Planning and Optimization

E1. Planning Tasks in Finite-Domain Representation

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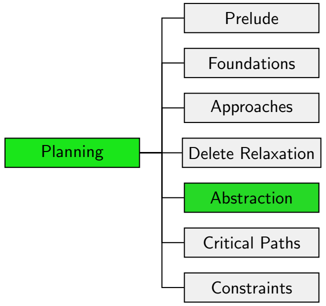
How We Continue

- The next class of heuristics we will consider are abstraction heuristics.



Planning 🟩

- Prelude
- Foundations
- Approaches
- Delete Relaxation
- Abstraction 🟩
- Critical Paths
- Constraints

- However, this requires some preparations.

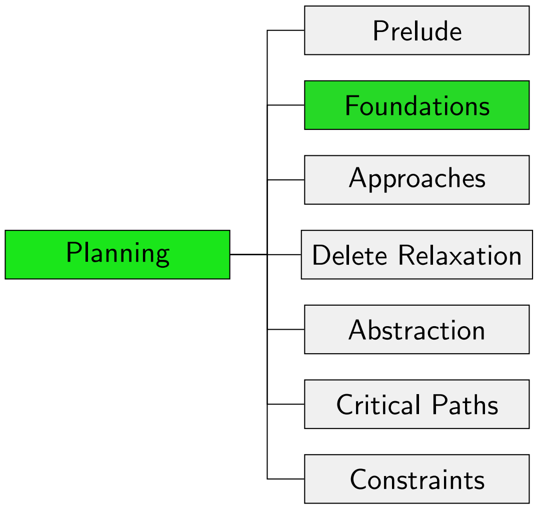
Back to Foundations: Finite-Domain Representation

- Abstraction heuristics benefit from a more compact task representation, called finite-domain representation.
- To understand the relationship to the propositional task representation, we need to know a special kind of invariants, namely mutexes.

⇝ We first get to know finite-domain representation (this chapter) and then speak about invariants and transformations between the representations (next chapter).

⇝ not specific to abstraction heuristics, but general foundations

Content of this Course



E1.1 Finite-Domain Representation

Finite-Domain State Variables

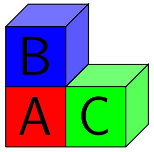
- So far, we used propositional (Boolean) state variables.
  ⇝ possible values **T** and **F**
- We now consider finite-domain variables.
  ⇝ every variable has a finite set of possible values
- A state is still an assignment to the state variables.

Example: $O(n^2)$ Boolean variables or $O(n)$ finite-domain variables with domain size $O(n)$ suffice for blocks world with n blocks.

Blocks World State with Propositional Variables

Example

$$\begin{aligned}
s(\textit{A-on-B}) &= \mathbf{F}\\
s(\textit{A-on-C}) &= \mathbf{F}\\
s(\textit{A-on-table}) &= \mathbf{T}\\
s(\textit{B-on-A}) &= \mathbf{T}\\
s(\textit{B-on-C}) &= \mathbf{F}\\
s(\textit{B-on-table}) &= \mathbf{F}\\
s(\textit{C-on-A}) &= \mathbf{F}\\
s(\textit{C-on-B}) &= \mathbf{F}\\
s(\textit{C-on-table}) &= \mathbf{T}
\end{aligned}$$



$\rightsquigarrow 2^9 = 512$ states

Note: it may be useful to add auxiliary state variables like *A-clear*.

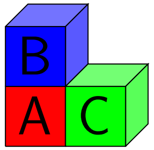
Blocks World State with Finite-Domain Variables

Example

Use three finite-domain state variables:

- *below-a*: $\{\mathrm{b}, \mathrm{c}, \mathrm{table}\}$
- *below-b*: $\{\mathrm{a}, \mathrm{c}, \mathrm{table}\}$
- *below-c*: $\{\mathrm{a}, \mathrm{b}, \mathrm{table}\}$

$$s(\mathit{below\text{-}a}) = \mathrm{table}$$
$$s(\mathit{below\text{-}b}) = \mathrm{a}$$
$$s(\mathit{below\text{-}c}) = \mathrm{table}$$



$\rightsquigarrow 3^3 = 27$ states

Note: it may be useful to add auxiliary state variables like *above-a*.

Advantage of Finite-Domain Representation

How many “useless” (physically impossible) states are there with these blocks world state representations?

- There are 13 physically possible states with three blocks:
  - all blocks on table: 1 state
  - all blocks in one stack: $3! = 6$ states
  - two block stacked, the other separate: $\binom{3}{2}2! = 6$
- With propositional variables, $2^9 - 13 = 499$ states are useless.
- With finite-domain variables, only $27 - 13 = 14$ are useless.

Although useless states are unreachable, they can introduce “shortcuts” in some heuristics and thus lead to worse heuristic estimates.

Finite-Domain State Variables

Definition (Finite-Domain State Variable)

A finite-domain state variable is a symbol v with an associated domain $\mathrm{dom}(v)$, which is a finite non-empty set of values.

Let V be a finite set of finite-domain state variables.

A state s over V is an assignment $s : V \to \bigcup_{v \in V} \mathrm{dom}(v)$ such that $s(v) \in \mathrm{dom}(v)$ for all $v \in V$.

A formula over V is a propositional logic formula whose atomic propositions are of the form $v = d$ where $v \in V$ and $d \in \mathrm{dom}(v)$.

Slightly extending propositional logic, we treat states s over finite-domain variables as logical interpretations where $s \models v = d$ iff $s(v) = d$.

Example: Finite-Domain State Variables

Example

Consider finite-domain variables $V = \{\mathit{location}, \mathit{bike}\}$ with $\mathrm{dom}(\mathit{location}) = \{\text{at-home}, \text{in-front-of-uni}, \text{in-lecture}\}$ and $\mathrm{dom}(\mathit{bike}) = \{\mathrm{locked}, \mathrm{unlocked}, \mathrm{stolen}\}$.

Consider state $s = \{\mathit{location} \mapsto \text{at-home}, \mathit{bike} \mapsto \mathit{locked}\}$.

Does $s \models (\mathit{location} = \text{at-home} \land \lnot \mathit{bike} = \mathrm{stolen})$ hold?

Reminder: Syntax of Operators

Definition (Operator)

An operator o over state variables V is an object with three properties:

- a precondition $pre(o)$, a formula over V
- an effect $eff(o)$ over V
- a cost $cost(o) \in \mathbb{R}_0^+$

Only necessary adaptation: What is an effect?

Example

$\langle location = \text{in-front-of-uni},$
$location := \text{in-lecture} \land (bike = \text{unlocked} \rhd bike := \text{stolen}), 1\rangle$

Syntax of Effects

Definition (Effect over Finite-Domain State Variables)

Effects over finite-domain state variables V are inductively defined as follows:

- $\top$ is an effect (empty effect).
- If $v \in V$ is a finite-domain state variable and $d \in \text{dom}(v)$, then $v := d$ is an effect (atomic effect).
- If e and e' are effects, then $(e \land e')$ is an effect (conjunctive effect).
- If χ is a formula over V and e is an effect, then $(\chi \rhd e)$ is an effect (conditional effect).

Parentheses can be omitted when this does not cause ambiguity.

only change compared to propositional case: atomic effects

Semantics of Effects: Effect Conditions

Definition (Effect Condition with Finite-Domain Representation)

Let $v := d$ be an atomic effect, and let e be an effect.

The effect condition $effcond(v := d, e)$ under which $v := d$ triggers given the effect e is a propositional formula defined as follows:

- $effcond(v := d, \top) = \bot$
- $effcond(v := d, v := d) = \top$
- $effcond(v := d, v' := d') = \bot$
  for atomic effects with $v' \neq v$ or $d' \neq d$
- $effcond(v := d, (e \land e')) =$
  $(effcond(v := d, e) \lor effcond(v := d, e'))$
- $effcond(v := d, (\chi \rhd e)) = (\chi \land effcond(v := d, e))$

Same definition as for propositional tasks,
we just use the adapted definition of atomic effects.

Conflicting Effects and Consistency Condition

- What should an effect of the form $v := a \land v := b$ mean?
- For finite-domain representations, the accepted semantics is to make this illegal, i.e., to make an operator inapplicable if it would lead to conflicting effects.

Definition (Consistency Condition)

Let e be an effect over finite-domain state variables V.

The consistency condition for e, $consist(e)$ is defined as

$$\bigwedge_{v \in V} \bigwedge_{d,d' \in \text{dom}(v), d \neq d'} \neg(effcond(v := d, e) \land effcond(v := d', e)).$$

How did we handle conflicting effects
in propositional planning tasks?

Semantics of Operators: Finite-Domain Case

Definition (Applicable, Resulting State)

Let V be a set of finite-domain state variables
and e be an effect over V.
If $s \models \mathit{consist}(e)$, the resulting state of applying e in s,
written $s[\![e]\!]$, is the state s' defined as follows for all $v \in V$:

$$s'(v) = \begin{cases} d & \text{if } s \models \mathit{effcond}(v := d, e) \text{ for some } d \in \mathrm{dom}(v) \\ s(v) & \text{otherwise} \end{cases}$$

Let o be an operator over V.
Operator o is applicable in s if $s \models \mathit{pre}(o) \wedge \mathit{consist}(\mathit{eff}(o))$.

If o is applicable in s, the resulting state of applying o in s,
written $s[\![o]\!]$, is the state $s[\![\mathit{eff}(o)]\!]$.

Applying Operators: Example

Example

$V = \{\mathit{location}, \mathit{bike}\}$ with
$\mathrm{dom}(\mathit{location}) = \{\text{at-home}, \text{in-front-of-uni}, \text{in-lecture}\}$ and
$\mathrm{dom}(\mathit{bike}) = \{\text{locked}, \text{unlocked}, \text{stolen}\}$.

State $s = \{\mathit{location} \mapsto \text{in-front-of-uni}, \mathit{bike} \mapsto \text{unlocked}\}$

$o = \langle \mathit{location} = \text{in-front-of-uni}, \mathit{location} := \text{at-home}, 1\rangle$
$o' = \langle \mathit{location} = \text{in-front-of-uni},$
$\quad \mathit{location} := \text{in-lecture} \wedge (\mathit{bike} = \text{unlocked} \rhd \mathit{bike} := \text{stolen}), 1\rangle$

What is $s[\![o]\!]$? What is $s[\![o']\!]$?

FDR Planning Tasks

Definition (Planning Task)

An FDR planning task (or planning task in finite-domain representation) is a 4-tuple $\Pi = \langle V, I, O, \gamma\rangle$ where

- V is a finite set of finite-domain state variables,
- I is an assignment for V called the initial state,
- O is a finite set of operators over V, and
- γ is a formula over V called the goal.

Apart from the variables, this is the same definition
as for propositional planning tasks,
but the underlying concepts have been adapted.

Mapping FDR Planning Tasks to Transition Systems

Definition (Transition System Induced by an FDR Planning Task)

The FDR planning task $\Pi = \langle V, I, O, \gamma\rangle$ induces
the transition system $\mathcal{T}(\Pi) = \langle S, L, c, T, s_0, S_\star\rangle$, where

- S is the set of all states over V,
- L is the set of operators O,
- $c(o) = \mathit{cost}(o)$ for all operators $o \in O$,
- $T = \{\langle s, o, s'\rangle \mid s \in S,\ o \text{ applicable in } s,\ s' = s[\![o]\!]\}$,
- $s_0 = I$, and
- $S_\star = \{s \in S \mid s \models \gamma\}$.

Exactly the same definition as for propositional planning tasks,
but the underlying concepts have been adapted.

E1.2 Equivalence and Normal Forms

Equivalence and Flat Operators

- The definitions of equivalent effects/operators and flat effects/operators apply equally to finite-domain representation.
- The same is true for the equivalence transformations.

You find the definitions and transformations in Chapter B4.

Conflict-Free Operators

Definition (Conflict-Free)

An effect e over finite-domain state variables V is called conflict-free if $\mathit{effcond}(v := d, e) \land \mathit{effcond}(v := d', e)$ is unsatisfiable for all $v \in V$ and $d, d' \in \mathrm{dom}(v)$ with $d \neq d'$.

An operator o is called conflict-free if $\mathit{eff}(o)$ is conflict-free.

Note: $\mathit{consist}(e) \equiv \top$ for conflict-free e.

Algorithm to make given operator o conflict-free:

- replace $\mathit{pre}(o)$ with $\mathit{pre}(o) \land \mathit{consist}(\mathit{eff}(o))$
- replace all atomic effects $v := d$ by $(\mathit{consist}(\mathit{eff}(o)) \rhd v := d)$

The resulting operator o' is conflict-free and $o \equiv o'$.

SAS$^+$ Operators and Planning Tasks

Definition (SAS$^+$ Operator)

An operator o of an FDR planning task is a SAS$^+$ operator if

- $\mathit{pre}(o)$ is a satisfiable conjunction of atoms, and
- $\mathit{eff}(o)$ is a conflict-free conjunction of atomic effects.

Definition (SAS$^+$ Planning Task)

An FDR planning task $\langle V, O, I, \gamma \rangle$ is a SAS$^+$ planning task if all operators $o \in O$ are SAS$^+$ operators and γ is a satisfiable conjunction of atoms.

Note: SAS$^+$ operators are conflict-free and flat.

SAS$^+$ Operators: Remarks

- Every SAS$^+$ operator is of the form

$$\langle v_1 = d_1 \land \dots \land v_n = d_n, \quad v'_1 := d'_1 \land \dots \land v'_m := d'_m \rangle$$

  where all v_i are distinct and all v'_j are distinct.
- Often, SAS$^+$ operators o are described via two sets of partial assignments:
  - the preconditions $\{v_1 \mapsto d_1, \dots, v_n \mapsto d_n\}$
  - the effects $\{v'_1 \mapsto d'_1, \dots, v'_m \mapsto d'_m\}$

SAS$^+$ vs. STRIPS

- SAS$^+$ is an analogue of STRIPS planning tasks for FDR, but there is no special role of “positive” conditions.
- Apart from this difference, all comments for STRIPS apply analogously.
- If all variable domains are binary, SAS$^+$ is essentially STRIPS with negation.

SAS$^+$

Derives from SAS = Simplified Action Structures
(Bäckström & Klein, 1991)

E1.3 Summary

Summary

- Planning tasks in finite-domain representation (FDR) are an alternative to propositional planning tasks.
- FDR tasks are often more compact (have fewer states).
- This makes many planning algorithms more efficient when working with a finite-domain representation.
- SAS$^+$ tasks are a restricted form of FDR tasks
  where only conjunctions of atoms are allowed
  in the preconditions, effects and goal.
  No conditional effects are allowed.