Planning and Optimization

B4. Equivalent Operators and Normal Forms

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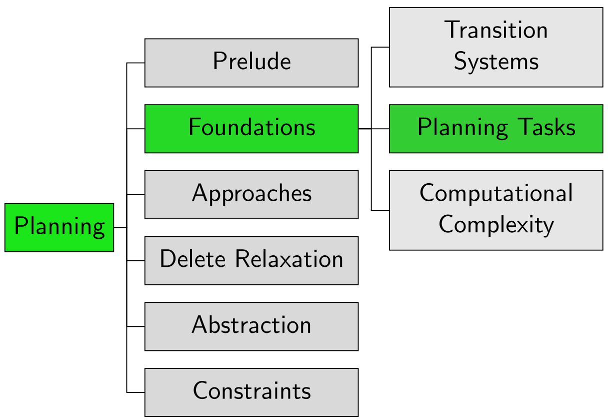
Content of the Course



B4.1 Reminder & Motivation

Reminder: Syntax of Effects

Definition (Effect)

Effects over propositional state variables V are inductively defined as follows:

- $\top$ is an effect (empty effect).
- If $v \in V$ is a propositional state variable, then v and $\neg v$ are effects (atomic effect).
- If e and e' are effects, then $(e \wedge e')$ is an effect (conjunctive effect).
- If χ is a formula over V and e is an effect, then $(\chi \rhd e)$ is an effect (conditional effect).

Arbitrary nesting of conjunctive and conditional effects, e.g. $c \wedge (a \rhd (\neg b \wedge (c \rhd (b \wedge \neg d \wedge \neg a)))) \wedge (\neg b \rhd \neg a)$

$\rightsquigarrow$ Can we make our life easier?

Reminder: Semantics of Effects

- *effcond*(ℓ, e): condition that must be true in the current state for the effect e to trigger the atomic effect ℓ
- add-after-delete semantics: if an operator with effect e is applied in state s and we have both $s \models effcond(v, e)$ and $s \models effcond(\neg v, e)$, then $s'(v) = \mathbf{T}$ in the resulting state s'.

This is a very subtle detail.

$\rightsquigarrow$ Can we make our life easier?

Motivation

Similarly to normal forms in propositional logic (DNF, CNF, NNF), we can define normal forms for effects, operators and planning tasks.

Among other things, we consider normal forms that avoid complicated nesting and subtleties of conflicts.

This is useful because algorithms (and proofs) then only need to deal with effects, operators and tasks in normal form.

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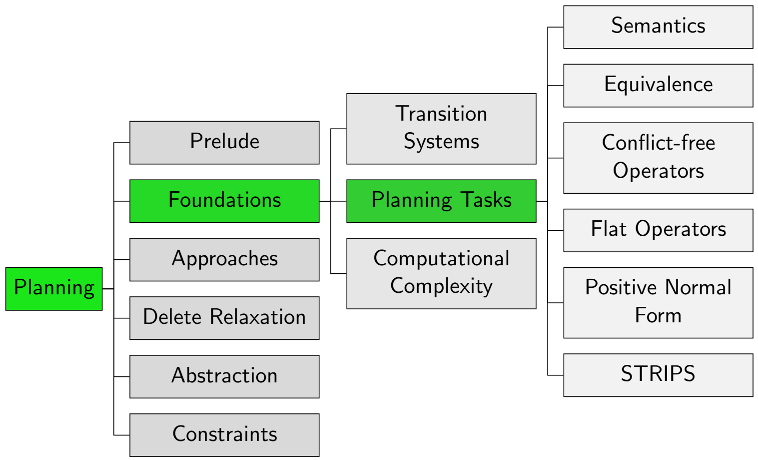

Notation: Applying Operator Sequences

Existing notation:

- We already write $s[\![o]\!]$ for the resulting state after applying operator o in state s.

New extended notation:

- For a sequence $\pi = \langle o_1, \dots, o_n \rangle$ of operators that are consecutively applicable in s, we write $s[\![\pi]\!]$ for $s[\![o_1]\!][\![o_2]\!] \dots [\![o_n]\!]$.
- This includes the case of an empty operator sequence: $s[\![\langle\rangle]\!] = s$

B4.2 Equivalence Transformations

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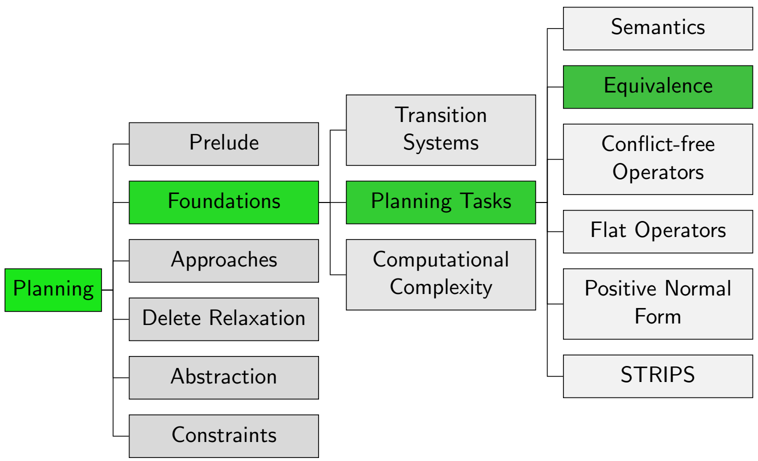

Equivalence of Operators and Effects: Definition

Definition (Equivalent Effects)

Two effects e and e' over state variables V are equivalent, written $e \equiv e'$, if $s[\![e]\!] = s[\![e']\!]$ for all states s.

Definition (Equivalent Operators)

Two operators o and o' over state variables V are equivalent, written $o \equiv o'$, if $cost(o) = cost(o')$ and for all states s, s' over V, o induces the transition $s \xrightarrow{o} s'$ iff o' induces the transition $s \xrightarrow{o'} s'$.

Equivalence of Operators and Effects: Theorem

Theorem

Let o and o' be operators with $pre(o) \equiv pre(o')$, $eff(o) \equiv eff(o')$ and $cost(o) = cost(o')$. Then $o \equiv o'$.

Note: The converse is not true. (Why not?)

Equivalence Transformations for Effects

$$
\begin{aligned}
e \land e' &\equiv e' \land e && (1)\\
(e \land e') \land e'' &\equiv e \land (e' \land e'') && (2)\\
\top \land e &\equiv e && (3)\\
\chi \rhd e &\equiv \chi' \rhd e \quad \text{if } \chi \equiv \chi' && (4)\\
\top \rhd e &\equiv e && (5)\\
\bot \rhd e &\equiv \top && (6)\\
\chi \rhd (\chi' \rhd e) &\equiv (\chi \land \chi') \rhd e && (7)\\
\chi \rhd (e \land e') &\equiv (\chi \rhd e) \land (\chi \rhd e') && (8)\\
(\chi \rhd e) \land (\chi' \rhd e) &\equiv (\chi \lor \chi') \rhd e && (9)
\end{aligned}
$$

B4.3 Conflict-Free Operators

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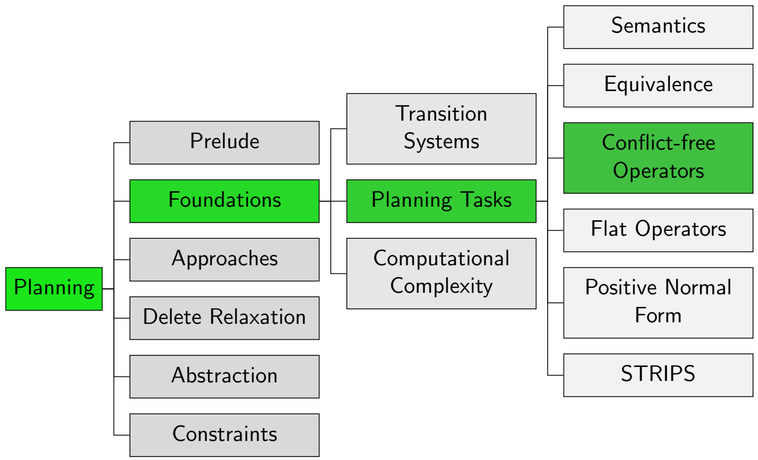

Conflict-Freeness: Motivation

- The add-after-delete semantics makes effects like $(a \rhd c) \land (b \rhd \neg c)$ somewhat unintuitive to interpret.

$\rightsquigarrow$ What happens in states where $a \land b$ is true?

- It would be nicer if $effcond(\ell, e)$ always were the condition under which the atomic effect ℓ actually materializes (because of add-after-delete, it is not)

$\rightsquigarrow$ introduce normal form where "complicated case" never arises

Conflict-Free Effects and Operators

Definition (Conflict-Free)

An effect e over propositional state variables V is called conflict-free if $effcond(v, e) \land effcond(\neg v, e)$ is unsatisfiable for all $v \in V$.

An operator o is called conflict-free if $eff(o)$ is conflict-free.

Making Operators Conflict-Free

- In general, testing whether an operator is conflict-free is a coNP-complete problem. (Why?)
- However, we do not necessarily need such a test. Instead, we can produce an equivalent conflict-free operator in polynomial time.
- Algorithm: given operator o, replace all atomic effects of the form $\neg v$ by $(\neg effcond(v, eff(o)) \rhd \neg v)$. The resulting operator o' is conflict-free and $o \equiv o'$. (Why?)

B4.4 Flat Effects

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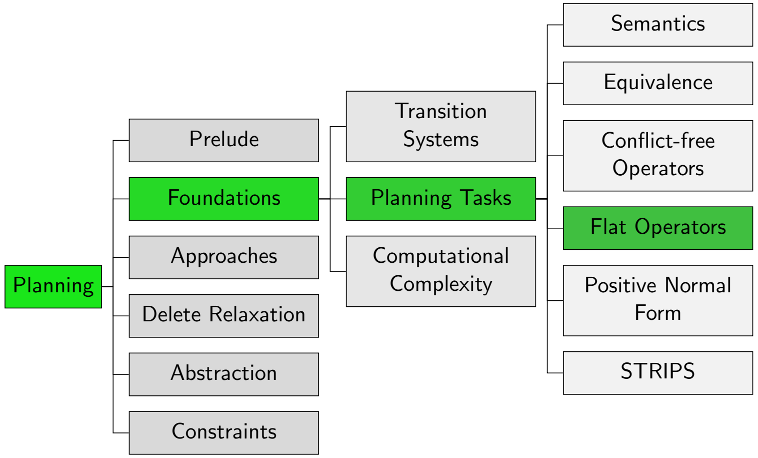

Flat Effects: Motivation

- CNF and DNF limit the nesting of connectives in propositional logic.
- For example, a CNF formula is
  - a conjunction of 0 or more subformulas,
  - each of which is a disjunction of 0 or more subformulas,
  - each of which is a literal.
- Similarly, we can define a normal form that limits the nesting of effects.
- This is useful because we then do not have to consider arbitrarily structured effects, e.g., when representing them in a planning algorithm.

Flat Effect

Definition (Flat Effect)

An effect is simple if it is either an atomic effect or of the form $(\chi \rhd e)$, where e is an atomic effect.

An effect e is flat if it is a conjunction of 0 or more simple effects, and none of these simple effects include the same atomic effect.

An operator o is flat if $\mathit{eff}(o)$ is flat.

Notes: analogously to CNF, we consider

- a single simple effect as a conjunction of 1 simple effect
- the empty effect as a conjunction of 0 simple effects

Flat Effect: Example

Example

Consider the effect

$$c \land (a \rhd (\neg b \land (c \rhd (b \land \neg d \land \neg a)))) \land (\neg b \rhd \neg a)$$

An equivalent flat (and conflict-free) effect is

$$\begin{gathered} c \land \\ ((a \land \neg c) \rhd \neg b) \land \\ ((a \land c) \rhd b) \land \\ ((a \land c) \rhd \neg d) \land \\ ((\neg b \lor (a \land c)) \rhd \neg a) \end{gathered}$$

Note: if we want, we can write c as $(\top \rhd c)$ to make the structure even more uniform, with each simple effect having a condition.

Producing Flat Operators

Theorem
For every operator, an equivalent flat operator and an equivalent flat, conflict-free operator can be computed in polynomial time.

Producing Flat Operators: Proof

Proof Sketch.
Replace the effect e over variables V by

$$\begin{aligned}&\bigwedge_{v \in V}(\mathit{effcond}(v, e) \rhd v)\\ \wedge &\bigwedge_{v \in V}(\mathit{effcond}(\neg v, e) \rhd \neg v),\end{aligned}$$

which is an equivalent flat effect.

To additionally obtain conflict-freeness, use

$$\begin{aligned}&\bigwedge_{v \in V}(\mathit{effcond}(v, e) \rhd v)\\ \wedge &\bigwedge_{v \in V}((\mathit{effcond}(\neg v, e) \wedge \neg \mathit{effcond}(v, e)) \rhd \neg v)\end{aligned}$$

instead.

(Conjuncts of the form $(\chi \rhd e)$ where $\chi \equiv \bot$ can be omitted to simplify the effect.)

B4.5 Summary

Summary

- Equivalences can be used to simplify operators and effects.
- In conflict-free operators, the "complicated case" of operator semantics does not arise.
- For flat operators, the only permitted nesting is atomic effects within conditional effects within conjunctive effects, and all atomic effects must be distinct.
- For flat, conflict-free operators, it is easy to determine the condition under which a given literal is made true by applying the operator in a given state.
- Every operator can be transformed into an equivalent flat and conflict-free one in polynomial time.