collection $\mathcal{C} = \{P_1, \dots, P_5\}$ with $P_1 = \{v_1, v_2, v_3\}$, $P_2 = \{v_1, v_2\}$, $P_3 = \{v_3\}$, $P_4 = \{v_4\}$ and $P_5 = \{v_5\}$.

There are operators affecting each individual variable, an operator that affects v_1 and v_2 and an operator that affects v_3, v_4 and v_5.

What are the maximal cliques in the compatibility graph for $\mathcal{C}$?

Answer: $\{P_1\}$, $\{P_2, P_3\}$, $\{P_2, P_4, P_5\}$

What is the canonical heuristic function $h^{\mathcal{C}}$?

Answer:

$$\begin{aligned} h^{\mathcal{C}} &= \max\{h^{P_1}, h^{P_2} + h^{P_3}, h^{P_2} + h^{P_4} + h^{P_5}\} \\ &= \max\{h^{\{v_1,v_2,v_3\}}, h^{\{v_1,v_2\}} + h^{\{v_3\}}, h^{\{v_1,v_2\}} + h^{\{v_4\}} + h^{\{v_5\}}\} \end{aligned}$$

How Good is the Canonical Heuristic Function?

- The canonical heuristic function is the best possible admissible heuristic we can derive from $\mathcal{C}$ using our additivity criterion.
- Even better heuristic estimates can be obtained from projection heuristics using a more general additivity criterion based on an idea called cost partitioning.

$\rightsquigarrow$ We will return to this topic in Part F.

E7.2 Dominated Additive Sets

Computing $h^{\mathcal{C}}$ Efficiently: Motivation

Consider
$h^{\mathcal{C}} = \max\{h^{\{v_1,v_2,v_3\}}, h^{\{v_1,v_2\}} + h^{\{v_3\}}, h^{\{v_1,v_2\}} + h^{\{v_4\}} + h^{\{v_5\}}\}$.

- We need to evaluate this expression for every search node.
- It is thus worth to spend some effort in precomputations to make the evaluation more efficient.

A naive implementation requires 5 PDB lookups (one for each pattern) and maximizes over 3 additive sets.

Can we do better?

Dominated Sum Theorem

Theorem (Dominated Sum)

Let $\{P_1, \dots, P_k\}$ be an additive pattern set for an FDR planning task Π, and let P be a pattern with $P_i \subseteq P$ for all $i \in \{1, \dots, k\}$. Then $\sum_{i=1}^{k} h^{P_i} \leq h^P$.

Proof.

Because $P_i \subseteq P$, all projections π_{P_i} are coarsenings of the projection π_P. Let $\mathcal{T}' := \mathcal{T}(\Pi)^{\pi_P}$.
We can view each h^{P_i} as an abstraction heuristic for solving $\mathcal{T}'$.

By the argumentation of the previous theorem, $\{P_1, \dots, P_k\}$ is an additive pattern set and hence $\sum_{i=1}^{k} h^{P_i}$ is an admissible heuristic for solving $\mathcal{T}'$. Hence, $\sum_{i=1}^{k} h^{P_i}$ is bounded by the optimal goal distances in $\mathcal{T}'$, which implies $\sum_{i=1}^{k} h^{P_i} \leq h^P$. □

Dominated Sum Corollary

Corollary (Dominated Sum)

Let $\{P_1, \dots, P_n\}$ and $\{Q_1, \dots, Q_m\}$ be additive pattern sets of an FDR planning task such that each pattern P_i is a subset of some pattern Q_j (not necessarily proper).

Then $\sum_{i=1}^{n} h^{P_i} \leq \sum_{j=1}^{m} h^{Q_j}$.

Proof.

$$\sum_{i=1}^{n} h^{P_i} \overset{(1)}{\leq} \sum_{j=1}^{m} \sum_{P_i \subseteq Q_j} h^{P_i} \overset{(2)}{\leq} \sum_{j=1}^{m} h^{Q_j},$$

where (1) holds because each P_i is contained in some Q_j and (2) follows from the dominated sum theorem. □

Dominance Pruning

- We can use the dominated sum corollary to simplify the representation of $h^{\mathcal{C}}$: sums that are dominated by other sums can be pruned.
- The dominance test can be performed in polynomial time.

Example

$$\begin{aligned} & \max\{h^{\{v_1,v_2,v_3\}}, h^{\{v_1,v_2\}} + h^{\{v_3\}}, h^{\{v_1,v_2\}} + h^{\{v_4\}} + h^{\{v_5\}}\} \\ = & \max\{h^{\{v_1,v_2,v_3\}}, h^{\{v_1,v_2\}} + h^{\{v_4\}} + h^{\{v_5\}}\} \end{aligned}$$

$\rightsquigarrow$ number of PDB lookups reduced from 5 to 4;
number of additive sets reduced from 3 to 2

E7.3 Redundant Patterns

Redundant Patterns

- The previous example shows that sometimes, not all patterns in a pattern collection are useful.
  - Pattern $\{v_3\}$ could be removed because it does not affect the heuristic value.
- In this section, we will show that certain patterns are never useful and should thus never be considered.
- Knowing about such redundant patterns is useful for algorithms that try to find good patterns automatically.

$\rightsquigarrow$ It allows us to focus on the useful ones.

Non-Goal Patterns

Theorem (Non-Goal Patterns are Trivial)

Let Π be a SAS$^+$ planning task, and let P be a pattern for Π such that no variable in P is mentioned in the goal formula of Π. Then $h^P(s) = 0$ for all states s.

Proof.

All states in the abstraction are goal states. $\square$

$\rightsquigarrow$ Patterns with no goal variables are redundant. They should not be included in a pattern collection.

Causal Graphs: Motivation

- For more interesting notions of redundancy, we need to introduce causal graphs.
- Causal graphs describe the dependency structure between the state variables of a planning task.
- Causal graphs are a general tool for analyzing planning tasks.
- They are used in many contexts besides abstraction heuristics.

Causal Graphs

Definition (Causal Graph)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be an FDR planning task.

The causal graph of Π, written $CG(\Pi)$, is the directed graph whose vertices are the state variables V and which has an arc $\langle u, v \rangle$ iff $u \neq v$ and there exists an operator $o \in O$ such that:

- u appears anywhere in o (in precondition, effect conditions or atomic effects), and
- v is modified by an effect of o.

Idea: an arc $\langle u, v \rangle$ in the causal graph indicates that variable u is in some way relevant for modifying the value of v

Causally Relevant Variables

Definition (Causally Relevant)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be an FDR planning task, and let $P \subseteq V$ be a pattern for Π.

We say that $v \in P$ is **causally relevant for P** if $CG(\Pi)$, restricted to the variables of P, contains a directed path from v to a variable $v' \in P$ that is mentioned in the goal formula γ.

Note: The definition implies that variables in P mentioned in the goal are always causally relevant for P.

Causally Irrelevant Variables are Useless

Theorem (Causally Irrelevant Variables are Useless)

Let $P \subseteq V$ be a pattern for an FDR planning task Π, and let $P' \subseteq P$ consist of all variables that are causally relevant for P.

Then $h^P(s) = h^{P'}(s)$ for all states s.

$\rightsquigarrow$ Patterns P where not all variables are causally relevant are redundant. The smaller subpattern P' should be used instead.

Causally Irrelevant Variables are Useless: Proof

Proof Sketch.

($\geq$): holds because π_P is a refinement of $\pi_{P'}$

($\leq$): Obvious if $h^{P'}(s) = \infty$; else, consider an optimal abstract plan $\langle o_1, \dots, o_n \rangle$ for $\pi_{P'}(s)$ in $\mathcal{T}(\Pi)^{\pi_{P'}}$.

W.l.o.g., each o_i modifies some variable in P'.
(Other o_i are redundant and can be omitted.)

Because P' includes all variables causally relevant for P, no variable in $P \setminus P'$ is mentioned in any o_i or in the goal.

Then the same abstract plan also is a solution for $\pi_P(s)$ in $\mathcal{T}(\Pi)^{\pi_P}$.
Hence, the optimal solution cost under abstraction π_P is no larger than under $\pi_{P'}$.

Causally Connected Patterns

Definition (Causally Connected)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be an FDR planning task, and let $P \subseteq V$ be a pattern for Π.

We say that P is causally connected if the subgraph of $CG(\Pi)$ induced by P is weakly connected (i.e., contains a path from every vertex to every other vertex, ignoring arc directions).

Disconnected Patterns are Decomposable

Theorem (Causally Disconnected Patterns are Decomposable)

Let $P \subseteq V$ be a pattern for a SAS^+ *planning task* Π *that is not causally connected, and let P_1, P_2 be a partition of P into non-empty subsets such that $CG(\Pi)$ contains no arc between the two sets.*

Then $h^P(s) = h^{P_1}(s) + h^{P_2}(s)$ for all states s.

$\rightsquigarrow$ Causally disconnected patterns P are redundant.
The smaller subpatterns P_1 and P_2 should be used instead.

Disconnected Patterns are Decomposable: Proof

Proof Sketch.

(≥): There is no arc between P_1 and P_2 in the causal graph, and thus there is no operator that affects both patterns.

Therefore, they are additive, and $h^P \geq h^{P_1} + h^{P_2}$ follows from the dominated sum theorem.

(≤): Obvious if $h^{P_1}(s) = \infty$ or $h^{P_2}(s) = \infty$. Else, consider optimal abstract plans ρ_1 for $\mathcal{T}(\Pi)^{\pi_{P_1}}$ and ρ_2 for $\mathcal{T}(\Pi)^{\pi_{P_2}}$.

Because the variables of the two projections do not interact, concatenating the two plans yields an abstract plan for $\mathcal{T}(\Pi)^{\pi_P}$.

Hence, the optimal solution cost under abstraction π_P is at most the sum of costs of ρ_1 and ρ_2, and thus $h^P \leq h^{P_1} + h^{P_2}$.

E7.4 Summary

Summary (1)

- When faced with multiple PDB heuristics (a pattern collection), we want to admissibly add their values where possible, and maximize where addition is inadmissible.
- A set of patterns is additive if each operator affects (i.e., assigns to a variable from) at most one pattern in the set.
- The canonical heuristic function is the best possible additive/maximizing combination for a given pattern collection given this additivity criterion.

Summary (2)

Not all patterns need to be considered, as some are redundant:

- Patterns should include a goal variable (else $h^P = 0$).
- Patterns should only include causally relevant variables (others can be dropped without affecting the heuristic value).
- Patterns should be causally connected (disconnected patterns can be split into smaller subpatterns at no loss).