

Planning and Optimization

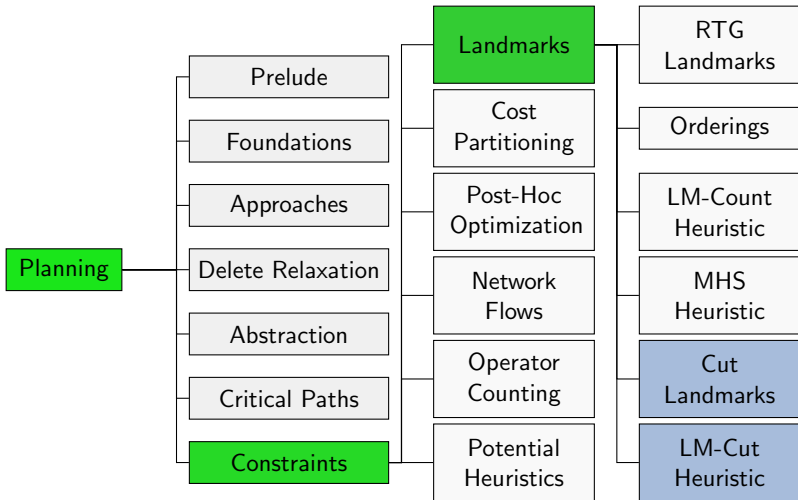
G5. Landmarks: Cut Landmarks & LM-Cut Heuristic

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Content of this Course



Roadmap for this Chapter

- We first introduce a new **normal form for delete-free STRIPS tasks** that simplifies later definitions.
- We then present a method that **computes disjunctive action landmarks** for such tasks.
- We conclude with the **LM-cut heuristic** that builds on this method.

i-g Form

Delete-Free STRIPS Planning Task in i-g Form (1)

In this chapter, we only consider **delete-free** STRIPS tasks in a special form:

Definition (i-g Form for Delete-free STRIPS)

A delete-free STRIPS planning task $\langle V, I, O, \gamma \rangle$ is in **i-g form** if

- V contains atoms i and g
- Initially exactly i is true: $I(v) = \mathbf{T}$ iff $v = i$
- g is the only goal atom: $\gamma = \{g\}$
- Every action has at least one precondition.

Transformation to i-g Form

Every delete-free STRIPS task $\Pi = \langle V, I, O, \gamma \rangle$ can easily be transformed into an analogous task in i-g form.

- If i or g are in V already, rename them everywhere.
- Add i and g to V .
- Add an operator $\langle \{i\}, \{v \in V \mid I(v) = \mathbf{T}\}, \{\}, 0 \rangle$.
- Add an operator $\langle \gamma, \{g\}, \{\}, 0 \rangle$.
- Replace all operator preconditions \mathbf{T} with i .
- Replace initial state and goal.

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- Replace all operator preconditions \mathbf{T} with i .
- Replace initial state and goal.

For the remainder of this chapter, we assume tasks in i-g form.

Example: Delete-Free Planning Task in i-g Form

Example

Consider a delete-relaxed STRIPS planning $\langle V, I, O, \gamma \rangle$ with $V = \{i, a, b, c, d, g\}$, $I = \{i \mapsto \mathbf{T}\} \cup \{v \mapsto \mathbf{F} \mid v \in V \setminus \{i\}\}$, $\gamma = g$ and operators

- $O_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 4 \rangle$,
- $O_{\text{green}} = \langle \{i\}, \{a, c\}, \{\}, 5 \rangle$,
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optimal solution?

Example: Delete-Free Planning Task in i-g Form

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Consider a delete-relaxed STRIPS planning $\langle V, I, O, \gamma \rangle$ with $V = \{i, a, b, c, d, g\}$, $I = \{i \mapsto \mathbf{T}\} \cup \{v \mapsto \mathbf{F} \mid v \in V \setminus \{i\}\}$, $\gamma = g$ and operators

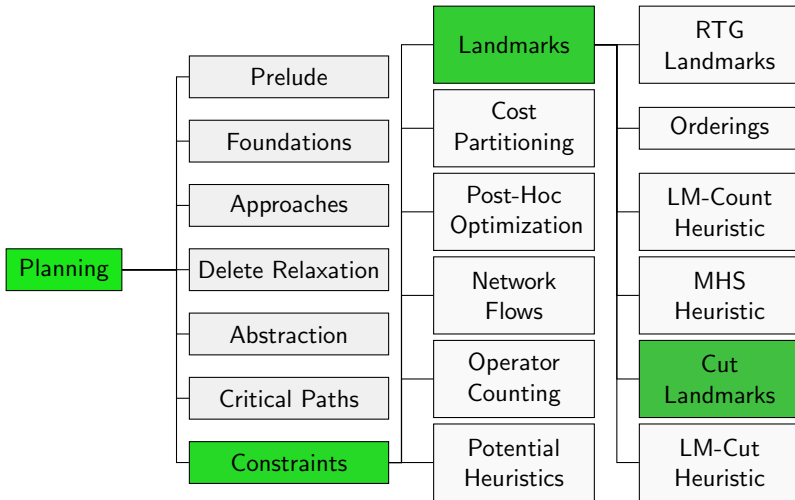
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optimal solution to reach g from i :

- plan: $\langle O_{\text{blue}}, O_{\text{black}}, O_{\text{red}}, O_{\text{orange}} \rangle$
- cost: $4 + 3 + 2 + 0 = 9$ ($= h^+(I)$ because plan is **optimal**)

Cut Landmarks

Content of this Course



Justification Graphs

Definition (Precondition Choice Function)

A **precondition choice function** (pcf) $P : O \rightarrow V$ for a delete-free STRIPS task $\Pi = \langle V, I, O, \gamma \rangle$ in i-g form maps each operator to one of its preconditions (i.e. $P(o) \in pre(o)$ for all $o \in O$).

Definition (Justification Graphs)

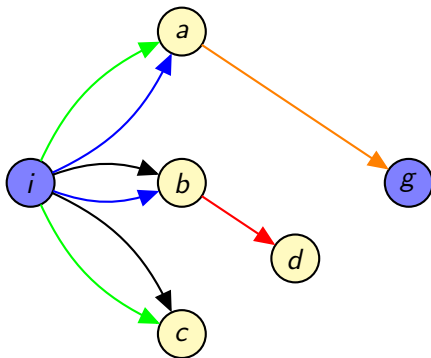
Let P be a pcf for $\langle V, I, O, \gamma \rangle$ in i-g form. The **justification graph** for P is the directed, edge-labeled graph $J = \langle V, E \rangle$, where

- the vertices are the variables from V , and
- E contains an edge $P(o) \xrightarrow{o} a$ for each $o \in O$, $a \in add(o)$.

Example: Justification Graph

Example (Precondition Choice Function)

$P(o_{\text{blue}}) = P(o_{\text{green}}) = P(o_{\text{black}}) = i$, $P(o_{\text{red}}) = b$, $P(o_{\text{orange}}) = a$



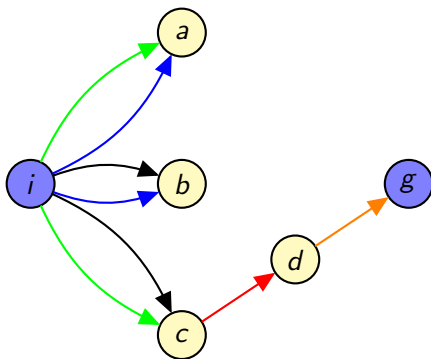
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Example: Justification Graph

Example (Precondition Choice Function)

$$P(O_{\text{blue}}) = P(O_{\text{green}}) = P(o_{\text{black}}) = i, P(O_{\text{red}}) = b, P(O_{\text{orange}}) = a$$

$$P'(O_{\text{blue}}) = P'(O_{\text{green}}) = P'(o_{\text{black}}) = i, P'(O_{\text{red}}) = c, P'(O_{\text{orange}}) = d$$



$$O_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 4 \rangle$$

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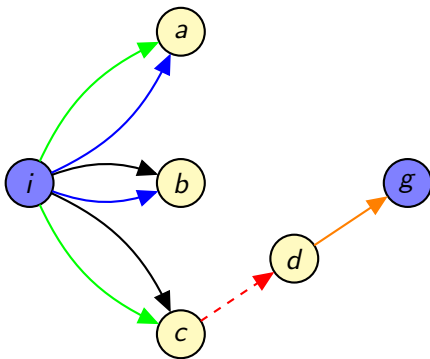
$$O_{\text{red}} = \langle \{b, c\}, \{d\}, \{\}, 2 \rangle$$

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Cuts

Definition (Cut)

A **cut** in a justification graph is a subset C of its edges such that all paths from i to g contain an edge from C .



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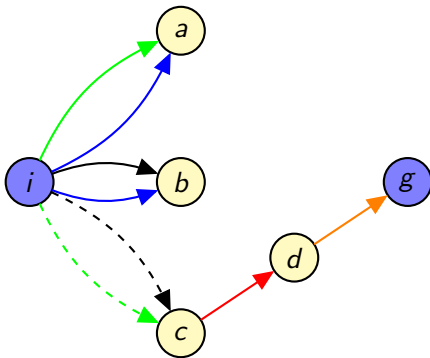
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Cuts are Disjunctive Action Landmarks

Theorem (Cuts are Disjunctive Action Landmarks)

Let P be a pcf for $\langle V, I, O, \gamma \rangle$ (in i-g form) and C be a *cut* in the justification graph for P .

The set of *edge labels* from C (formally $\{o \mid \langle v, o, v' \rangle \in C\}$) is a *disjunctive action landmark* for I .

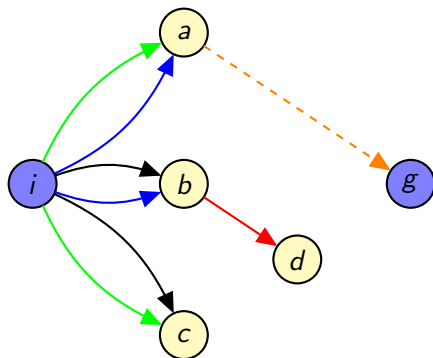
Proof idea:

- The justification graph corresponds to a simpler problem where some preconditions (those not picked by the pcf) are ignored.
- Cuts are landmarks for this simplified problem.
- Hence they are also landmarks for the original problem.

Example: Cuts in Justification Graphs

Example (Landmarks)

- $L_1 = \{O_{\text{orange}}\}$ (cost = 0)

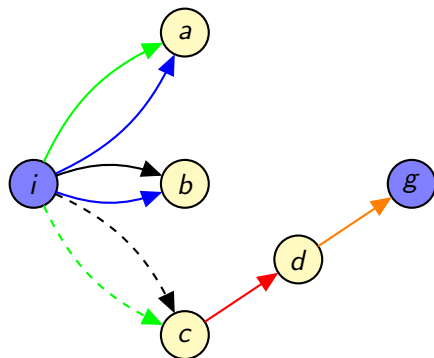


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Example: Cuts in Justification Graphs

Example (Landmarks)

- $L_1 = \{O_{\text{orange}}\}$ (cost = 0)
- $L_2 = \{O_{\text{green}}, O_{\text{black}}\}$ (cost = 3)

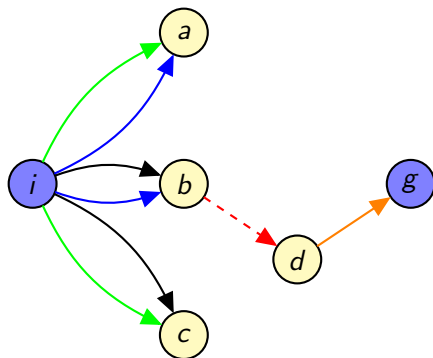


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Example: Cuts in Justification Graphs

Example (Landmarks)

- $L_1 = \{O_{\text{orange}}\}$ (cost = 0)
- $L_2 = \{O_{\text{green}}, O_{\text{black}}\}$ (cost = 3)
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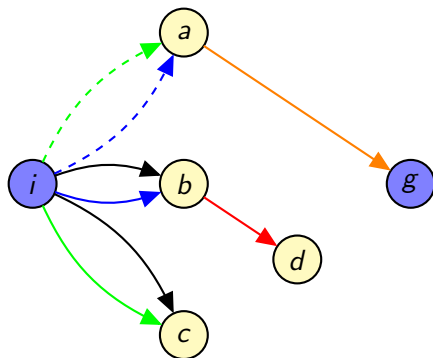


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Example: Cuts in Justification Graphs

Example (Landmarks)

- $L_1 = \{O_{\text{orange}}\}$ (cost = 0)
- $L_2 = \{O_{\text{green}}, O_{\text{black}}\}$ (cost = 3)
- $L_3 = \{O_{\text{red}}\}$ (cost = 2)
- $L_4 = \{O_{\text{green}}, O_{\text{blue}}\}$ (cost = 4)



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Power of Cuts in Justification Graphs

- Which landmarks can be computed with the cut method?

Power of Cuts in Justification Graphs

- Which landmarks can be computed with the cut method?
- **all interesting ones!**

Proposition (perfect hitting set heuristics)

Let \mathcal{L} be the set of **all** “cut landmarks” of a given planning task with initial state I . Then $h^{MHS}(\mathcal{L}) = h^+(I)$.

↪ Hitting set heuristic for \mathcal{L} is **perfect**.

Power of Cuts in Justification Graphs

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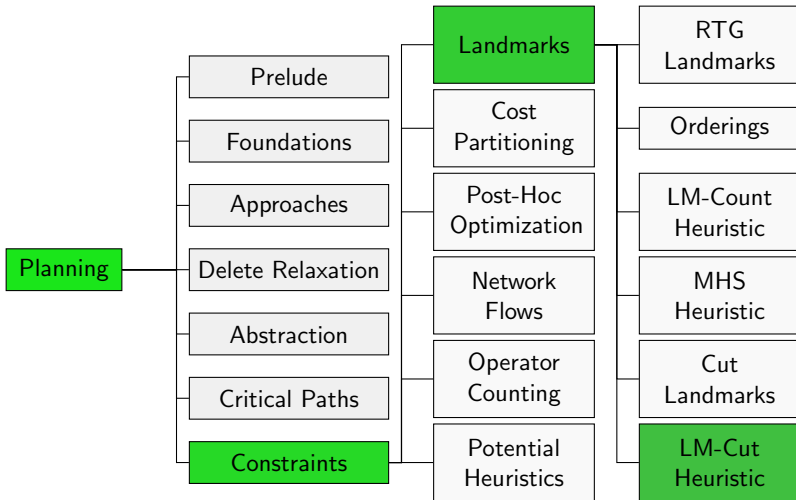
↪ Hitting set heuristic for \mathcal{L} is **perfect**.

Proof idea:

- Show 1:1 correspondence of hitting sets H for \mathcal{L} and plans, i.e., each hitting set for \mathcal{L} corresponds to a plan, and vice versa.

The LM-Cut Heuristic

Content of this Course



LM-Cut Heuristic: Motivation

- In general, there are exponentially many pcfs, hence computing all relevant landmarks is not tractable.
- The **LM-cut heuristic** is a method that chooses pcfs and computes cuts in a **goal-oriented** way.
- As a side effect, it computes a
 - a cost partitioning over multiple instances of h^{\max} that is also
 - a **saturated cost partitioning** over disjunctive action landmarks.

↪ next week

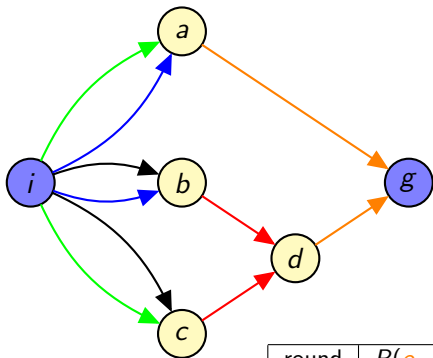
LM-Cut Heuristic

$h^{\text{LM-cut}}$: Helmert & Domshlak (2009)

Initialize $h^{\text{LM-cut}}(I) := 0$. Then iterate:

- 1 Compute h^{max} values of the variables. Stop if $h^{\text{max}}(g) = 0$.
- 2 Compute justification graph G for the P that chooses preconditions with maximal h^{max} value
- 3 Determine the goal zone V_g of G that consists of all nodes that have a zero-cost path to g .
- 4 Compute the cut L that contains the labels of all edges $\langle v, o, v' \rangle$ such that $v \notin V_g$, $v' \in V_g$ and v can be reached from i without traversing a node in V_g .
It is guaranteed that $\text{cost}(L) > 0$.
- 5 Increase $h^{\text{LM-cut}}(I)$ by $\text{cost}(L)$.
- 6 Decrease $\text{cost}(o)$ by $\text{cost}(L)$ for all $o \in L$.

Example: Computation of LM-Cut

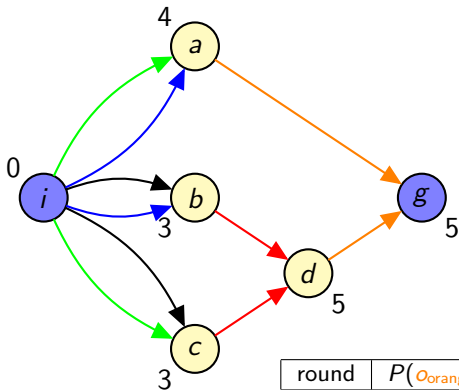


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 $O_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

| round | $P(O_{\text{orange}})$ | $P(O_{\text{red}})$ | landmark | cost |
|------------------------|------------------------|---------------------|----------|------|
| | | | | |
| | | | | |
| | | | | |
| $h^{\text{LM-cut}}(I)$ | | | | 0 |

Example: Computation of LM-Cut

1 Compute h^{\max} values of the variables

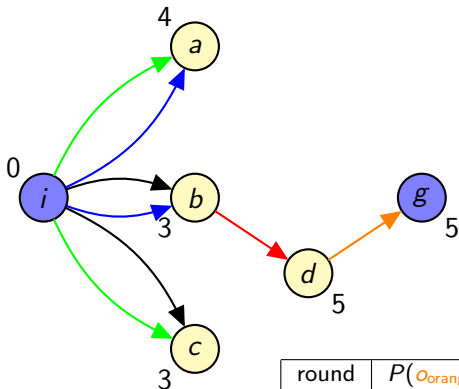


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- $O_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

| round | $P(O_{\text{orange}})$ | $P(O_{\text{red}})$ | landmark | cost |
|------------------------|------------------------|---------------------|----------|------|
| 1 | | | | |
| | | | | |
| | | | | |
| $h^{\text{LM-cut}}(I)$ | | | | 0 |

Example: Computation of LM-Cut

2 Compute justification graph

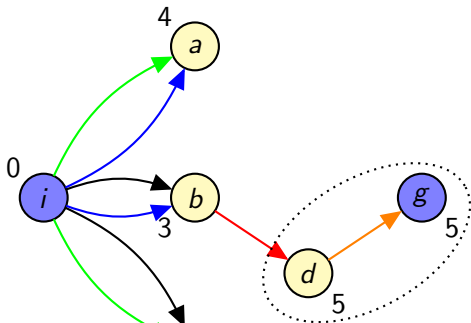


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- $O_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

| round | $P(O_{\text{orange}})$ | $P(O_{\text{red}})$ | landmark | cost |
|------------------------|------------------------|---------------------|----------|------|
| 1 | d | b | | |
| | | | | |
| | | | | |
| $h^{\text{LM-cut}}(I)$ | | | | 0 |

Example: Computation of LM-Cut

3 Determine goal zone

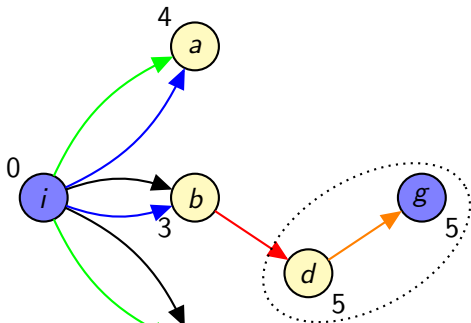


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| round | $P(O_{\text{orange}})$ | $P(O_{\text{red}})$ | landmark | cost |
|------------------------|------------------------|---------------------|----------|------|
| 1 | d | b | | |
| | | | | |
| | | | | |
| $h^{\text{LM-cut}}(I)$ | | | | 0 |

Example: Computation of LM-Cut

④ Compute cut

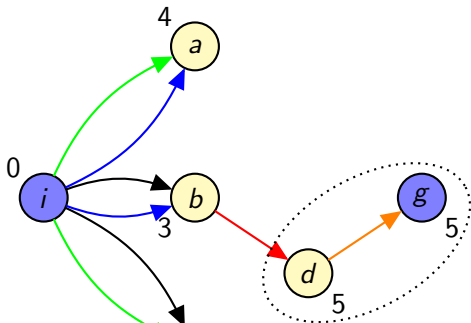


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 O_{\text{orange}} &= \langle \{a, d\}, \{g\}, \{\}, 0 \rangle
 \end{aligned}$$

| round | $P(O_{\text{orange}})$ | $P(O_{\text{red}})$ | landmark | cost |
|------------------------|------------------------|---------------------|----------------------|------|
| 1 | d | b | $\{O_{\text{red}}\}$ | 2 |
| | | | | |
| | | | | |
| $h^{\text{LM-cut}}(I)$ | | | | 0 |

Example: Computation of LM-Cut

- 5 Increase $h^{\text{LM-cut}}(l)$ by $\text{cost}(L)$

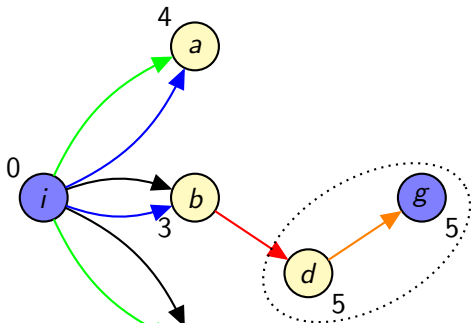


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 $O_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

| round | $P(O_{\text{orange}})$ | $P(O_{\text{red}})$ | landmark | cost |
|------------------------|------------------------|---------------------|----------------------|------|
| 1 | d | b | $\{O_{\text{red}}\}$ | 2 |
| | | | | |
| | | | | |
| $h^{\text{LM-cut}}(l)$ | | | | 2 |

Example: Computation of LM-Cut

- 6 Decrease $cost(o)$ by $cost(L)$ for all $o \in L$

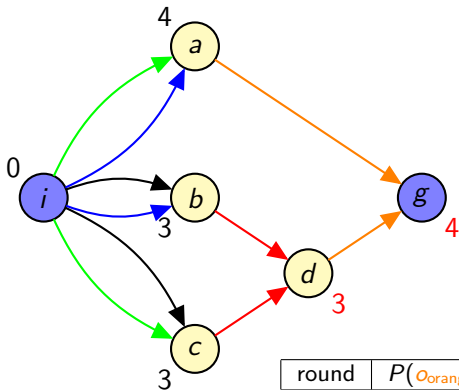


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 \end{aligned}$$

| round | $P(O_{\text{orange}})$ | $P(O_{\text{red}})$ | landmark | cost |
|------------------------|------------------------|---------------------|----------------------|------|
| 1 | d | b | $\{O_{\text{red}}\}$ | 2 |
| | | | | |
| | | | | |
| $h^{\text{LM-cut}}(I)$ | | | | 2 |

Example: Computation of LM-Cut

1 Compute h^{\max} values of the variables

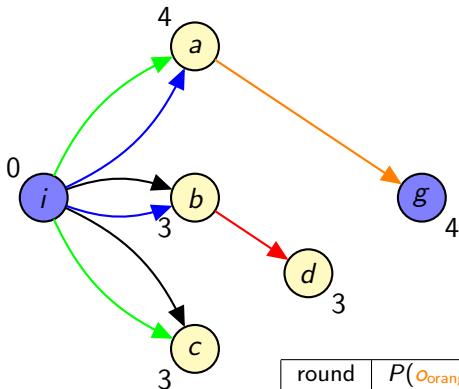


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 \end{aligned}$$

| round | $P(O_{\text{orange}})$ | $P(O_{\text{red}})$ | landmark | cost |
|------------------------|------------------------|---------------------|----------------------|------|
| 1 | d | b | $\{O_{\text{red}}\}$ | 2 |
| 2 | | | | |
| | | | | |
| $h^{\text{LM-cut}}(I)$ | | | | 2 |

Example: Computation of LM-Cut

2 Compute justification graph

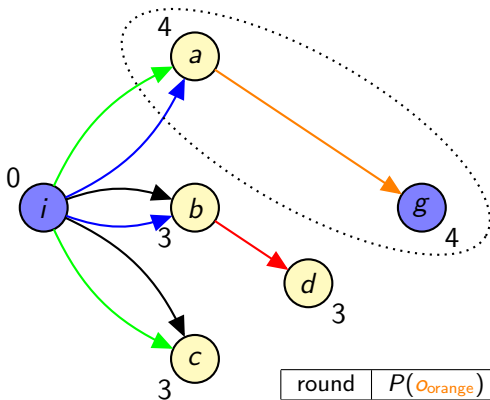


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 \end{aligned}$$

| round | $P(O_{\text{orange}})$ | $P(O_{\text{red}})$ | landmark | cost |
|------------------------|------------------------|---------------------|----------------------|------|
| 1 | d | b | $\{O_{\text{red}}\}$ | 2 |
| 2 | a | b | | |
| | | | | |
| $h^{\text{LM-cut}}(I)$ | | | | 2 |

Example: Computation of LM-Cut

3 Determine goal zone

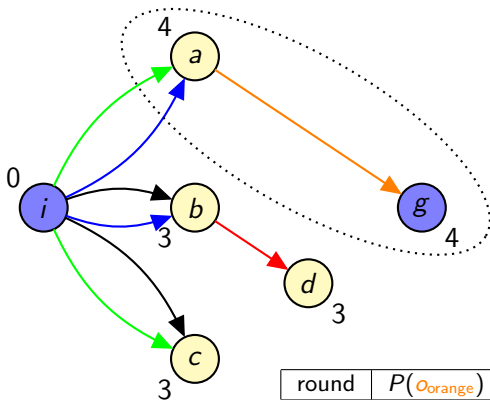


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 \end{aligned}$$

| round | $P(O_{\text{orange}})$ | $P(O_{\text{red}})$ | landmark | cost |
|------------------------|------------------------|---------------------|----------------------|------|
| 1 | d | b | $\{O_{\text{red}}\}$ | 2 |
| 2 | a | b | | |
| | | | | |
| $h^{\text{LM-cut}}(I)$ | | | | 2 |

Example: Computation of LM-Cut

④ Compute cut

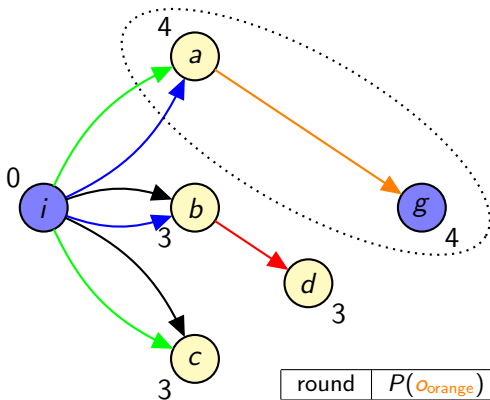


$$\begin{aligned}
 O_{\text{blue}} &= \langle \{i\}, \{a, b\}, \{\}, 4 \rangle \\
 O_{\text{green}} &= \langle \{i\}, \{a, c\}, \{\}, 5 \rangle \\
 O_{\text{black}} &= \langle \{i\}, \{b, c\}, \{\}, 3 \rangle \\
 O_{\text{red}} &= \langle \{b, c\}, \{d\}, \{\}, 0 \rangle \\
 O_{\text{orange}} &= \langle \{a, d\}, \{g\}, \{\}, 0 \rangle
 \end{aligned}$$

| round | $P(O_{\text{orange}})$ | $P(O_{\text{red}})$ | landmark | cost |
|------------------------|------------------------|---------------------|---|------|
| 1 | d | b | $\{O_{\text{red}}\}$ | 2 |
| 2 | a | b | $\{O_{\text{green}}, O_{\text{blue}}\}$ | 4 |
| | | | | |
| $h^{\text{LM-cut}}(I)$ | | | | 2 |

Example: Computation of LM-Cut

5 Increase $h^{\text{LM-cut}}(l)$ by $\text{cost}(L)$

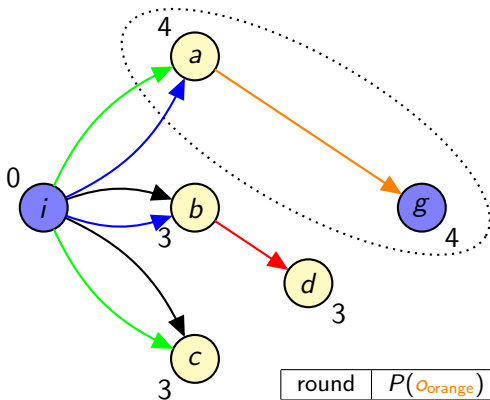


$O_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 4 \rangle$
 $O_{\text{green}} = \langle \{i\}, \{a, c\}, \{\}, 5 \rangle$
 $O_{\text{black}} = \langle \{i\}, \{b, c\}, \{\}, 3 \rangle$
 $O_{\text{red}} = \langle \{b, c\}, \{d\}, \{\}, 0 \rangle$
 $O_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

| round | $P(O_{\text{orange}})$ | $P(O_{\text{red}})$ | landmark | cost |
|------------------------|------------------------|---------------------|---|------|
| 1 | d | b | $\{O_{\text{red}}\}$ | 2 |
| 2 | a | b | $\{O_{\text{green}}, O_{\text{blue}}\}$ | 4 |
| | | | | |
| $h^{\text{LM-cut}}(l)$ | | | | 6 |

Example: Computation of LM-Cut

- 6 Decrease $cost(o)$ by $cost(L)$ for all $o \in L$

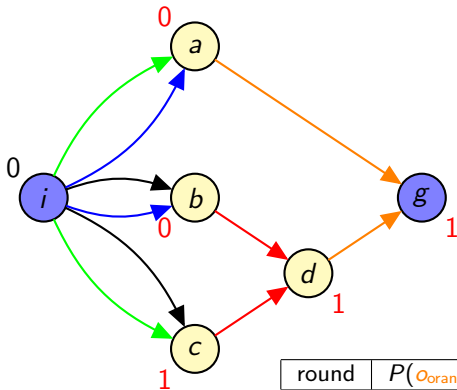


$$\begin{aligned}
 O_{\text{blue}} &= \langle \{i\}, \{a, b\}, \{\}, 0 \rangle \\
 O_{\text{green}} &= \langle \{i\}, \{a, c\}, \{\}, 1 \rangle \\
 O_{\text{black}} &= \langle \{i\}, \{b, c\}, \{\}, 3 \rangle \\
 O_{\text{red}} &= \langle \{b, c\}, \{d\}, \{\}, 0 \rangle \\
 O_{\text{orange}} &= \langle \{a, d\}, \{g\}, \{\}, 0 \rangle
 \end{aligned}$$

| round | $P(O_{\text{orange}})$ | $P(O_{\text{red}})$ | landmark | cost |
|------------------------|------------------------|---------------------|---|------|
| 1 | d | b | $\{O_{\text{red}}\}$ | 2 |
| 2 | a | b | $\{O_{\text{green}}, O_{\text{blue}}\}$ | 4 |
| | | | | |
| $h^{\text{LM-cut}}(I)$ | | | | 6 |

Example: Computation of LM-Cut

1 Compute h^{\max} values of the variables

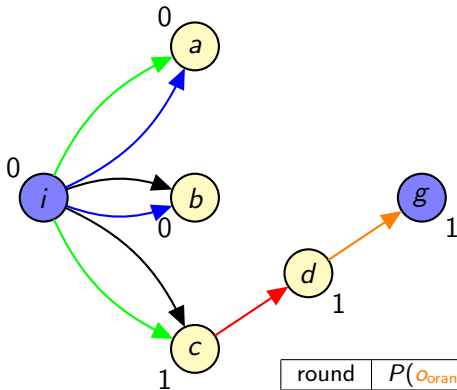


$$\begin{aligned}
 O_{\text{blue}} &= \langle \{i\}, \{a, b\}, \{\}, 0 \rangle \\
 O_{\text{green}} &= \langle \{i\}, \{a, c\}, \{\}, 1 \rangle \\
 O_{\text{black}} &= \langle \{i\}, \{b, c\}, \{\}, 3 \rangle \\
 O_{\text{red}} &= \langle \{b, c\}, \{d\}, \{\}, 0 \rangle \\
 O_{\text{orange}} &= \langle \{a, d\}, \{g\}, \{\}, 0 \rangle
 \end{aligned}$$

| round | $P(O_{\text{orange}})$ | $P(O_{\text{red}})$ | landmark | cost |
|------------------------|------------------------|---------------------|---|------|
| 1 | d | b | $\{O_{\text{red}}\}$ | 2 |
| 2 | a | b | $\{O_{\text{green}}, O_{\text{blue}}\}$ | 4 |
| 3 | | | | |
| $h^{\text{LM-cut}}(I)$ | | | | 6 |

Example: Computation of LM-Cut

2 Compute justification graph

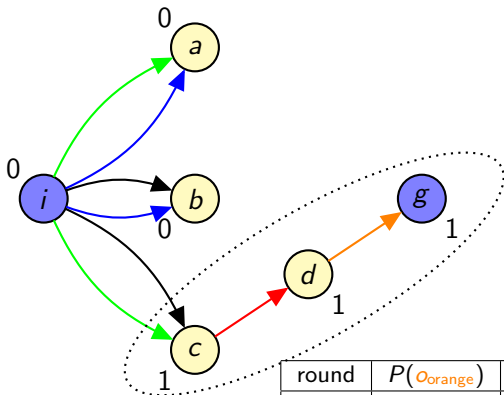


$$\begin{aligned}
 O_{\text{blue}} &= \langle \{i\}, \{a, b\}, \{\}, 0 \rangle \\
 O_{\text{green}} &= \langle \{i\}, \{a, c\}, \{\}, 1 \rangle \\
 O_{\text{black}} &= \langle \{i\}, \{b, c\}, \{\}, 3 \rangle \\
 O_{\text{red}} &= \langle \{b, c\}, \{d\}, \{\}, 0 \rangle \\
 O_{\text{orange}} &= \langle \{a, d\}, \{g\}, \{\}, 0 \rangle
 \end{aligned}$$

| round | $P(O_{\text{orange}})$ | $P(O_{\text{red}})$ | landmark | cost |
|------------------------|------------------------|---------------------|---|------|
| 1 | d | b | $\{O_{\text{red}}\}$ | 2 |
| 2 | a | b | $\{O_{\text{green}}, O_{\text{blue}}\}$ | 4 |
| 3 | d | c | | |
| $h^{\text{LM-cut}}(I)$ | | | | 6 |

Example: Computation of LM-Cut

3 Determine goal zone

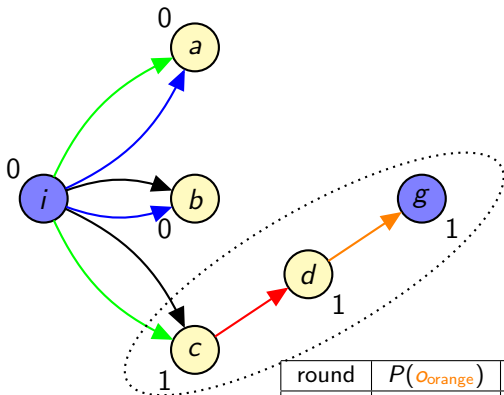


$$\begin{aligned}
 O_{\text{blue}} &= \langle \{i\}, \{a, b\}, \{\}, 0 \rangle \\
 O_{\text{green}} &= \langle \{i\}, \{a, c\}, \{\}, 1 \rangle \\
 O_{\text{black}} &= \langle \{i\}, \{b, c\}, \{\}, 3 \rangle \\
 O_{\text{red}} &= \langle \{b, c\}, \{d\}, \{\}, 0 \rangle \\
 O_{\text{orange}} &= \langle \{a, d\}, \{g\}, \{\}, 0 \rangle
 \end{aligned}$$

| round | $P(O_{\text{orange}})$ | $P(O_{\text{red}})$ | landmark | cost |
|------------------------|------------------------|---------------------|---|------|
| 1 | d | b | $\{O_{\text{red}}\}$ | 2 |
| 2 | a | b | $\{O_{\text{green}}, O_{\text{blue}}\}$ | 4 |
| 3 | d | c | | |
| $h^{\text{LM-cut}}(I)$ | | | | 6 |

Example: Computation of LM-Cut

4 Compute cut

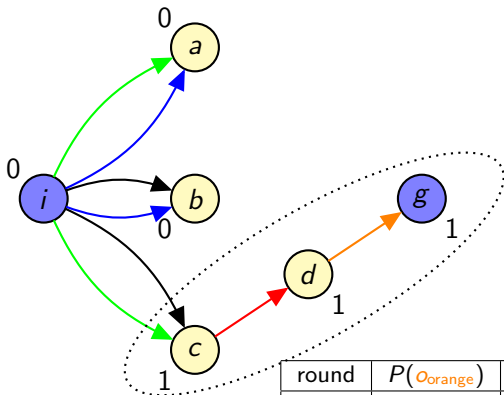


$$\begin{aligned}
 O_{\text{blue}} &= \langle \{i\}, \{a, b\}, \{\}, 0 \rangle \\
 O_{\text{green}} &= \langle \{i\}, \{a, c\}, \{\}, 1 \rangle \\
 O_{\text{black}} &= \langle \{i\}, \{b, c\}, \{\}, 3 \rangle \\
 O_{\text{red}} &= \langle \{b, c\}, \{d\}, \{\}, 0 \rangle \\
 O_{\text{orange}} &= \langle \{a, d\}, \{g\}, \{\}, 0 \rangle
 \end{aligned}$$

| round | $P(O_{\text{orange}})$ | $P(O_{\text{red}})$ | landmark | cost |
|------------------------|------------------------|---------------------|--|----------|
| 1 | d | b | $\{O_{\text{red}}\}$ | 2 |
| 2 | a | b | $\{O_{\text{green}}, O_{\text{blue}}\}$ | 4 |
| 3 | d | c | $\{O_{\text{green}}, O_{\text{black}}\}$ | 1 |
| $h^{\text{LM-cut}}(I)$ | | | | 6 |

Example: Computation of LM-Cut

5 Increase $h^{\text{LM-cut}}(l)$ by $\text{cost}(L)$

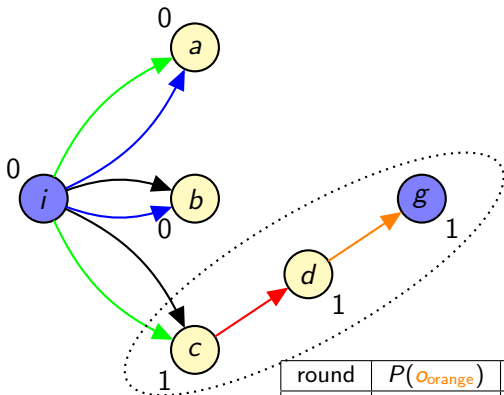


- $O_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 0 \rangle$
- $O_{\text{green}} = \langle \{i\}, \{a, c\}, \{\}, 1 \rangle$
- $O_{\text{black}} = \langle \{i\}, \{b, c\}, \{\}, 3 \rangle$
- $O_{\text{red}} = \langle \{b, c\}, \{d\}, \{\}, 0 \rangle$
- $O_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

| round | $P(O_{\text{orange}})$ | $P(O_{\text{red}})$ | landmark | cost |
|------------------------|------------------------|---------------------|--|------|
| 1 | d | b | $\{O_{\text{red}}\}$ | 2 |
| 2 | a | b | $\{O_{\text{green}}, O_{\text{blue}}\}$ | 4 |
| 3 | d | c | $\{O_{\text{green}}, O_{\text{black}}\}$ | 1 |
| $h^{\text{LM-cut}}(l)$ | | | | 7 |

Example: Computation of LM-Cut

- 6 Decrease $cost(o)$ by $cost(L)$ for all $o \in L$

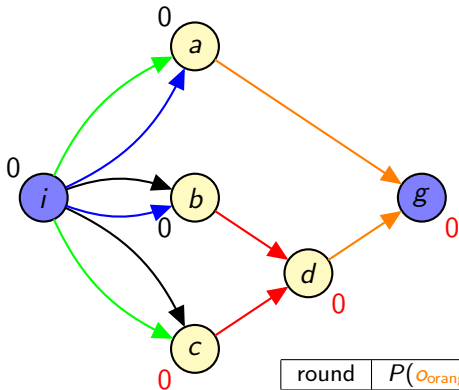


$$\begin{aligned}
 O_{\text{blue}} &= \langle \{i\}, \{a, b\}, \{\}, 0 \rangle \\
 O_{\text{green}} &= \langle \{i\}, \{a, c\}, \{\}, 0 \rangle \\
 O_{\text{black}} &= \langle \{i\}, \{b, c\}, \{\}, 2 \rangle \\
 O_{\text{red}} &= \langle \{b, c\}, \{d\}, \{\}, 0 \rangle \\
 O_{\text{orange}} &= \langle \{a, d\}, \{g\}, \{\}, 0 \rangle
 \end{aligned}$$

| round | $P(O_{\text{orange}})$ | $P(O_{\text{red}})$ | landmark | cost |
|------------------------|------------------------|---------------------|--|------|
| 1 | d | b | $\{O_{\text{red}}\}$ | 2 |
| 2 | a | b | $\{O_{\text{green}}, O_{\text{blue}}\}$ | 4 |
| 3 | d | c | $\{O_{\text{green}}, O_{\text{black}}\}$ | 1 |
| $h^{\text{LM-cut}}(I)$ | | | | 7 |

Example: Computation of LM-Cut

- 1 Compute h^{\max} values of the variables. Stop if $h^{\max}(g) = 0$.



$$\begin{aligned}
 O_{\text{blue}} &= \langle \{i\}, \{a, b\}, \{\}, 0 \rangle \\
 O_{\text{green}} &= \langle \{i\}, \{a, c\}, \{\}, 0 \rangle \\
 O_{\text{black}} &= \langle \{i\}, \{b, c\}, \{\}, 2 \rangle \\
 O_{\text{red}} &= \langle \{b, c\}, \{d\}, \{\}, 0 \rangle \\
 O_{\text{orange}} &= \langle \{a, d\}, \{g\}, \{\}, 0 \rangle
 \end{aligned}$$

| round | $P(O_{\text{orange}})$ | $P(O_{\text{red}})$ | landmark | cost |
|------------------------|------------------------|---------------------|--|------|
| 1 | d | b | $\{O_{\text{red}}\}$ | 2 |
| 2 | a | b | $\{O_{\text{green}}, O_{\text{blue}}\}$ | 4 |
| 3 | d | c | $\{O_{\text{green}}, O_{\text{black}}\}$ | 1 |
| $h^{\text{LM-cut}}(I)$ | | | | |

Properties of LM-Cut Heuristic

Theorem

Let $\langle V, I, O, \gamma \rangle$ be a delete-free STRIPS task in i-g normal form.
The **LM-cut heuristic is admissible**: $h^{\text{LM-cut}}(I) \leq h^*(I)$.

Proof omitted.

If Π is not delete-free, we can compute $h^{\text{LM-cut}}$ on Π^+ .
Then $h^{\text{LM-cut}}$ is bounded by h^+ .

Summary

Summary

- **Cuts** in **justification graphs** are a general method to find disjunctive action landmarks.
- The minimum hitting set over **all cut landmarks** is a **perfect heuristic** for delete-free planning tasks.
- The **LM-cut heuristic** is an admissible heuristic based on these ideas.