

# Discrete Mathematics in Computer Science

## B1. Tuples & Cartesian Product

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# Tuples and the Cartesian Product

# Motivation

- A **set** is an **unordered collection** of **distinct** objects.
- We often need a more structured way of representation.
  - A person is associated with a name, address, phone number.
  - A set of persons makes sense in many contexts.
  - Representing the associated data as a set rather not.
- We could for example want to
  - directly access the name of a person, or
  - have a separate billing and delivery address for some order, but in general, these can be the same.
- **Tuples** are mathematical building blocks that support this.

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- A **set** is an **unordered collection** of **distinct** objects.
- A **tuple** is an **ordered sequence** of objects.

# Tuples

- ***k*-tuple**: ordered sequence of  $k$  objects ( $k \in \mathbb{N}_0$ )
- written  $(o_1, \dots, o_k)$  or  $\langle o_1, \dots, o_k \rangle$
- unlike sets, **order matters** ( $\langle 1, 2 \rangle \neq \langle 2, 1 \rangle$ )
- objects may occur multiple times in a tuple

German:  $k$ -Tupel, Komponente, (geordnetes) Paar, Tripel, Quadrupel

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- objects may occur multiple times in a tuple
- objects contained in tuples are called **components**
- terminology:
  - $k = 2$ : (ordered) pair
  - $k = 3$ : triple
  - more rarely: quadruple, quintuple, sextuple, septuple, ...
- if  $k$  is clear from context (or does not matter), often just called **tuple**

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# Equality of Tuples

## Definition (Equality of Tuples)

Two  $n$ -tuples  $t = \langle o_1, \dots, o_n \rangle$  and  $t' = \langle o'_1, \dots, o'_n \rangle$  are **equal** ( $t = t'$ ) if for  $i \in \{1, \dots, n\}$  it holds that  $o_i = o'_i$ .



# Cartesian Product

## Definition (Cartesian Product and Cartesian Power)

Let  $S_1, \dots, S_n$  be sets. The **Cartesian product**  $S_1 \times \dots \times S_n$  is the following set of  $n$ -tuples:

$$S_1 \times \dots \times S_n = \{ \langle x_1, \dots, x_n \rangle \mid x_1 \in S_1, x_2 \in S_2, \dots, x_n \in S_n \}.$$

**René Descartes:** French mathematician and philosopher (1596–1650)

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$A \times B =$

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The  $k$ -ary **Cartesian power** of a set  $S$  (with  $k \in \mathbb{N}_1$ ) is the set  $S^k = \{ \langle o_1, \dots, o_k \rangle \mid o_i \in S \text{ for all } i \in \{1, \dots, k\} \} = \underbrace{S \times \dots \times S}_{k \text{ times}}.$

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**Example:**  $A = \{a, b\}$ ,  $B = \{1, 2, 3\}$

$A^2 =$

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## (Non-)properties of the Cartesian Product

The Cartesian product is

- **not commutative**, in most cases  $A \times B \neq B \times A$ .
- **not associative**, in most cases  $(A \times B) \times C \neq A \times (B \times C)$

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Why? Exceptions?

# Questions



Questions?

# Summary

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- A  **$k$ -tuple** is an **ordered sequence** of  $k$  objects, called the **components** of the tuple.
- 2-tuples are also called **pairs** and 3-tuples **triples**.
- The **Cartesian Product**  $S_1 \times \cdots \times S_n$  of set  $S_1, \dots, S_n$  is the set of all tuples  $\langle o_1, \dots, o_n \rangle$ , where for all  $i \in \{1, \dots, n\}$  component  $o_i$  is an element of  $S_i$ .