Planning and Optimization

B1. Transition Systems and Propositional Logic

Malte Helmert and Gabriele Röger

Universität Basel

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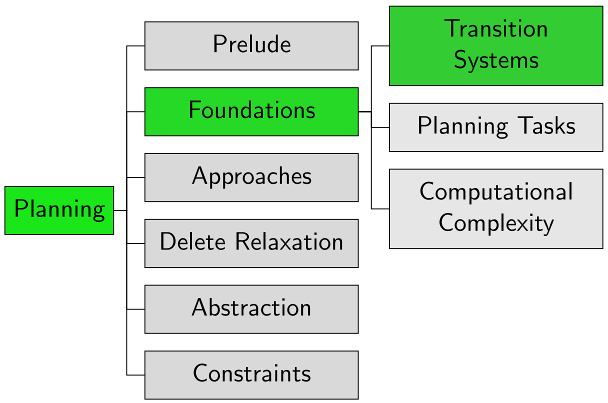
Content of the Course



Next Steps

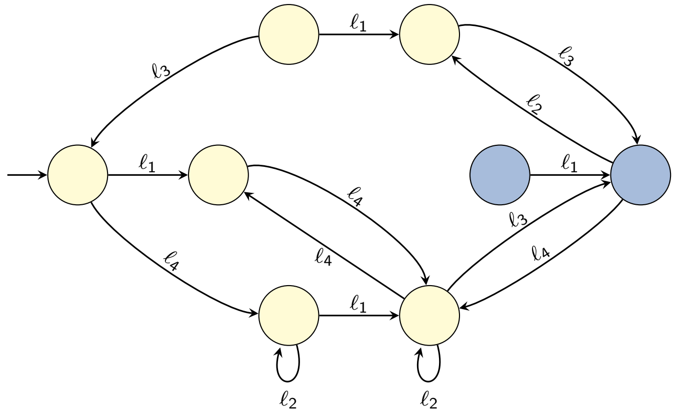
Our next steps are to formally define our problem:

- introduce a mathematical model for planning tasks: transition systems
  ⇝ Chapter B1
- introduce compact representations for planning tasks suitable as input for planning algorithms
  ⇝ Chapter B2

B1.1 Transition Systems

Transition System Example

Transition systems are often depicted as directed arc-labeled graphs with decorations to indicate the initial state and goal states.



$$c(\ell_1) = 1,\ c(\ell_2) = 1,\ c(\ell_3) = 5,\ c(\ell_4) = 0$$

Transition Systems

Definition (Transition System)

A transition system is a 6-tuple $\mathcal{T} = \langle S, L, c, T, s_0, S_\star \rangle$ where

- S is a finite set of states,
- L is a finite set of (transition) labels,
- $c : L \to \mathbb{R}_0^+$ is a label cost function,
- $T \subseteq S \times L \times S$ is the transition relation,
- $s_0 \in S$ is the initial state, and
- $S_\star \subseteq S$ is the set of goal states.

We say that $\mathcal{T}$ has the transition $\langle s, \ell, s' \rangle$ if $\langle s, \ell, s' \rangle \in T$.

We also write this as $s \xrightarrow{\ell} s'$, or $s \to s'$ when not interested in ℓ.

Note: Transition systems are also called state spaces.

Deterministic Transition Systems

Definition (Deterministic Transition System)

A transition system is called deterministic if for all states s and all labels ℓ, there is at most one state s' with $s \xrightarrow{\ell} s'$.

Example: previously shown transition system

Transition System Terminology (1)

We use common terminology from graph theory:

- s' **successor** of s if $s \to s'$
- s **predecessor** of s' if $s \to s'$

Transition System Terminology (2)

We use common terminology from graph theory:

- s' **reachable** from s if there exists a sequence of transitions $s^0 \xrightarrow{\ell_1} s^1, \dots, s^{n-1} \xrightarrow{\ell_n} s^n$ s.t. $s^0 = s$ and $s^n = s'$
  - Note: $n = 0$ possible; then $s = s'$
  - $s^0, \dots, s^n$ is called **(state) path** from s to s'
  - $\ell_1, \dots, \ell_n$ is called **(label) path** from s to s'
  - $s^0 \xrightarrow{\ell_1} s^1, \dots, s^{n-1} \xrightarrow{\ell_n} s^n$ is called **trace** from s to s'
  - **length** of path/trace is n
  - **cost** of label path/trace is $\sum_{i=1}^{n} c(\ell_i)$

Transition System Terminology (3)

We use common terminology from graph theory:

- s' **reachable** (without reference state) means reachable from initial state s_0
- **solution** or **goal path** from s: path from s to some $s' \in S_\star$
  - if s is omitted, $s = s_0$ is implied
- transition system **solvable** if a goal path from s_0 exists

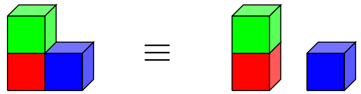
B1.2 Example: Blocks World

Running Example: Blocks World

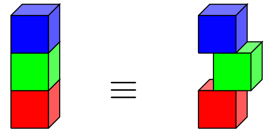
- Throughout the course, we occasionally use the blocks world domain as an example.
- In the blocks world, a number of different blocks are arranged on a table.
- Our job is to rearrange them according to a given goal.

Blocks World Rules (1)

Location on the table does not matter.

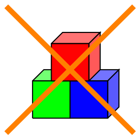


Location on a block does not matter.

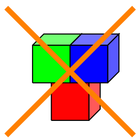


Blocks World Rules (2)

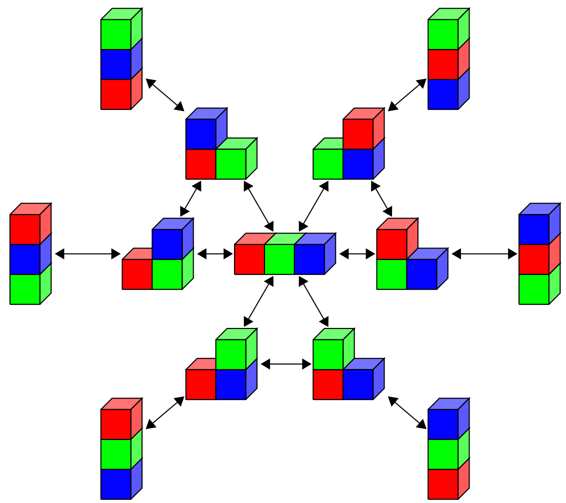
At most one block may be below a block.



At most one block may be on top of a block.



Blocks World Transition System for Three Blocks



Labels omitted for clarity. All label costs are 1. Initial/goal states not marked.

Blocks World Computational Properties

blocks	states	blocks	states
1	1	10	58941091
2	3	11	824073141
3	13	12	12470162233
4	73	13	202976401213
5	501	14	3535017524403
6	4051	15	65573803186921
7	37633	16	1290434218669921
8	394353	17	26846616451246353
9	4596553	18	588633468315403843

- Finding solutions is possible in linear time in the number of blocks: move everything onto the table, then construct the goal configuration.
- Finding a shortest solution is NP-complete given a compact description of the problem.

The Need for Compact Descriptions

- We see from the blocks world example that transition systems are often far too large to be directly used as inputs to planning algorithms.
- We therefore need compact descriptions of transition systems.
- For this purpose, we will use propositional logic, which allows expressing information about 2^n states as logical formulas over n state variables.

B1.3 Reminder: Propositional Logic

More on Propositional Logic

Need to Catch Up?

- This section is a reminder. We assume you are already well familiar with propositional logic.
- If this is not the case, we recommend Chapters D1–D4 of the Discrete Mathematics in Computer Science course: `https://dmi.unibas.ch/en/studies/computer-science/courses-in-fall-semester-2023/lecture-discrete-mathematics-in-computer-science/`
  - Videos for these chapters are available on request.

Syntax of Propositional Logic

Definition (Logical Formula)

Let A be a set of atomic propositions.

The logical formulas over A are constructed by finite application of the following rules:

- $\top$ and $\bot$ are logical formulas (truth and falsity).
- For all $a \in A$, a is a logical formula (atom).
- If φ is a logical formula, then so is $\neg\varphi$ (negation).
- If φ and ψ are logical formulas, then so are $(\varphi \lor \psi)$ (disjunction) and $(\varphi \land \psi)$ (conjunction).

Syntactical Conventions for Propositional Logic

Abbreviations:

- $(\varphi \to \psi)$ is short for $(\neg\varphi \lor \psi)$ (implication)
- $(\varphi \leftrightarrow \psi)$ is short for $((\varphi \to \psi) \land (\psi \to \varphi))$ (equijunction)
- parentheses omitted when not necessary:
  - $(\neg)$ binds more tightly than binary connectives
  - $(\land)$ binds more tightly than $(\lor)$, which binds more tightly than $(\to)$, which binds more tightly than $(\leftrightarrow)$

Semantics of Propositional Logic

Definition (Interpretation, Model)

An interpretation of propositions A is a function $I : A \to \{\mathbf{T}, \mathbf{F}\}$.

Define the notation $I \models \varphi$ (I satisfies φ; I is a model of φ; φ is true under I) for interpretations I and formulas φ by

- $I \models \top$
- $I \not\models \bot$
- $I \models a$ iff $I(a) = \mathbf{T}$ (for all $a \in A$)
- $I \models \neg\varphi$ iff $I \not\models \varphi$
- $I \models (\varphi \lor \psi)$ iff $(I \models \varphi$ or $I \models \psi)$
- $I \models (\varphi \land \psi)$ iff $(I \models \varphi$ and $I \models \psi)$

Note: Interpretations are also called valuations or truth assignments.

Propositional Logic Terminology (1)

- A logical formula φ is satisfiable if there is at least one interpretation I such that $I \models \varphi$.
- Otherwise it is unsatisfiable.
- A logical formula φ is valid or a tautology if $I \models \varphi$ for all interpretations I.
- A logical formula ψ is a logical consequence of a logical formula φ, written $\varphi \models \psi$, if $I \models \psi$ for all interpretations I with $I \models \varphi$.
- Two logical formulas φ and ψ are logically equivalent, written $\varphi \equiv \psi$, if $\varphi \models \psi$ and $\psi \models \varphi$.

Question: How to phrase these in terms of models?

Propositional Logic Terminology (2)

- A logical formula that is a proposition a or a negated proposition $\neg a$ for some atomic proposition $a \in A$ is a literal.
- A formula that is a disjunction of literals is a clause. This includes unit clauses ℓ consisting of a single literal and the empty clause $\bot$ consisting of zero literals.
- A formula that is a conjunction of literals is a monomial. This includes unit monomials ℓ consisting of a single literal and the empty monomial $\top$ consisting of zero literals.

Normal forms:

- negation normal form (NNF)
- conjunctive normal form (CNF)
- disjunctive normal form (DNF)

B1.4 Summary

Summary

- Transition systems are (typically huge) directed graphs that encode how the state of the world can change.
- Propositional logic allows us to compactly describe complex information about large sets of interpretations as logical formulas.