# Planning and Optimization

## E2. Invariants and Mutexes


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E2.1 Invariants

E2.2 Computing Invariants

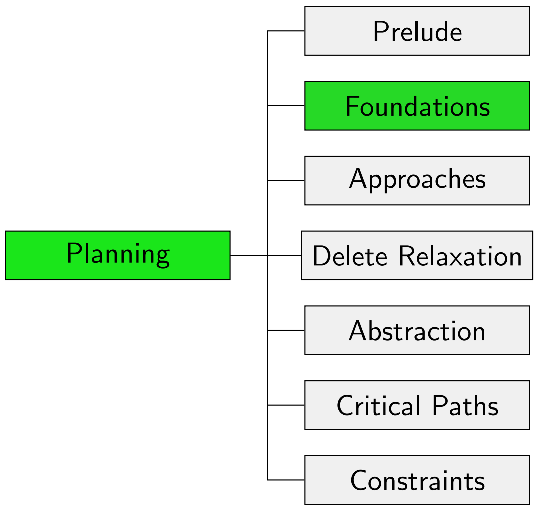
E2.3 Mutexes

E2.4 Reformulation

E2.5 Summary




# Content of this Course








# E2.1 Invariants

## Invariants

- When we as humans reason about planning tasks, we implicitly make use of "obvious" properties of these tasks.
  - Example: we are never in two places at the same time
- We can represent such properties as logical formulas $\varphi$ that are true in all reachable states.
  - Example: $\varphi = \neg(\textit{at-uni} \land \textit{at-home})$
- Such formulas are called invariants of the task.





## Invariants: Definition

**Definition (Invariant)**

An invariant of a planning task $\Pi$ with state variables $V$ is a logical formula $\varphi$ over $V$ such that $s \models \varphi$ for all reachable states $s$ of $\Pi$.





# E2.2 Computing Invariants





## Computing Invariants

How does an automated planner come up with invariants?

- Theoretically, testing if a formula $\varphi$ is an invariant is as hard as planning itself.
  $\rightsquigarrow$ proof idea: a planning task is unsolvable iff the negation of its goal is an invariant
- Still, many practical invariant synthesis algorithms exist.
- To remain efficient (= polynomial-time), these algorithms only compute a subset of all useful invariants.
  $\rightsquigarrow$ sound, but not complete
- Empirically, they tend to at least find the "obvious" invariants of a planning task.

## Invariant Synthesis Algorithms

Most algorithms for generating invariants are based on the generate-test-repair approach:

- Generate: Suggest some invariant candidates, e.g., by enumerating all possible formulas $\varphi$ of a certain size.
- Test: Try to prove that $\varphi$ is indeed an invariant. Usually done inductively:
  1. Test that initial state satisfies $\varphi$.
  2. Test that if $\varphi$ is true in the current state, it remains true after applying a single operator.
- Repair: If invariant test fails, replace candidate $\varphi$ by a weaker formula, ideally exploiting why the proof failed.





## Invariant Synthesis: References

We will not cover invariant synthesis algorithms in this course.

Literature on invariant synthesis:

- DISCOPLAN (Gerevini & Schubert, 1998)
- TIM (Fox & Long, 1998)
- Edelkamp & Helmert's algorithm (1999)
- Bonet & Geffner's algorithm (2001)
- Rintanen's algorithm (2008)
- Rintanen's algorithm for schematic invariants (2017)





## Exploiting Invariants

Invariants have many uses in planning:

- Regression search (C2–C3): Prune subgoals that violate (are inconsistent with) invariants.
- Planning as satisfiability (C4–C5): Add invariants to a SAT encoding of a planning task to get tighter constraints.
- Proving unsolvability: If $\varphi$ is an invariant such that $\varphi \land \gamma$ is unsatisfiable, the planning task with goal $\gamma$ is unsolvable.
- Finite-Domain Reformulation: Derive a more compact FDR representation (equivalent, but with fewer states) from a given propositional planning task.

We now discuss the last point because it connects to our discussion of propositional vs. FDR planning tasks.



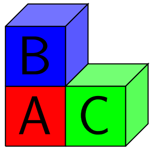


# E2.3 Mutexes

# Reminder: Blocks World (Propositional Variables)

Example

$s(A\text{-}on\text{-}B) = \mathbf{F}$
$s(A\text{-}on\text{-}C) = \mathbf{F}$
$s(A\text{-}on\text{-}table) = \mathbf{T}$
$s(B\text{-}on\text{-}A) = \mathbf{T}$
$s(B\text{-}on\text{-}C) = \mathbf{F}$
$s(B\text{-}on\text{-}table) = \mathbf{F}$
$s(C\text{-}on\text{-}A) = \mathbf{F}$
$s(C\text{-}on\text{-}B) = \mathbf{F}$
$s(C\text{-}on\text{-}table) = \mathbf{T}$




$\rightsquigarrow 2^9 = 512$ states

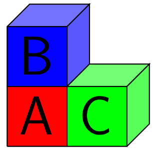




# Reminder: Blocks World (Finite-Domain Variables)

Example
Use three finite-domain state variables:
- *below-a*: $\{\text{b}, \text{c}, \text{table}\}$
- *below-b*: $\{\text{a}, \text{c}, \text{table}\}$
- *below-c*: $\{\text{a}, \text{b}, \text{table}\}$

$s(below\text{-}a) = \text{table}$
$s(below\text{-}b) = \text{a}$
$s(below\text{-}c) = \text{table}$




$\rightsquigarrow 3^3 = 27$ states





# Task Reformulation

- Common modeling languages (like PDDL) often give us propositional tasks.
- More compact FDR tasks are often desirable.
- Can we do an automatic reformulation?





# Mutexes

Invariants that take the form of binary clauses are called mutexes because they express that certain variable assignments cannot be simultaneously true (are mutually exclusive).

Example (Blocks World)
The invariant $\neg A\text{-}on\text{-}B \vee \neg A\text{-}on\text{-}C$ states that *A-on-B* and *A-on-C* are mutex.

We say that a set of literals is a mutex group if every subset of two literals is a mutex.

Example (Blocks World)
$\{A\text{-}on\text{-}B, A\text{-}on\text{-}C, A\text{-}on\text{-}table\}$ is a mutex group.

## Encoding Mutex Groups as Finite-Domain Variables

Let $G = \{\ell_1, \dots, \ell_n\}$ be a mutex group over $n$ different propositional state variables $V_G = \{v_1, \dots, v_n\}$.

Then a single finite-domain state variable $v_G$ with $\mathrm{dom}(v_G) = \{\ell_1, \dots, \ell_n, \mathrm{none}\}$ can replace the $n$ variables $V_G$:

- $s(v_G) = \ell_i$ represents situations where (exactly) $\ell_i$ is true
- $s(v_G) = \mathrm{none}$ represents situations where all $\ell_i$ are false

Note: We can omit the "none" value if $\ell_1 \vee \dots \vee \ell_n$ is an invariant.





## Mutex Covers

**Definition (Mutex Cover)**

A mutex cover for a propositional planning task $\Pi$ is a set of mutex groups $\{G_1, \dots, G_n\}$ where each variable of $\Pi$ occurs in exactly one group $G_i$.

A mutex cover is positive if all literals in all groups are positive.

Note: always exists (use trivial group $\{v\}$ if $v$ otherwise uncovered)





## Positive Mutex Covers

In the following, we stick to positive mutex covers for simplicity.

If we have $\neg v$ in $G$ for some group $G$ in the cover, we can reformulate the task to use an "opposite" variable $\hat{v}$ instead, as in the conversion to positive normal form (Chapter B5).





# E2.4 Reformulation

## Mutex-Based Reformulation of Propositional Tasks

Given a conflict-free propositional planning task $\Pi$ with positive mutex cover $\{G_1, \dots, G_n\}$:

- In all conditions where variable $v \in G_i$ occurs, replace $v$ with $v_{G_i} = v$.
- In all effects $e$ where variable $v \in G_i$ occurs,
  - Replace all atomic add effects $v$ with $v_{G_i} := v$
  - Replace all atomic delete effects $\neg v$ with $(v_{G_i} = v \land \neg \bigvee_{v' \in G_i \setminus \{v\}} \mathit{effcond}(v', e)) \rhd v_{G_i} := \text{none}$

This results in an FDR planning task $\Pi'$ that is equivalent to $\Pi$ (without proof).

Note: the conditional effects encoding delete effects can often be simplified away to an unconditional or empty effect.





## And Back?

- It can also be useful to reformulate an FDR task into a propositional task.
- For example, we might want positive normal form, which requires a propositional task.
- Key idea: each variable/value combination $v = d$ becomes a separate propositional state variable $\langle v, d\rangle$





## Converting FDR Tasks into Propositional Tasks

**Definition (Induced Propositional Planning Task)**

Let $\Pi = \langle V, I, O, \gamma\rangle$ be a conflict-free FDR planning task.
The induced propositional planning task $\Pi'$ is the propositional planning task $\Pi' = \langle V', I', O', \gamma'\rangle$, where

- $V' = \{\langle v, d\rangle \mid v \in V, d \in \text{dom}(v)\}$
- $I'(\langle v, d\rangle) = \mathbf{T}$ iff $I(v) = d$
- $O'$ and $\gamma'$ are obtained from $O$ and $\gamma$ by
  - replacing each atomic formula $v = d$ by the proposition $\langle v, d\rangle$
  - replacing each atomic effect $v := d$ by the effect $\langle v, d\rangle \land \bigwedge_{d' \in \text{dom}(v) \setminus \{d\}} \neg\langle v, d'\rangle$.

Notes:

- Again, simplifications are often possible to avoid introducing so many delete effects.
- $\text{SAS}^+$ tasks induce STRIPS tasks.





# E2.5 Summary

## Summary (1)

- Invariants are common properties of all reachable states, expressed as formulas.
- A number of algorithms for computing invariants exist.
- These algorithms will not find all useful invariants (which is too hard), but try to find some useful subset with reasonable (polynomial) computational effort.





## Summary (2)

- Mutexes are invariants that express that certain literals are mutually exclusive.
- Mutex covers provide a way to convert a set of propositional state variables into a potentially much smaller set of finite-domain state variables.
- Using mutex covers, we can reformulate propositional tasks as more compact FDR tasks.
- Conversely, we can reformulate FDR tasks as propositional tasks by introducing propositions for each variable/value pair.