Planning and Optimization

E6. Pattern Databases: Introduction

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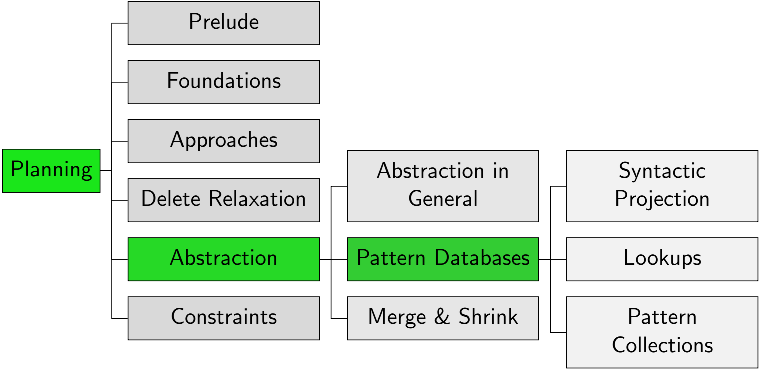
Content of the Course



Projections and Pattern Database Heuristics

Pattern Database Heuristics

- The most commonly used abstraction heuristics in search and planning are pattern database (PDB) heuristics.
- PDB heuristics were originally introduced for the 15-puzzle (Culberson & Schaeffer, 1996) and for Rubik's cube (Korf, 1997).
- The first use for domain-independent planning is due to Edelkamp (2001).
- Since then, much research has focused on the theoretical properties of pattern databases, how to use pattern databases more effectively, how to find good patterns, etc.
- Pattern databases are a research area both in planning and in (domain-specific) heuristic search.
- For many search problems, pattern databases are the most effective admissible heuristics currently known.

Pattern Database Heuristics Informally

Pattern Databases: Informally

A pattern database heuristic for a planning task
is an abstraction heuristic where

- some aspects of the task are represented in the abstraction with perfect precision, while
- all other aspects of the task are not represented at all.

This is achieved by projecting the task onto the variables
that describe the aspects that are represented.

Example (15-Puzzle)

- Choose a subset T of tiles (the pattern).
- Faithfully represent the locations of T in the abstraction.
- Assume that all other tiles and the blank can be anywhere in the abstraction.

Projections

Formally, pattern database heuristics are abstraction heuristics induced by a particular class of abstractions called projections.

Definition (Projection)

Let Π be an FDR planning task with variables V and states S. Let $P \subseteq V$, and let S' be the set of states over P.

The projection $\pi_P : S \rightarrow S'$ is defined as $\pi_P(s) := s|_P$, (where $s|_P(v) := s(v)$ for all $v \in P$).

We call P the pattern of the projection π_P.

In other words, π_P maps two states s_1 and s_2 to the same abstract state iff they agree on all variables in P.

Pattern Database Heuristics

Abstraction heuristics based on projections are called pattern database (PDB) heuristics.

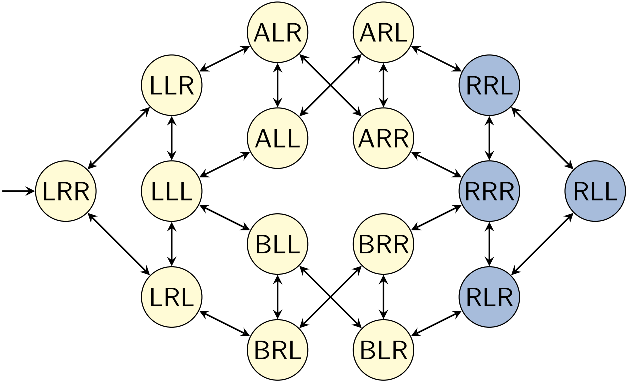
Definition (Pattern Database Heuristic)

The abstraction heuristic induced by π_P is called a pattern database heuristic or PDB heuristic.
We write h^P as a shorthand for h^{π_P}.

Why are they called pattern database heuristics?

- Heuristic values for PDB heuristics are traditionally stored in a 1-dimensional table (array) called a pattern database (PDB). Hence the name "PDB heuristic".
- The word pattern database alludes to endgame databases for 2-player games (in particular chess and checkers).

Example: Transition System

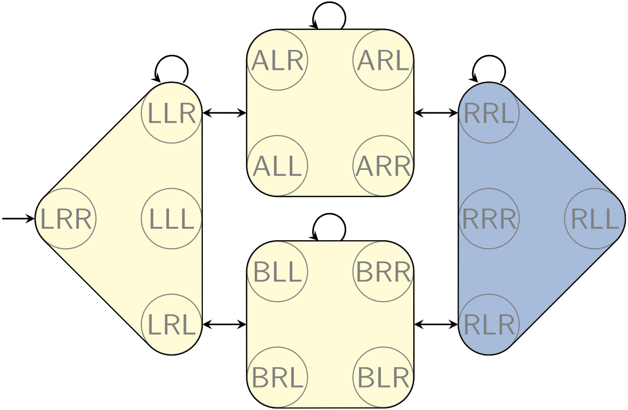


Logistics problem with one package, two trucks, two locations:

- state variable package: $\{L, R, A, B\}$
- state variable truck A: $\{L, R\}$
- state variable truck B: $\{L, R\}$

Example: Projection (1)

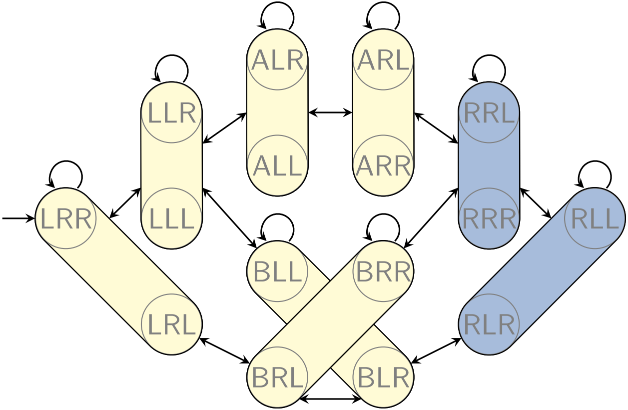
Abstraction induced by $\pi_{\{\text{package}\}}$:



$$h^{\{\text{package}\}}(\text{LRR}) = 2$$

Example: Projection (2)

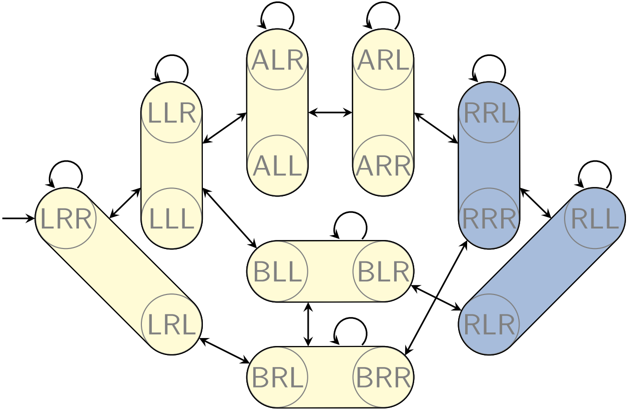
Abstraction induced by $\pi_{\{\text{package},\text{truck A}\}}$:



$$h^{\{\text{package},\text{truck A}\}}(\text{LRR}) = 2$$

Example: Projection (2)

Abstraction induced by $\pi_{\{\text{package},\text{truck A}\}}$:



$$h^{\{\text{package},\text{truck A}\}}(\text{LRR}) = 2$$

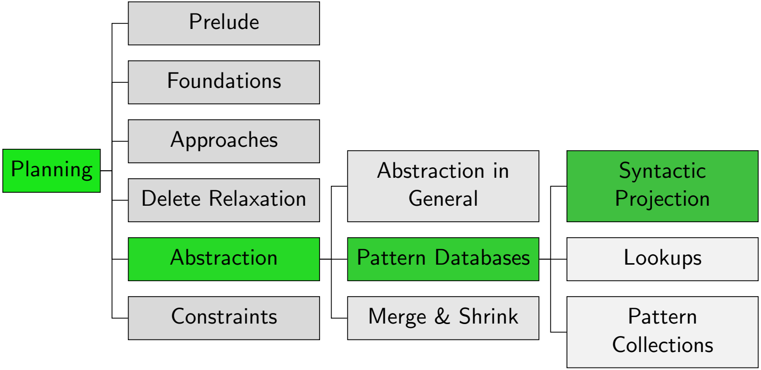
Pattern Databases: Chapter Overview

In the following, we will discuss:

- how to implement PDB heuristics
  $\rightsquigarrow$ this chapter
- how to effectively make use of multiple PDB heuristics
  $\rightsquigarrow$ Chapter E7
- how to find good patterns for PDB heuristics
  $\rightsquigarrow$ Chapter E8

Implementing PDBs: Precomputation

Content of the Course



Pattern Database Implementation

Assume we are given a pattern P for a planning task Π.
How do we implement h^P?

1. In a precomputation step, we compute a graph representation for the abstraction $\mathcal{T}(\Pi)^{\pi_P}$ and compute the abstract goal distance for each abstract state.
2. During search, we use the precomputed abstract goal distances in a lookup step.

Precomputation Step

Let Π be a planning task and P a pattern.
Let $\mathcal{T} = \mathcal{T}(\Pi)$ and $\mathcal{T}' = \mathcal{T}^{\pi_P}$.

- We want to compute a graph representation of $\mathcal{T}'$.
- $\mathcal{T}'$ is defined through an abstraction of $\mathcal{T}$.
  - For example, each concrete transition induces an abstract transition.
- However, we cannot compute $\mathcal{T}'$ by iterating over all transitions of $\mathcal{T}$.
  - This would take time $\Omega(\|\mathcal{T}\|)$.
  - This is prohibitively long (or else we could solve the task using uniform-cost search or similar techniques).
- Hence, we need a way of computing $\mathcal{T}'$ in time which is polynomial only in $\|\Pi\|$ and $\|\mathcal{T}'\|$.

Syntactic Projections

Definition (Syntactic Projection)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be an FDR planning task, and let $P \subseteq V$ be a subset of its variables.
The syntactic projection $\Pi|_P$ of Π to P is the FDR planning task $\langle P, I|_P, \{o|_P \mid o \in O\}, \gamma|_P \rangle$, where

- $\varphi|_P$ for formula φ is defined as the formula obtained from φ by replacing all atoms $(v = d)$ with $v \notin P$ by $\top$, and
- $o|_P$ for operator o is defined by replacing all formulas φ occurring in the precondition or effect conditions of o with $\varphi|_P$ and all atomic effects $(v := d)$ with $v \notin P$ with the empty effect $\top$.

Put simply, $\Pi|_P$ throws away all information not pertaining to variables in P.

Idea

$$
\begin{array}{ccc}
\Pi & \xrightarrow{\text{induced TS}} & \mathcal{T}(\Pi) \\
 & & \Big\downarrow \text{abstract TS} \\
 & & \mathcal{T}(\Pi)^{\pi_P}
\end{array}
$$

Idea

$$
\begin{tikzcd}
\Pi \arrow[r, "\text{induced TS}"] \arrow[dd, "\text{syntactic projection}"'] & \mathcal{T}(\Pi) \arrow[d, "\text{abstract TS}"] \\
 & \mathcal{T}(\Pi)^{\pi_P} \\
\Pi|_P \arrow[r, "\text{induced TS}"] & \mathcal{T}(\Pi|_P)
\end{tikzcd}
$$

Idea

$$
\begin{tikzcd}
\Pi \arrow[r, "\text{induced TS}"] \arrow[dd, "\text{syntactic projection}"'] & \mathcal{T}(\Pi) \arrow[d, "\text{abstract TS}"] \\
 & \mathcal{T}(\Pi)^{\pi_P} \\
\Pi|_P \arrow[r, "\text{induced TS}"] & \mathcal{T}(\Pi|_P)
\end{tikzcd}
$$

relationship?

- $\Pi|_P$ can be computed in linear time in $\|\Pi\|$.
- If $\mathcal{T}(\Pi|_P)$ was "equivalent" to $\mathcal{T}(\Pi)^{\pi_P}$ this would give us an efficient way to compute $\mathcal{T}(\Pi)^{\pi_P}$.

Idea

$$\begin{array}{ccc}
\Pi & \xrightarrow{\text{induced TS}} & \mathcal{T}(\Pi) \\
\text{syntactic projection} \downarrow & & \downarrow \text{abstract TS} \\
 & & \mathcal{T}(\Pi)^{\pi_P} \\
 & & \textcolor{red}{\text{relationship?}} \\
\Pi|_P & \xrightarrow{\text{induced TS}} & \mathcal{T}(\Pi|_P)
\end{array}$$

- $\Pi|_P$ can be computed in linear time in $\|\Pi\|$.
- If $\mathcal{T}(\Pi|_P)$ was "equivalent" to $\mathcal{T}(\Pi)^{\pi_P}$ this would give us an efficient way to compute $\mathcal{T}(\Pi)^{\pi_P}$.
- What do we mean with "equivalent"?
- Is this actually the case?

Isomorphic Transition Systems

Isomorphic = equivalent up to renaming

Definition (Isomorphic Transition Systems)

Let $\mathcal{T} = \langle S, L, c, T, s_0, S_\star \rangle$ and $\mathcal{T}' = \langle S', L', c', T', s'_0, S'_\star \rangle$ be transition systems.

We say that $\mathcal{T}$ is isomorphic to $\mathcal{T}'$, in symbols $\mathcal{T} \sim \mathcal{T}'$, if there exist bijective functions $\varphi : S \to S'$ and $\lambda : L \to L'$ such that:

- $s \xrightarrow{\ell} t \in T$ iff $\varphi(s) \xrightarrow{\lambda(\ell)} \varphi(t) \in T'$,
- $c'(\lambda(\ell)) = c(\ell)$ for all $\ell \in L$,
- $\varphi(s_0) = s'_0$, and
- $s \in S_\star$ iff $\varphi(s) \in S'_\star$.

$(\sim)$ is a an equivalence relation. Two isomorphic transition systems are interchangeable for all practical intents and purposes.

Equivalence Theorem for Syntactic Projections

Theorem (Syntactic Projections vs. Projections)

Let Π be a SAS$^+$ *task, and let P be a pattern for Π.*
Then $\mathcal{T}(\Pi)^{\pi_P} \sim \mathcal{T}(\Pi|_P)$.

Proof.

$\rightsquigarrow$ exercises $\square$

PDB Computation

Using the equivalence theorem, we can compute pattern databases for SAS$^+$ tasks Π and patterns P:

Computing Pattern Databases

def compute-PDB(Π, P):
  Compute $\Pi' := \Pi|_P$.
  Compute $\mathcal{T}' := \mathcal{T}(\Pi')$.
  Perform a backward uniform-cost search from the goal
    states of $\mathcal{T}'$ to compute all abstract goal distances.
  $PDB :=$ a table containing all goal distances in $\mathcal{T}'$
  return PDB

The algorithm runs in polynomial time and space
in terms of $\|\Pi\| + |PDB|$.

Generalizations of the Equivalence Theorem

- The restriction to SAS^+ tasks is necessary.
- We can slightly generalize the result if we allow general negation-free formulas, but still forbid conditional effects.
  - In that case, the weighted graph of $\mathcal{T}(\Pi)^{\pi_P}$ is isomorphic to a subgraph of the weighted graph of $\mathcal{T}(\Pi|_P)$.
  - This means that we can use $\mathcal{T}(\Pi|_P)$ to derive an admissible estimate of h^P.
- With negations in conditions or with conditional effects, not even this weaker result holds.

Going Beyond SAS$^+$ Tasks

- Most practical implementations of PDB heuristics are limited to SAS$^+$ tasks (or modest generalizations).
- One way to avoid the issues with general FDR tasks is to convert them to equivalent SAS$^+$ tasks.
- However, most direct conversions can exponentially increase the task size in the worst case.

$\rightsquigarrow$ We will only consider SAS$^+$ tasks in the chapters dealing with pattern databases.

Implementing PDBs: Lookup

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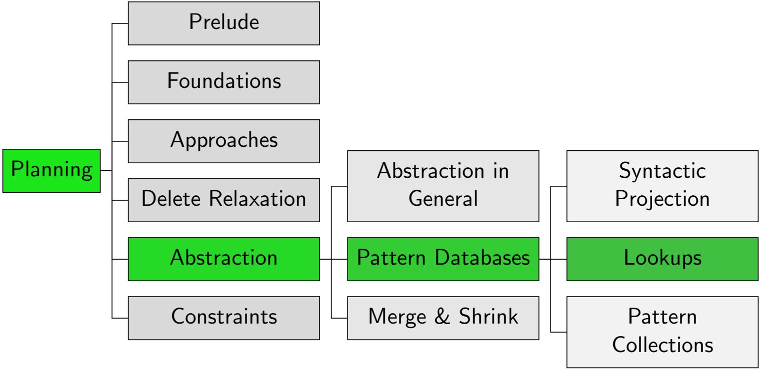


Lookup Step: Overview

- During search, the PDB is the only piece of information necessary to represent h^P. (It is not necessary to store the abstract transition system itself at this point.)
- Hence, the space requirements for PDBs during search are linear in the number of abstract states S': there is one table entry for each abstract state.
- During search, $h^P(s)$ is computed by mapping $\pi_P(s)$ to a natural number in the range $\{0, \ldots, |S'| - 1\}$ using a perfect hash function, then looking up the table entry for this number.

Lookup Step: Algorithm

Let $P = \{v_1, \dots, v_k\}$ be the pattern.

- We assume that all variable domains are natural numbers counted from 0, i.e., $\mathrm{dom}(v) = \{0, 1, \dots, |\mathrm{dom}(v)| - 1\}$.
- For all $i \in \{1, \dots, k\}$, we precompute $N_i := \prod_{j=1}^{i-1} |\mathrm{dom}(v_j)|$.

Then we can look up heuristic values as follows:

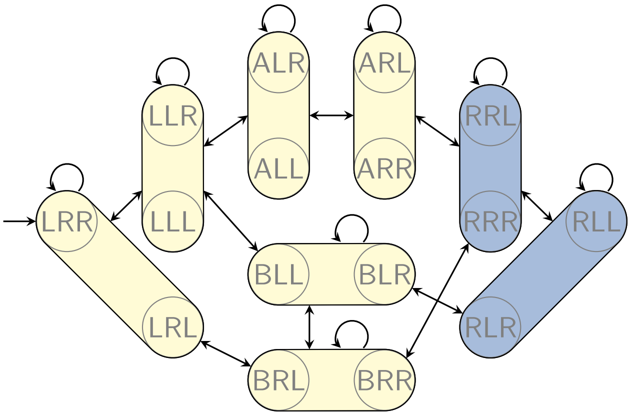
Computing Pattern Database Heuristics

def PDB-heuristic(s):
    $\mathit{index} := \sum_{i=1}^{k} N_i s(v_i)$
    return $\mathit{PDB}[\mathit{index}]$

- This is a very fast operation: it can be performed in $O(k)$.
- For comparison, most relaxation heuristics need time $O(\|\Pi\|)$ per state.

Lookup Step: Example (1)

Abstraction induced by $\pi_{\{\text{package},\text{truck A}\}}$:



Lookup Step: Example (2)

- $P = \{v_1, v_2\}$ with $v_1 =$ package, $v_2 =$ truck A.
- $\text{dom}(v_1) = \{\text{L}, \text{R}, \text{A}, \text{B}\} \approx \{0, 1, 2, 3\}$
- $\text{dom}(v_2) = \{\text{L}, \text{R}\} \approx \{0, 1\}$

$\rightsquigarrow$ $N_1 = \prod_{j=1}^{0} |\text{dom}(v_j)| = 1$, $N_2 = \prod_{j=1}^{1} |\text{dom}(v_j)| = 4$

$\rightsquigarrow$ $index(s) = 1 \cdot s(\text{package}) + 4 \cdot s(\text{truck A})$

Pattern database:

abstract state	LL	RL	AL	BL	LR	RR	AR	BR
index	0	1	2	3	4	5	6	7
value	2	0	2	1	2	0	1	1

Summary

Summary

- Pattern database (PDB) heuristics are abstraction heuristics based on projection to a subset of variables.
- For SAS^+ tasks, they can easily be implemented via syntactic projections of the task representation.
- PDBs are lookup tables that store heuristic values, indexed by perfect hash values for projected states.
- PDB values can be looked up very fast, in time $O(k)$ for a projection to k variables.