

Planning and Optimization

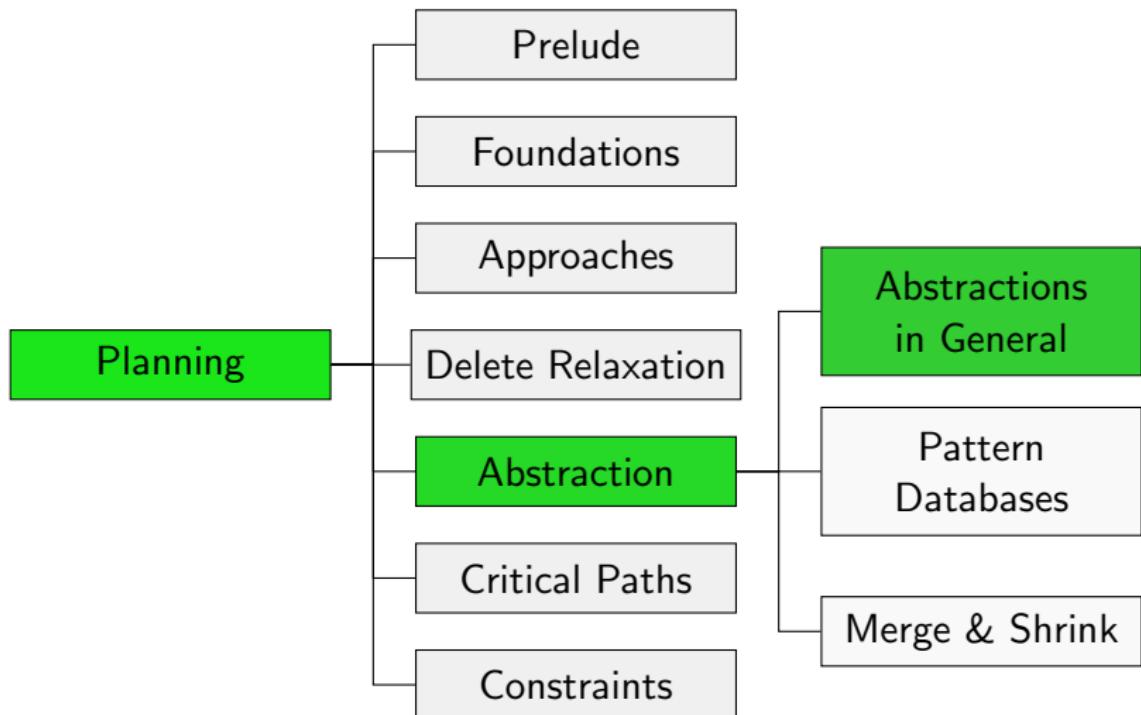
E3. Abstractions: Introduction

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Content of this Course



Introduction

Coming Up with Heuristics in a Principled Way

General Procedure for Obtaining a Heuristic

Solve a simplified version of the problem.

Major ideas for heuristics in the planning literature:

- delete relaxation
- **abstraction**
- critical paths
- landmarks
- network flows
- potential heuristics

Heuristics based on **abstraction** are among the most prominent techniques for **optimal planning**.

Abstracting a Transition System

Abstracting a transition system means **dropping some distinctions** between states, while **preserving the transition behaviour** as much as possible.

- An abstraction of a transition system \mathcal{T} is defined by an **abstraction mapping α** that defines which states of \mathcal{T} should be distinguished and which ones should not.
- From \mathcal{T} and α , we compute an **abstract transition system \mathcal{T}^α** which is similar to \mathcal{T} , but smaller.
- The **abstract goal distances** (goal distances in \mathcal{T}^α) are used as heuristic estimates for goal distances in \mathcal{T} .

Abstracting a Transition System: Example

example from domain-specific heuristic search:

Example (15-Puzzle)

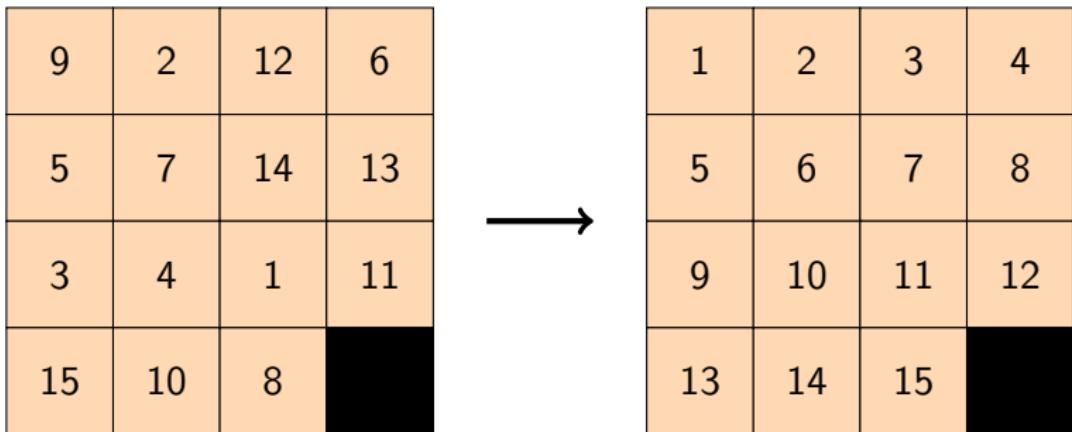
A **15-puzzle** state is given by a permutation $\langle b, t_1, \dots, t_{15} \rangle$ of $\{1, \dots, 16\}$, where b denotes the blank position and the other components denote the positions of the 15 tiles.

One possible **abstraction mapping** ignores the precise location of tiles 8–15, i.e., two states are distinguished iff they differ in the position of the blank or one of the tiles 1–7:

$$\alpha(\langle b, t_1, \dots, t_{15} \rangle) = \langle b, t_1, \dots, t_7 \rangle$$

The heuristic values for this abstraction correspond to the cost of moving tiles 1–7 to their goal positions.

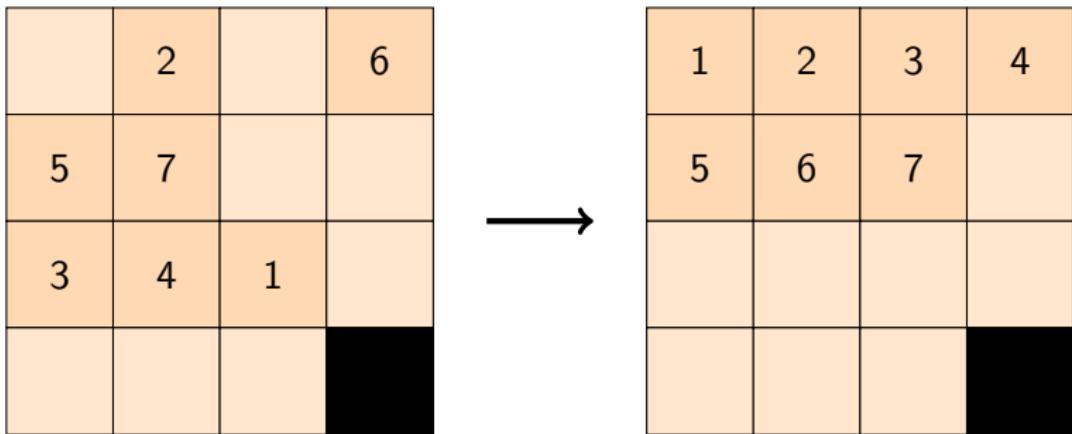
Abstraction Example: 15-Puzzle



real state space:

- $16! = 20922789888000 \approx 2 \cdot 10^{13}$ states
- $\frac{16!}{2} = 10461394944000 \approx 10^{13}$ reachable states

Abstraction Example: 15-Puzzle



abstract state space:

- $16 \cdot 15 \cdot \dots \cdot 9 = 518918400 \approx 5 \cdot 10^8$ states
- $16 \cdot 15 \cdot \dots \cdot 9 = 518918400 \approx 5 \cdot 10^8$ reachable states

Computing the Abstract Transition System

Given \mathcal{T} and α , how do we compute \mathcal{T}^α ?

Requirement

We want to obtain an **admissible heuristic**.

Hence, $h^*(\alpha(s))$ (in the abstract state space \mathcal{T}^α) should never overestimate $h^*(s)$ (in the concrete state space \mathcal{T}).

An easy way to achieve this is to ensure that **all solutions in \mathcal{T} are also present in \mathcal{T}^α** :

- If s is a goal state in \mathcal{T} , then $\alpha(s)$ is a goal state in \mathcal{T}^α .
- If \mathcal{T} has a transition from s to t , then \mathcal{T}^α has a transition from $\alpha(s)$ to $\alpha(t)$ with the same cost.

Computing the Abstract Transition System: Example

Example (15-Puzzle)

In the running example:

- \mathcal{T} has the unique goal state $\langle 16, 1, 2, \dots, 15 \rangle$.
 - ~~~ \mathcal{T}^α has the unique goal state $\langle 16, 1, 2, \dots, 7 \rangle$.
- Let x and y be neighbouring positions in the 4×4 grid.
 \mathcal{T} has a transition from $\langle x, t_1, \dots, t_{i-1}, y, t_{i+1}, \dots, t_{15} \rangle$ to $\langle y, t_1, \dots, t_{i-1}, x, t_{i+1}, \dots, t_{15} \rangle$ for all $i \in \{1, \dots, 15\}$.
 - ~~~ \mathcal{T}^α has a transition from $\langle x, t_1, \dots, t_{i-1}, y, t_{i+1}, \dots, t_7 \rangle$ to $\langle y, t_1, \dots, t_{i-1}, x, t_{i+1}, \dots, t_7 \rangle$ for all $i \in \{1, \dots, 7\}$.
 - ~~~ Moreover, \mathcal{T}^α has a transition from $\langle x, t_1, \dots, t_7 \rangle$ to $\langle y, t_1, \dots, t_7 \rangle$ if $y \notin \{t_1, \dots, t_7\}$.

Practical Requirements

Practical Requirements for Abstractions

To be useful in practice, an abstraction heuristic must be efficiently computable. This gives us two requirements for α :

- For a given state s , the **abstract state** $\alpha(s)$ must be efficiently computable.
- For a given abstract state $\alpha(s)$, the **abstract goal distance** $h^*(\alpha(s))$ must be efficiently computable.

There are a number of ways of achieving these requirements:

- **pattern database heuristics** (Culberson & Schaeffer, 1996)
- domain abstractions (Hernádvölgyi and Holte, 2000)
- **merge-and-shrink abstractions** (Dräger, Finkbeiner & Podelski, 2006)
- Cartesian abstractions (Ball, Podelski & Rajamani, 2001)
- structural patterns (Katz & Domshlak, 2008)

Practical Requirements for Abstractions: Example

Example (15-Puzzle)

In our running example, α can be very efficiently computed: just project the given 16-tuple to its first 8 components.

To compute abstract goal distances efficiently during search, the most common approach is to precompute **all abstract goal distances** prior to search by performing a backward uniform-cost search from the abstract goal state(s). These distances are then stored in a table (requires ≈ 495 MiB RAM).

During search, computing $h^*(\alpha(s))$ is just a table lookup.

This heuristic is an example of a **pattern database heuristic**.

Multiple Abstractions

Multiple Abstractions

- One important practical question is how to come up with a suitable abstraction mapping α .
- Indeed, there is usually a **huge number of possibilities**, and it is important to pick good abstractions (i.e., ones that lead to informative heuristics).
- However, it is generally **not necessary to commit to a single abstraction**.

Combining Multiple Abstractions

Maximizing several abstractions:

- Each abstraction mapping gives rise to an admissible heuristic.
- By computing the **maximum** of several admissible heuristics, we obtain another admissible heuristic which **dominates** the component heuristics.
- Thus, we can always compute several abstractions and maximize over the individual abstract goal distances.

Adding several abstractions:

- In some cases, we can even compute the **sum** of individual estimates and still stay admissible.
- Summation often leads to **much higher estimates** than maximization, so it is important to understand **under which conditions** summation of heuristics is **admissible**.

Maximizing Several Abstractions: Example

Example (15-Puzzle)

- mapping to tiles 1–7 was arbitrary
~~ can use **any subset** of tiles
- with the same amount of memory required for the tables for the mapping to tiles 1–7, we could store the tables for **nine different abstractions** to six tiles and the blank
- use **maximum** of individual estimates

Adding Several Abstractions: Example

9	2	12	6
5	7	14	13
3	4	1	11
15	10	8	

9	2	12	6
5	7	14	13
3	4	1	11
15	10	8	

- **1st abstraction:** ignore precise location of 8–15
- **2nd abstraction:** ignore precise location of 1–7

↝ Is the **sum** of the abstraction heuristics **admissible**?

Adding Several Abstractions: Example

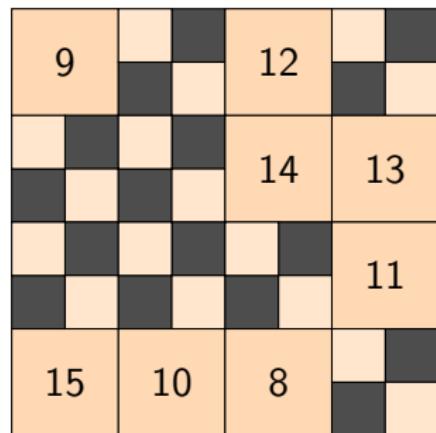
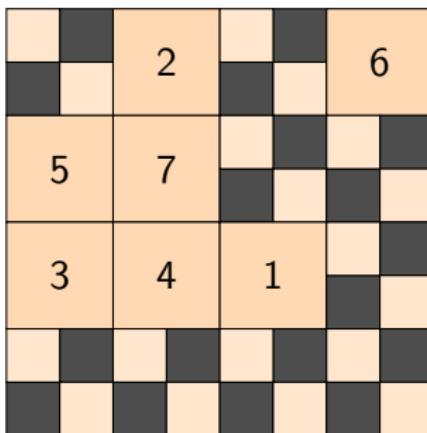
	2		6
5	7		
3	4	1	

9		12	
		14	13
			11
15	10	8	

- **1st abstraction:** ignore precise location of 8–15
- **2nd abstraction:** ignore precise location of 1–7

↝ The **sum** of the abstraction heuristics is **not admissible**.

Adding Several Abstractions: Example



- 1st abstraction: ignore precise location of 8–15 and blank
- 2nd abstraction: ignore precise location of 1–7 and blank

↝ The **sum** of the abstraction heuristics is **admissible**.

Outlook

Our Plan for the Next Lectures

In the following, we take a deeper look at abstractions and their use for admissible heuristics.

In the next two chapters, we **formally introduce** abstractions and abstraction heuristics and study some of their most important properties.

Afterwards, we discuss some particular classes of abstraction heuristics in detail, namely

- **pattern database heuristics** and
- **merge-and-shrink abstractions.**

Summary

Summary

- **Abstraction** is one of the principled ways of deriving heuristics for planning tasks and transition systems in general.
- The key idea is to map states to a smaller **abstract transition system** \mathcal{T}^α by means of an **abstraction function** α .
- Goal distances in \mathcal{T}^α are then used as **admissible estimates** for goal distances in the original transition system.
- To be **practical**, we must be able to **compute abstraction functions** and **determine abstract goal distances** efficiently.
- Often, **multiple abstractions** are used.
They can always be **maximized** admissibly.
- **Adding** abstraction heuristics is not always admissible.
When it is, it leads to a stronger heuristic than maximizing.