Planning and Optimization

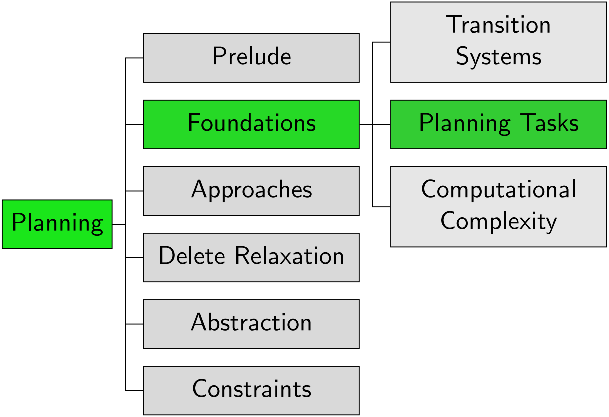
E2. Invariants and Mutexes

Malte Helmert and Gabriele Röger

Universität Basel

November 4, 2024

Content of the Course



Invariants

Invariants

- When we as humans reason about planning tasks, we implicitly make use of "obvious" properties of these tasks.
  - Example: we are never in two places at the same time
- We can represent such properties as logical formulas φ that are true in all reachable states.
  - Example: $\varphi = \neg(\mathit{at\text{-}uni} \wedge \mathit{at\text{-}home})$
- Such formulas are called invariants of the task.

Invariants: Definition

Definition (Invariant)

An invariant of a planning task Π with state variables V is a logical formula φ over V such that $s \models \varphi$ for all reachable states s of Π.

Computing Invariants

Computing Invariants

How does an automated planner come up with invariants?

- Theoretically, testing if a formula φ is an invariant is as hard as planning itself.
  $\rightsquigarrow$ proof idea: a planning task is unsolvable iff the negation of its goal is an invariant
- Still, many practical invariant synthesis algorithms exist.
- To remain efficient ($=$ polynomial-time), these algorithms only compute a subset of all useful invariants.
  $\rightsquigarrow$ sound, but not complete
- Empirically, they tend to at least find the "obvious" invariants of a planning task.

Invariant Synthesis Algorithms

Most algorithms for generating invariants are based on the generate-test-repair approach:

- Generate: Suggest some invariant candidates, e.g., by enumerating all possible formulas φ of a certain size.
- Test: Try to prove that φ is indeed an invariant. Usually done inductively:
  1. Test that initial state satisfies φ.
  2. Test that if φ is true in the current state, it remains true after applying a single operator.
- Repair: If invariant test fails, replace candidate φ by a weaker formula, ideally exploiting why the proof failed.

Invariant Synthesis: References

We will not cover invariant synthesis algorithms in this course.

Literature on invariant synthesis:

- DISCOPLAN (Gerevini & Schubert, 1998)
- TIM (Fox & Long, 1998)
- Edelkamp & Helmert's algorithm (1999)
- Bonet & Geffner's algorithm (2001)
- Rintanen's algorithm (2008)
- Rintanen's algorithm for schematic invariants (2017)

Exploiting Invariants

Invariants have many uses in planning:

- Regression search (C2–C3):
  Prune subgoals that violate (are inconsistent with) invariants.
- Planning as satisfiability (C4–C5):
  Add invariants to a SAT encoding of a planning task
  to get tighter constraints.
- Proving unsolvability:
  If φ is an invariant such that $\varphi \wedge \gamma$ is unsatisfiable,
  the planning task with goal γ is unsolvable.
- Finite-Domain Reformulation:
  Derive a more compact FDR representation (equivalent, but
  with fewer states) from a given propositional planning task.

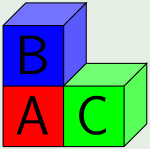
We now discuss the last point because it connects
to our discussion of propositional vs. FDR planning tasks.

Mutexes

Reminder: Blocks World (Propositional Variables)

Example

$$s(A\text{-}on\text{-}B) = \mathbf{F}$$
$$s(A\text{-}on\text{-}C) = \mathbf{F}$$
$$s(A\text{-}on\text{-}table) = \mathbf{T}$$
$$s(B\text{-}on\text{-}A) = \mathbf{T}$$
$$s(B\text{-}on\text{-}C) = \mathbf{F}$$
$$s(B\text{-}on\text{-}table) = \mathbf{F}$$
$$s(C\text{-}on\text{-}A) = \mathbf{F}$$
$$s(C\text{-}on\text{-}B) = \mathbf{F}$$
$$s(C\text{-}on\text{-}table) = \mathbf{T}$$



$\rightsquigarrow 2^9 = 512$ states

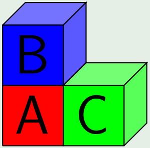
Reminder: Blocks World (Finite-Domain Variables)

Example

Use three finite-domain state variables:

- *below-a*: $\{\mathrm{b}, \mathrm{c}, \mathrm{table}\}$
- *below-b*: $\{\mathrm{a}, \mathrm{c}, \mathrm{table}\}$
- *below-c*: $\{\mathrm{a}, \mathrm{b}, \mathrm{table}\}$

$$s(\textit{below-a}) = \mathrm{table}$$
$$s(\textit{below-b}) = \mathrm{a}$$
$$s(\textit{below-c}) = \mathrm{table}$$



$\rightsquigarrow 3^3 = 27$ states

Task Reformulation

- Common modeling languages (like PDDL) often give us propositional tasks.
- More compact FDR tasks are often desirable.
- Can we do an automatic reformulation?

Mutexes

Invariants that take the form of binary clauses are called mutexes because they express that certain variable assignments cannot be simultaneously true (are mutually exclusive).

Example (Blocks World)

The invariant $\neg A\text{-}on\text{-}B \vee \neg A\text{-}on\text{-}C$ states that $A\text{-}on\text{-}B$ and $A\text{-}on\text{-}C$ are mutex.

We say that a set of literals is a mutex group if every subset of two literals is a mutex.

Example (Blocks World)

$\{A\text{-}on\text{-}B, A\text{-}on\text{-}C, A\text{-}on\text{-}table\}$ is a mutex group.

Encoding Mutex Groups as Finite-Domain Variables

Let $G = \{\ell_1, \dots, \ell_n\}$ be a mutex group over n different propositional state variables $V_G = \{v_1, \dots, v_n\}$.

Then a single finite-domain state variable v_G with $\mathrm{dom}(v_G) = \{\ell_1, \dots, \ell_n, \mathrm{none}\}$ can replace the n variables V_G:

- $s(v_G) = \ell_i$ represents situations where (exactly) ℓ_i is true
- $s(v_G) = \mathrm{none}$ represents situations where all ℓ_i are false

Note: We can omit the "none" value if $\ell_1 \vee \dots \vee \ell_n$ is an invariant.

Mutex Covers

Definition (Mutex Cover)

A mutex cover for a propositional planning task Π is a set of mutex groups $\{G_1, \dots, G_n\}$ where each variable of Π occurs in exactly one group G_i.

A mutex cover is positive if all literals in all groups are positive.

Note: always exists (use trivial group $\{v\}$ if v otherwise uncovered)

Positive Mutex Covers

In the following, we stick to positive mutex covers for simplicity.

If we have $\neg v$ in G for some group G in the cover, we can reformulate the task to use an "opposite" variable $\hat{v}$ instead, as in the conversion to positive normal form (Chapter B5).

Reformulation

Mutex-Based Reformulation of Propositional Tasks

Given a conflict-free propositional planning task Π
with positive mutex cover $\{G_1, \dots, G_n\}$:

- In all conditions where variable $v \in G_i$ occurs,
  replace v with $v_{G_i} = v$.
- In all effects e where variable $v \in G_i$ occurs,
  - Replace all atomic add effects v with $v_{G_i} := v$
  - Replace all atomic delete effects $\neg v$ with
    $(v_{G_i} = v \wedge \neg \bigvee_{v' \in G_i \setminus \{v\}} \mathit{effcond}(v', e)) \rhd v_{G_i} := \text{none}$

This results in an FDR planning task Π' that is equivalent to Π
(without proof).

Note: the conditional effects encoding delete effects
can often be simplified away to an unconditional or empty effect.

And Back?

- It can also be useful to reformulate an FDR task into a propositional task.
- For example, we might want positive normal form, which requires a propositional task.
- Key idea: each variable/value combination $v = d$ becomes a separate propositional state variable $\langle v, d \rangle$

Converting FDR Tasks into Propositional Tasks

Definition (Induced Propositional Planning Task)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a conflict-free FDR planning task.
The induced propositional planning task Π'
is the propositional planning task $\Pi' = \langle V', I', O', \gamma' \rangle$, where

- $V' = \{\langle v, d \rangle \mid v \in V, d \in \mathrm{dom}(v)\}$
- $I'(\langle v, d \rangle) = \mathbf{T}$ iff $I(v) = d$
- O' and γ' are obtained from O and γ by
  - replacing each atomic formula $v = d$ by the proposition $\langle v, d \rangle$
  - replacing each atomic effect $v := d$ by the effect $\langle v, d \rangle \wedge \bigwedge_{d' \in \mathrm{dom}(v) \setminus \{d\}} \neg \langle v, d' \rangle$.

Notes:

- Again, simplifications are often possible to avoid introducing so many delete effects.
- SAS$^+$ tasks induce STRIPS tasks.

Summary

Summary (1)

- Invariants are common properties of all reachable states, expressed as formulas.
- A number of algorithms for computing invariants exist.
- These algorithms will not find all useful invariants (which is too hard), but try to find some useful subset with reasonable (polynomial) computational effort.

Summary (2)

- Mutexes are invariants that express that certain literals are mutually exclusive.
- Mutex covers provide a way to convert a set of propositional state variables into a potentially much smaller set of finite-domain state variables.
- Using mutex covers, we can reformulate propositional tasks as more compact FDR tasks.
- Conversely, we can reformulate FDR tasks as propositional tasks by introducing propositions for each variable/value pair.