Planning and Optimization

F1. Critical Path Heuristics: h^m

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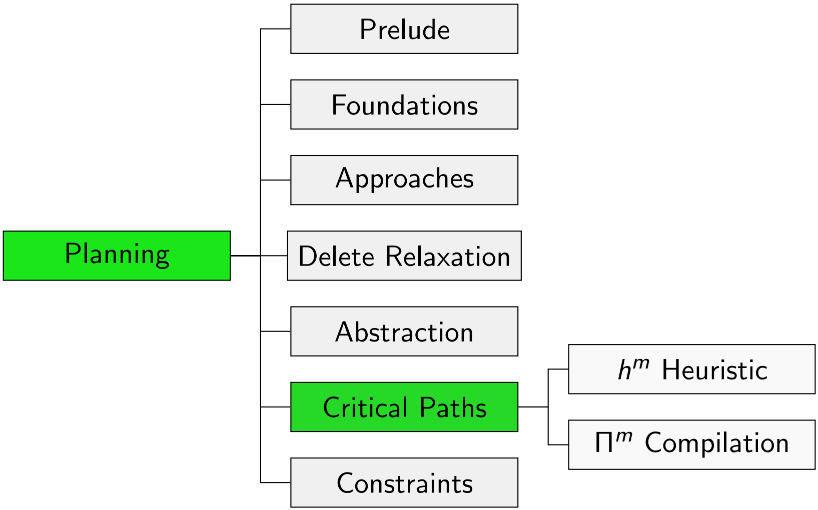
Content of this Course



Set Representation

In This (and the Next) Chapter...

- ... we consider only STRIPS, and ...
- ... we focus on backward search and regression.

Set Representation of STRIPS Planning Tasks

For a more convenient notation, we will use a set representation of STRIPS planning task...

Three differences:

- Represent conjunctions of variables as sets of variables.
- Use two sets to represent add and delete effects of operators separately.
- Represent states as sets of the true variables.

Reminder: STRIPS Operators in Set Representation

- Every STRIPS operator is of the form

$$\langle v_1 \wedge \cdots \wedge v_p, \quad a_1 \wedge \cdots \wedge a_q \wedge \neg d_1 \wedge \cdots \wedge \neg d_r, c\rangle$$

  where v_i, a_j, d_k are state variables and c is the cost.
- The same operator o in set representation is $\langle pre(o), add(o), del(o), cost(o)\rangle$, where
  - $pre(o) = \{v_1, \ldots, v_p\}$ are the preconditions,
  - $add(o) = \{a_1, \ldots, a_q\}$ are the add effects,
  - $del(o) = \{d_1, \ldots, d_r\}$ are the delete effects, and
  - $cost(o) = c$ is the operator cost.
- Since STRIPS operators must be conflict-free, $add(o) \cap del(o) = \emptyset$

STRIPS Planning Tasks in Set Representation

A STRIPS planning task in set representation is given as a tuple $\langle V, I, O, G \rangle$, where

- V is a finite set of state variables,
- $I \subseteq V$ is the initial state,
- O is a finite set of STRIPS operators in set representation,
- $G \subseteq V$ is the goal.

STRIPS Planning Tasks in Set Representation

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- $I \subseteq V$ is the initial state,
- O is a finite set of STRIPS operators in set representation,
- $G \subseteq V$ is the goal.

The corresponding planning task in the previous notation is $\langle V, I', O', \gamma \rangle$, where

- $I'(v) = \mathbf{T}$ iff $v \in I$,
- $O' = \{\langle \bigwedge_{v \in pre(o)} v, \bigwedge_{v \in add(o)} v \wedge \bigwedge_{v \in del(o)} \neg v, cost(o) \rangle \mid o \in O\}$,
- $\gamma = \bigwedge_{v \in G} v$.

Reminder: STRIPS Regression

Definition (STRIPS Regression)

Let $\varphi = \varphi_1 \wedge \cdots \wedge \varphi_n$ be a conjunction of atoms, and let o be a STRIPS operator which adds the atoms $a_1, \dots, a_k$ and deletes the atoms $d_1, \dots, d_l$.

The STRIPS regression of φ with respect to o is

$$sregr(\varphi, o) := \begin{cases} \bot & \text{if } \varphi_i = d_j \text{ for some } i, j \\ pre(o) \wedge \bigwedge(\{\varphi_1, \dots, \varphi_n\} \setminus \{a_1, \dots, a_k\}) & \text{else} \end{cases}$$

Note: $sregr(\varphi, o)$ is again a conjunction of atoms, or $\bot$.

STRIPS Regression in Set Representation

Definition (STRIPS Regression)

Let A be a set of atoms, and let o be a STRIPS operator $o = \langle pre(o), add(o), del(o), cost(o)\rangle$.

The STRIPS regression of A with respect to o is

$$sregr(A, o) := \begin{cases} \bot & \text{if } A \cap del(o) \neq \emptyset \\ pre(o) \cup (A \setminus add(o)) & \text{otherwise} \end{cases}$$

Note: $sregr(A, o)$ is again a set of atoms, or $\bot$.

Perfect Regression Heuristic

Perfect Regression Heuristic

Definition (Perfect Regression Heuristic)

For a STRIPS planning task $\langle V, I, O, G\rangle$ the perfect regression heuristic r^* for state s and variable set $A \subseteq V$ is defined as the (point-wise) greatest fixed-point solution of the equations:

$$
\begin{aligned}
r^*(s, A) &= 0 && \text{if } A \subseteq s \\
r^*(s, A) &= \min_{(B,o)\in R(A,O)} [cost(o) + r^*(s, B)] && \text{otherwise}
\end{aligned}
$$

$$R(A, O) = \{(B, o) \mid o \in O, B = sregr(A, o) \neq \bot\}$$

Perfect Regression Heuristic r^* vs. Perfect Heuristic h^*

Theorem

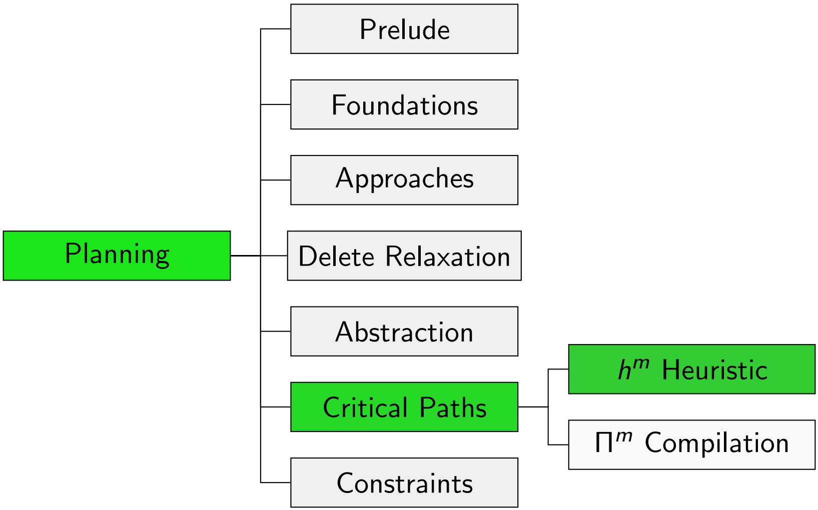
For a STRIPS planning task $\langle V, I, O, G\rangle$ it holds for each state s that $h^*(s) = r^*(s, G)$.

Intuition: We can extract a path from the operators in the minimizing pairs (B, o), starting from the goal.

$\rightsquigarrow$ r^* cannot be computed efficiently.

Critical Path Heuristics

Content of this Course



Running Example

We will use the following running example throughout this chapter: $\Pi = \langle V, I, \{o_1, o_2, o_3\}, G\rangle$ with

$$\begin{aligned}
V &= \{a, b, c\} \\
I &= \{a\} \\
o_1 &= \langle\{a, b\}, \{c\}, \{b\}, 1\rangle \\
o_2 &= \langle\{a\}, \{b\}, \{a\}, 2\rangle \\
o_3 &= \langle\{b\}, \{a\}, \emptyset, 2\rangle \\
G &= \{a, b, c\}
\end{aligned}$$

Optimal plan o_2, o_3, o_1, o_2, o_3 has cost 9.

Simplified Relaxed Task Graph

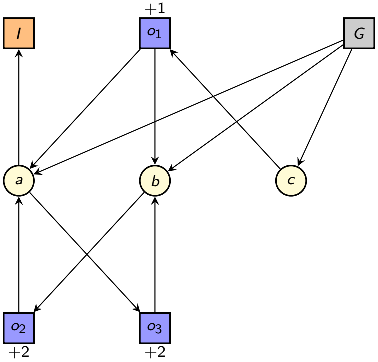
Definition

For a STRIPS planning task $\Pi = \langle V, I, O, \gamma \rangle$, the simplified relaxed task graph $sRTG(\Pi^+)$ is the AND/OR graph $\langle N_{\text{and}} \cup N_{\text{or}}, A, type \rangle$ with

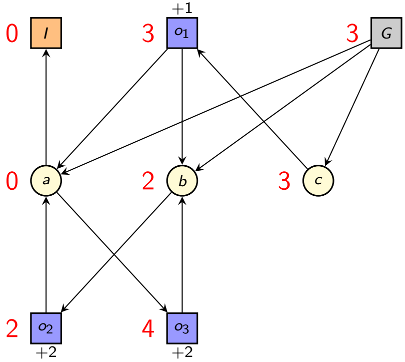
- $N_{\text{and}} = \{n_o \mid o \in O\} \cup \{v_I, v_G\}$ with $type(n) = \wedge$ for all $n \in N_{\text{and}}$,
- $N_{\text{or}} = \{n_v \mid v \in V\}$ with $type(n) = \vee$ for all $n \in N_{\text{or}}$, and
- $A = \{\langle n_a, n_o \rangle \mid o \in O, a \in add(o)\} \cup \{\langle n_o, n_p \rangle \mid o \in O, p \in pre(o)\} \cup \{\langle n_v, n_I \rangle \mid v \in I\} \cup \{\langle n_G, n_v \rangle \mid v \in \gamma\}$

Like RTG but without extra nodes to support arbitrary conditions.

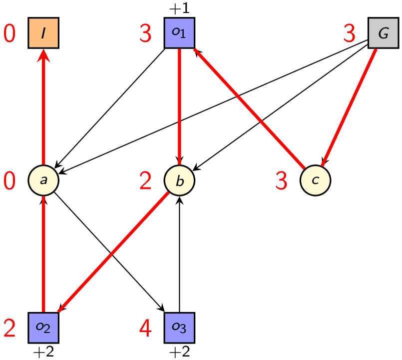
$h^{\max}$ in Simplified RTG



$h^{\max}$ in Simplified RTG



$h^{\max}$ in Simplified RTG



The critical path justifies the heuristic estimate $h^{\max}(I) = 3$

$h^{\max}$ as Critical Path Heuristic

Definition ($h^{\max}$ Heuristic)

For a STRIPS planning task $\langle V, I, O, G\rangle$ the heuristic $h^{\max}$ for state s and variable set $A \subseteq V$ is defined as the (point-wise) greatest fixed-point solution of $h^{\max}(s, A) =$

$$\begin{cases} 0 & \text{if } A \subseteq s \\ \min_{\langle B,o\rangle \in R(A,O)} [cost(o) + h^{\max}(s, B)] & \text{if } |A| \leq 1 \text{ and } A \not\subseteq s \\ \max_{v \in A} h^{\max}(s, \{v\}) & \text{otherwise} \end{cases}$$

$$R(A, O) = \{\langle B, o\rangle \mid o \in O, B = sregr(A, o) \neq \bot\}$$

Estimate $r^*(s, A)$ as cost of most expensive $v \in A$.

For STRIPS tasks, this definition specifies the same heuristic $h^{\max}$ as in the chapter on relaxation heuristics.

Critical Path Heuristics

Definition (h^m Heuristics)

For a STRIPS planning task $\langle V, I, O, G\rangle$ and $m \in \mathbb{N}_1$ the heuristic h^m for state s and variable set $A \subseteq V$ is defined as the (point-wise) greatest fixed-point solution of

$$h^m(s, A) = \begin{cases} 0 & \text{if } A \subseteq s \\ \min_{\langle B,o\rangle \in R(A,O)}[cost(o) + h^m(s, B)] & \text{if } |A| \leq m \text{ and } A \not\subseteq s \\ \max_{B \subseteq A, 1 \leq |B| \leq m} h^m(s, B) & \text{otherwise} \end{cases}$$

$$R(A, O) = \{\langle B, o\rangle \mid o \in O, B = sregr(A, o) \neq \bot\}$$

Estimate $r^*(s, A)$ as cost of most expensive $B \subseteq A$ with $|B| \leq m$.

Computation

Critical Path Heuristics: Computation

Definition (h^m Heuristics)

For a STRIPS planning task $\langle V, I, O, G\rangle$ and $m \in \mathbb{N}_1$ the heuristic h^m for state s and variable set $A \subseteq V$ is defined as the (point-wise) greatest fixed-point solution of

$$h^m(s, A) = \begin{cases} 0 & \text{if } A \subseteq s \\ \min_{\langle B,o\rangle \in R(A,O)} [cost(o) + h^m(s, B)] & \text{if } |A| \leq m \text{ and } A \not\subseteq s \\ \max_{B \subseteq A, 1 \leq |B| \leq m} h^m(s, B) & \text{otherwise} \end{cases}$$

$$R(A, O) = \{\langle B, o\rangle \mid o \in O, B = sregr(A, o) \neq \bot\}$$

Cheap to evaluate given $h^m(s, B)$ for all $B \subseteq V$ with $1 \leq |B| \leq m$.
We precompute these values.

h^m Precomputation (1)

For value m and state s of task with variables V and operators O

Computing h^m Values for Variable Sets up to Size m

$S := \{A \subseteq V \mid |A| \leq m\}$
Associate a *cost* attribute with each set $A \in S$.
for all sets $A \in S$:
  if $A \subseteq s$ **then** $A.cost := 0$
  else $A.cost := \infty$
while no fixed point is reached:
  Choose a variable set A from S.
  $newcost := \min_{\langle B,o\rangle \in R(A,O)}[cost(o) + currentcost(B, S)]$
  if $newcost < A.cost$ **then** $A.cost := newcost$

currentcost(B,S)

if $|B| \leq m$ **then return** $B.cost$ **else return** $\max_{A \in S, A \subseteq B} A.cost$

h^m Precomputation (2)

- Fixed point reached $\Rightarrow A.cost = h^m(s, A)$ for all $A \in S$.
- Intuition:
  - cost values satisfy h^m equations, and
  - no larger values can satisfy the equations: initialized to ∞ and values are only reduced if it is otherwise impossible to satisfy an equation.

h^m Precomputation (2)

- Fixed point reached $\Rightarrow$ $A.cost = h^m(s, A)$ for all $A \in S$.
- Intuition:
  - cost values satisfy h^m equations, and
  - no larger values can satisfy the equations: initialized to ∞ and values are only reduced if it is otherwise impossible to satisfy an equation.
- With suitable data structures, we can choose A in each iteration so that it directly gets assigned its final value (Generalized Dijkstra's algorithm).
- With such a strategy, the runtime is polynomial for fixed m.
- Runtime is exponential in m $\rightsquigarrow$ h^m typically used with $m \leq 3$

Example with $m = 1$ to Initial State

$$R(\{a\}, \{o_1, o_2, o_3\}) = \{(\{a, b\}, o_1), (\{b\}, o_3)\}$$
$$R(\{b\}, \{o_1, o_2, o_3\}) = \{(\{a\}, o_2), (\{b\}, o_3)\}$$
$$R(\{c\}, \{o_1, o_2, o_3\}) = \{(\{a, b\}, o_1), (\{a, c\}, o_2), (\{b, c\}, o_3)\}$$

Example with $m = 1$ to Initial State

$$R(\{a\}, \{o_1, o_2, o_3\}) = \{(\{a, b\}, o_1), (\{b\}, o_3)\}$$
$$R(\{b\}, \{o_1, o_2, o_3\}) = \{(\{a\}, o_2), (\{b\}, o_3)\}$$
$$R(\{c\}, \{o_1, o_2, o_3\}) = \{(\{a, b\}, o_1), (\{a, c\}, o_2), (\{b, c\}, o_3)\}$$

	$\{a\}$	$\{b\}$	$\{c\}$
cost	0	∞	∞

$\{b\}$: $\min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2$

Example with $m = 1$ to Initial State

$$R(\{a\}, \{o_1, o_2, o_3\}) = \{(\{a, b\}, o_1), (\{b\}, o_3)\}$$
$$R(\{b\}, \{o_1, o_2, o_3\}) = \{(\{a\}, o_2), (\{b\}, o_3)\}$$
$$R(\{c\}, \{o_1, o_2, o_3\}) = \{(\{a, b\}, o_1), (\{a, c\}, o_2), (\{b, c\}, o_3)\}$$

	$\{a\}$	$\{b\}$	$\{c\}$
cost	0	2	∞

$\{b\}$: $\min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2$

$\{c\}$: $\min\{1 + \max\{\{a\}.cost, \{b\}.cost\},$
$2 + \max\{\{a\}.cost, \{c\}.cost\},$
$2 + \max\{\{b\}.cost, \{c\}.cost\}\} = 3$

Example with $m = 1$ to Initial State

$$R(\{a\}, \{o_1, o_2, o_3\}) = \{(\{a, b\}, o_1), (\{b\}, o_3)\}$$
$$R(\{b\}, \{o_1, o_2, o_3\}) = \{(\{a\}, o_2), (\{b\}, o_3)\}$$
$$R(\{c\}, \{o_1, o_2, o_3\}) = \{(\{a, b\}, o_1), (\{a, c\}, o_2), (\{b, c\}, o_3)\}$$

	$\{a\}$	$\{b\}$	$\{c\}$
cost	0	2	3

$\{b\}$: $\min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2$

$\{c\}$: $\min\{1 + \max\{\{a\}.cost, \{b\}.cost\},$
$2 + \max\{\{a\}.cost, \{c\}.cost\},$
$2 + \max\{\{b\}.cost, \{c\}.cost\}\} = 3$

Example with $m = 1$ to Initial State

	$\{a\}$	$\{b\}$	$\{c\}$
cost	0	2	3

$\{b\}$: $\min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2$

$\{c\}$: $\min\{1 + \max\{\{a\}.cost, \{b\}.cost\},$
$2 + \max\{\{a\}.cost, \{c\}.cost\},$
$2 + \max\{\{b\}.cost, \{c\}.cost\}\} = 3$

Fixed point reached

Example with $m = 1$ to Initial State

	$\{a\}$	$\{b\}$	$\{c\}$
cost	0	2	3

$$
\begin{aligned}
\{b\}: \quad & \min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2 \\
\{c\}: \quad & \min\{1 + \max\{\{a\}.cost, \{b\}.cost\}, \\
& \quad 2 + \max\{\{a\}.cost, \{c\}.cost\}, \\
& \quad 2 + \max\{\{b\}.cost, \{c\}.cost\}\} = 3
\end{aligned}
$$

Fixed point reached

$$
\begin{aligned}
h^1(I, \{a, b, c\}) &= \max\{h^1(I, \{a\}), h^1(I, \{b\}), h^1(I, \{c\})\} \\
&= \max\{0, 2, 3\} = 3
\end{aligned}
$$

Example with $m = 2$ to Initial State

	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$cost$	0	∞	∞	∞	∞	∞

$\{b\}$: $\min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2$

Example with $m = 2$ to Initial State

	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
cost	0	2	∞	∞	∞	∞

$$\begin{aligned} \{b\}: &\quad \min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2 \\ \{a, b\}: &\quad \min\{2 + \{b\}.cost\} = 4 \end{aligned}$$

Example with $m = 2$ to Initial State

	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
cost	0	2	∞	4	∞	∞

$\{b\}$: $\min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2$

$\{a, b\}$: $\min\{2 + \{b\}.cost\} = 4$

$\{c\}$: $\min\{1 + \{a, b\}.cost, 2 + \{a, c\}.cost, 2 + \{b, c\}.cost\} = 5$

Example with $m = 2$ to Initial State

	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
cost	0	2	5	4	∞	∞

$$\{b\}: \quad \min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2$$
$$\{a, b\}: \quad \min\{2 + \{b\}.cost\} = 4$$
$$\{c\}: \quad \min\{1 + \{a, b\}.cost, 2 + \{a, c\}.cost, 2 + \{b, c\}.cost\} = 5$$
$$\{a, c\}: \quad \min\{1 + \{a, b\}.cost, 2 + \{b, c\}.cost\} = 5$$

Example with $m = 2$ to Initial State

	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
cost	0	2	5	4	5	∞

$\{b\}$: $\min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2$

$\{a, b\}$: $\min\{2 + \{b\}.cost\} = 4$

$\{c\}$: $\min\{1 + \{a, b\}.cost, 2 + \{a, c\}.cost, 2 + \{b, c\}.cost\} = 5$

$\{a, c\}$: $\min\{1 + \{a, b\}.cost, 2 + \{b, c\}.cost\} = 5$

$\{b, c\}$: $\min\{2 + \{a, c\}.cost, 2 + \{b, c\}.cost\} = 7$

Example with $m = 2$ to Initial State

	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
cost	0	2	5	4	5	7

$$\begin{aligned}
\{b\}: &\quad \min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2 \\
\{a, b\}: &\quad \min\{2 + \{b\}.cost\} = 4 \\
\{c\}: &\quad \min\{1 + \{a, b\}.cost, 2 + \{a, c\}.cost, 2 + \{b, c\}.cost\} = 5 \\
\{a, c\}: &\quad \min\{1 + \{a, b\}.cost, 2 + \{b, c\}.cost\} = 5 \\
\{b, c\}: &\quad \min\{2 + \{a, c\}.cost, 2 + \{b, c\}.cost\} = 7
\end{aligned}$$

$$\begin{aligned}
h^2(I, \{a, b, c\}) = \max\{& h^2(I, \{a\}), h^2(I, \{b\}), h^2(I, \{c\}) \\
& h^2(I, \{a, b\}), h^2(I, \{a, c\}), h^2(I, \{b, c\})\} \\
= \max\{& 0, 2, 5, 4, 5, 7\} = 7
\end{aligned}$$

Summary

Summary

- Critical path heuristic h^m estimates the cost of reaching a set ($\hat{=}$ conjunction) of variables as the cost of reaching the most expensive subset of size at most m.
- h^m computation is polynomial for fixed m.
- h^m computation is exponential in m.
- In practice, we use $m \in \{1, 2, 3\}$.