Planning and Optimization

D2. Delete Relaxation: Properties of Relaxed Planning Tasks

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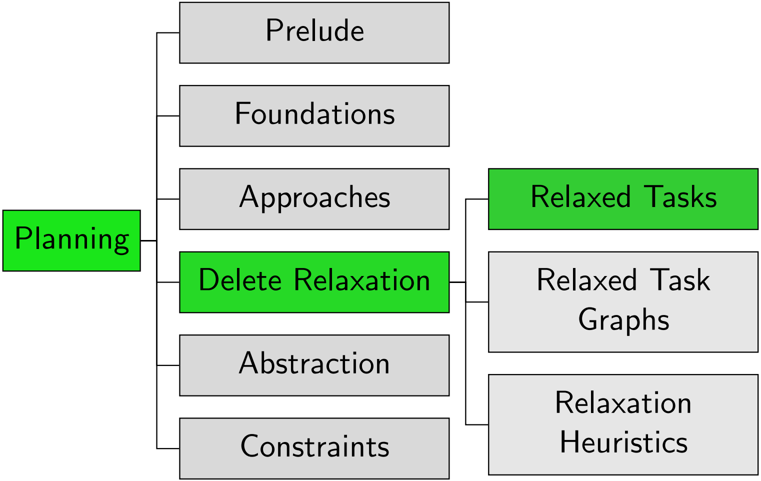
Content of the Course



D2.1 The Domination Lemma

On-Set and Dominating States

Definition (On-Set)

The on-set of an interpretation s is the set of propositional variables that are true in s, i.e., $on(s) = s^{-1}(\{\mathbf{T}\})$.

$\rightsquigarrow$ for states of propositional planning tasks: states can be viewed as sets of (true) state variables

Definition (Dominate)

An interpretation s' dominates an interpretation s if $on(s) \subseteq on(s')$.

$\rightsquigarrow$ all state variables true in s are also true in s'

Domination Lemma (1)

Lemma (Domination)

Let s and s' be interpretations of a set of propositional variables V, and let χ be a propositional formula over V which does not contain negation symbols.

If $s \models \chi$ and s' dominates s, then $s' \models \chi$.

Proof.

Proof by induction over the structure of χ.

- Base case $\chi = \top$: then $s' \models \top$.
- Base case $\chi = \bot$: then $s \not\models \bot$.

...

Domination Lemma (2)

Proof (continued).

- Base case $\chi = v \in V$: if $s \models v$, then $v \in on(s)$. With $on(s) \subseteq on(s')$, we get $v \in on(s')$ and hence $s' \models v$.
- Inductive case $\chi = \chi_1 \land \chi_2$: by induction hypothesis, our claim holds for the proper subformulas χ_1 and χ_2 of χ.

$$
\begin{aligned}
s \models \chi \quad &\Longrightarrow \quad s \models \chi_1 \land \chi_2 \\
&\Longrightarrow \quad s \models \chi_1 \text{ and } s \models \chi_2 \\
&\overset{\text{I.H. (twice)}}{\Longrightarrow} \quad s' \models \chi_1 \text{ and } s' \models \chi_2 \\
&\Longrightarrow \quad s' \models \chi_1 \land \chi_2 \\
&\Longrightarrow \quad s' \models \chi.
\end{aligned}
$$

- Inductive case $\chi = \chi_1 \lor \chi_2$: analogous

□

D2.2 The Relaxation Lemma

Add Sets and Delete Sets

Definition (Add Set and Delete Set for an Effect)

Consider a propositional planning task with state variables V.
Let e be an effect over V, and let s be a state over V.
The add set of e in s, written $addset(e, s)$,
and the delete set of e in s, written $delset(e, s)$,
are defined as the following sets of state variables:

$$addset(e, s) = \{v \in V \mid s \models effcond(v, e)\}$$
$$delset(e, s) = \{v \in V \mid s \models effcond(\neg v, e)\}$$

Note: For all states s and operators o applicable in s, we have $on(s[\![o]\!]) = (on(s) \setminus delset(eff(o), s)) \cup addset(eff(o), s)$.

Relaxation Lemma

For this and the following chapters on delete relaxation, we assume implicitly that we are working with propositional planning tasks in positive normal form.

Lemma (Relaxation)

Let s be a state, and let s' be a state that dominates s.

1. *If o is an operator applicable in s, then o^+ is applicable in s' and $s'[\![o^+]\!]$ dominates $s[\![o]\!]$.*
2. *If π is an operator sequence applicable in s, then π^+ is applicable in s' and $s'[\![\pi^+]\!]$ dominates $s[\![\pi]\!]$.*
3. *If additionally π leads to a goal state from state s, then π^+ leads to a goal state from state s'.*

Proof of Relaxation Lemma (1)

Proof.

Let V be the set of state variables.

Part 1: Because o is applicable in s, we have $s \models \mathit{pre}(o)$.

Because $\mathit{pre}(o)$ is negation-free and s' dominates s, we get $s' \models \mathit{pre}(o)$ from the domination lemma.

Because $\mathit{pre}(o^+) = \mathit{pre}(o)$, this shows that o^+ is applicable in s'.

...

Proof of Relaxation Lemma (2)

Proof (continued).

To prove that $s'[\![o^+]\!]$ dominates $s[\![o]\!]$,
we first compare the relevant add sets:

$$\begin{aligned} \mathit{addset}(\mathit{eff}(o), s) &= \{v \in V \mid s \models \mathit{effcond}(v, \mathit{eff}(o))\} \\ &= \{v \in V \mid s \models \mathit{effcond}(v, \mathit{eff}(o^+))\} && (1) \\ &\subseteq \{v \in V \mid s' \models \mathit{effcond}(v, \mathit{eff}(o^+))\} && (2) \\ &= \mathit{addset}(\mathit{eff}(o^+), s'), \end{aligned}$$

where (1) uses $\mathit{effcond}(v, \mathit{eff}(o)) \equiv \mathit{effcond}(v, \mathit{eff}(o^+))$
and (2) uses the dominance lemma (note that effect conditions are negation-free for operators in positive normal form). ...

Proof of Relaxation Lemma (3)

Proof (continued).

We then get:

$$\begin{aligned} on(s[\![o]\!]) &= (on(s) \setminus delset(eff(o), s)) \cup addset(eff(o), s) \\ &\subseteq on(s) \cup addset(eff(o), s) \\ &\subseteq on(s') \cup addset(eff(o^+), s') \\ &= on(s'[\![o^+]\!]), \end{aligned}$$

and thus $s'[\![o^+]\!]$ dominates $s[\![o]\!]$.

This conclude the proof of Part 1. ...

Proof of Relaxation Lemma (4)

Proof (continued).

Part 2: by induction over $n = |\pi|$

Base case: $\pi = \langle\rangle$
The empty plan is trivially applicable in s', and
$s'[\![\langle\rangle^+]\!] = s'$ dominates $s[\![\langle\rangle]\!] = s$ by prerequisite.

Inductive case: $\pi = \langle o_1, \dots, o_{n+1}\rangle$
By the induction hypothesis, $\langle o_1^+, \dots, o_n^+\rangle$ is applicable in s',
and $t' = s'[\![\langle o_1^+, \dots, o_n^+\rangle]\!]$ dominates $t = s[\![\langle o_1, \dots, o_n\rangle]\!]$.
Also, o_{n+1} is applicable in t.

Using Part 1, o_{n+1}^+ is applicable in t' and $s'[\![\pi^+]\!] = t'[\![o_{n+1}^+]\!]$
dominates $s[\![\pi]\!] = t[\![o_{n+1}]\!]$.

This concludes the proof of Part 2. ...

Proof of Relaxation Lemma (5)

Proof (continued).

Part 3: Let γ be the goal formula.

From Part 2, we obtain that $t' = s'[\![\pi^+]\!]$ dominates $t = s[\![\pi]\!]$.
By prerequisite, t is a goal state and hence $t \models \gamma$.

Because the task is in positive normal form, γ is negation-free, and hence $t' \models \gamma$ because of the domination lemma.

Therefore, t' is a goal state. □

D2.3 Consequences

Consequences of the Relaxation Lemma

- The relaxation lemma is the main technical result that we will use to study delete relaxation.
- Next, we show two further properties of delete relaxation that will be useful for us.
- They are direct consequences of the relaxation lemma.

Consequences of the Relaxation Lemma (1)

Corollary (Relaxation Preserves Plans and Leads to Dominance)

Let π be an operator sequence that is applicable in state s. Then π^+ is applicable in s and $s[\![\pi^+]\!]$ dominates $s[\![\pi]\!]$. If π is a plan for Π, then π^+ is a plan for Π^+.

Proof.

Apply relaxation lemma with $s' = s$. □

- ⇝ Relaxations of plans are relaxed plans.
- ⇝ Delete relaxation is no harder to solve than original task.
- ⇝ Optimal relaxed plans are never more expensive than optimal plans for original tasks.

Consequences of the Relaxation Lemma (2)

Corollary (Relaxation Preserves Dominance)

Let s be a state, let s' be a state that dominates s, and let π^+ be a relaxed operator sequence applicable in s. Then π^+ is applicable in s' and $s'[\![\pi^+]\!]$ dominates $s[\![\pi^+]\!]$.

Proof.

Apply relaxation lemma with π^+ for π, noting that $(\pi^+)^+ = \pi^+$. □

- ⇝ If there is a relaxed plan starting from state s, the same plan can be used starting from a dominating state s'.
- ⇝ Dominating states are always "better" in relaxed tasks.

D2.4 Monotonicity

Monotonicity of Relaxed Planning Tasks

Lemma (Monotonicity)

Let s be a state in which relaxed operator o^+ is applicable. Then $s[\![o^+]\!]$ dominates s.

Proof.

Since relaxed operators only have positive effects, we have $on(s) \subseteq on(s) \cup addset(eff(o^+), s) = on(s[\![o^+]\!])$. □

⇝ Together with our previous results, this means that making a transition in a relaxed planning task never hurts.

Finding Relaxed Plans

Using the theory we developed, we are now ready to study the problem of finding plans for relaxed planning tasks.

$\rightsquigarrow$ next chapter

D2.5 Summary

Summary

- With positive normal form, having more true variables is good.
- We can formalize this as dominance between states.
- It follows that delete relaxation is a simplification: it is never harder to solve a relaxed task than the original one.
- In delete-relaxed tasks, applying an operator always takes us to a dominating state and therefore never hurts.