Planning and Optimization

D7. Delete Relaxation: Analysis of $h^{\max}$ and h^{add}

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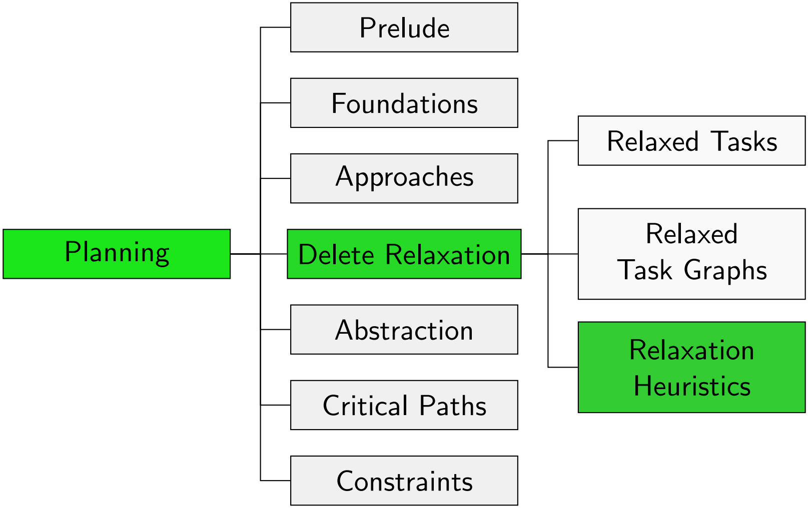
Content of this Course



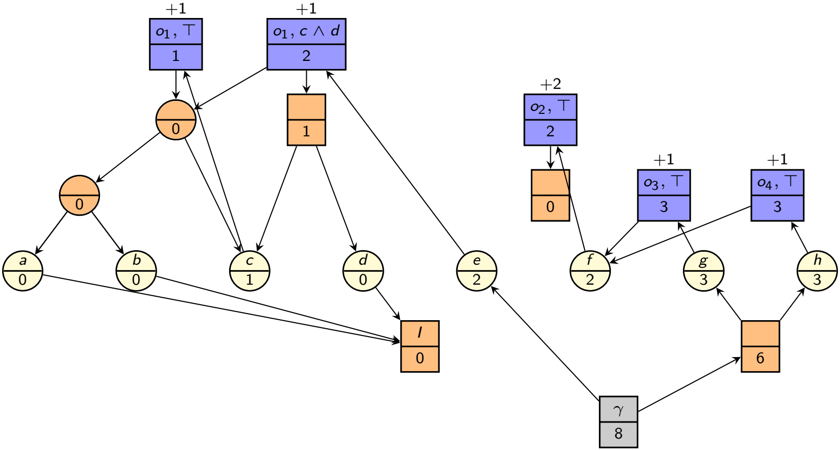
Choice Functions

Motivation

- In this chapter, we analyze the behaviour of h^{max} and h^{add} more deeply.
- Our goal is to understand their shortcomings.
  - In the next chapter we then used this understanding to devise an improved heuristic.
- As a preparation for our analysis, we need some further definitions that concern choices in AND/OR graphs.
- The key observation is that if we want to establish the value of a certain node n, we can to some extent choose how we want to achieve the OR nodes that are relevant to achieving n.

Preview: Choice Function & Best Achievers

Preserve at most one outgoing arc of each OR node,
but node values may not change.



(precondition of o_1 modified to $c \vee (a \vee b)$)

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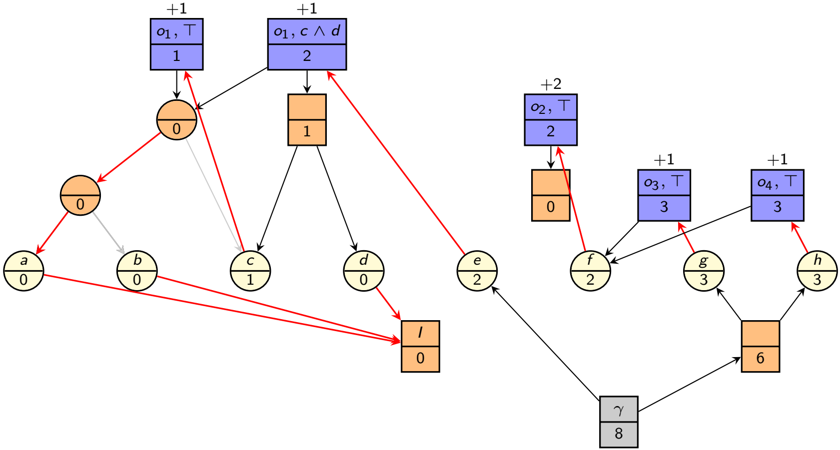


(precondition of o_1 modified to $c \vee (a \vee b)$)

Choice Functions

Definition (Choice Function)

Let G be an AND/OR graph with nodes N and OR nodes $N_\vee$.

A choice function for G is a function $f : N' \to N$ defined on some set $N' \subseteq N_\vee$ such that $f(n) \in succ(n)$ for all $n \in N'$.

- In words, choice functions select (at most) one successor for each OR node of G.
- Intuitively, $f(n)$ selects by which disjunct n is achieved.
- If $f(n)$ is undefined for a given n, the intuition is that n is not achieved.

Reduced Graphs

Once we have decided how to achieve an OR node,
we can remove the other alternatives:

Definition (Reduced Graph)

Let G be an AND/OR graph, and let f be a choice function for G defined on nodes N'.

The reduced graph for f is the subgraph of G where all outgoing arcs of OR nodes are removed except for the chosen arcs $\langle n, f(n) \rangle$ with $n \in N'$.

Best Achievers

Choice Functions Induced by $h^{\max}$ and h^{add}

Which choices do $h^{\max}$ and h^{add} make?

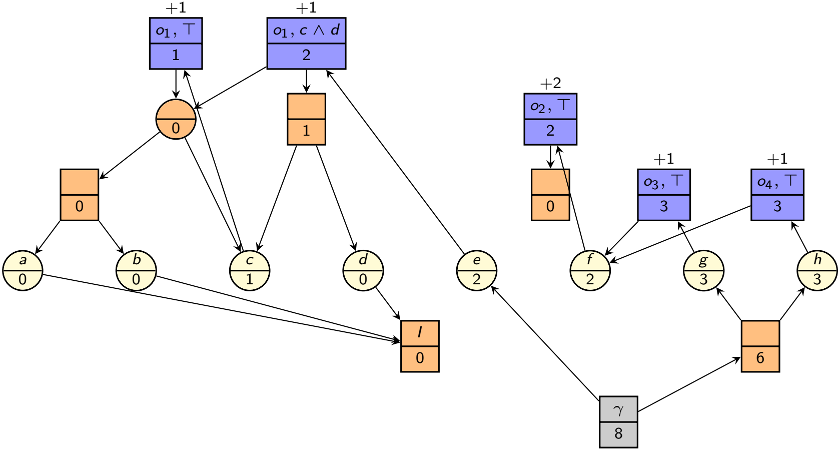
- At every OR node n, we set the cost of n to the minimum of the costs of the successors of n.
- The motivation for this is to achieve n via the successor that can be achieved most cheaply according to our cost estimates.

$\rightsquigarrow$ This corresponds to defining a choice function f with $f(n) \in \arg\min_{n' \in N'} n'.cost$ for all reached OR nodes n, where $N' \subseteq succ(n)$ are all successors of n processed before n.

- The successors chosen by this cost function are called best achievers (according to $h^{\max}$ or h^{add}).
- Note that the best achiever function f is in general not well-defined because there can be multiple minimizers. We assume that ties are broken arbitrarily.

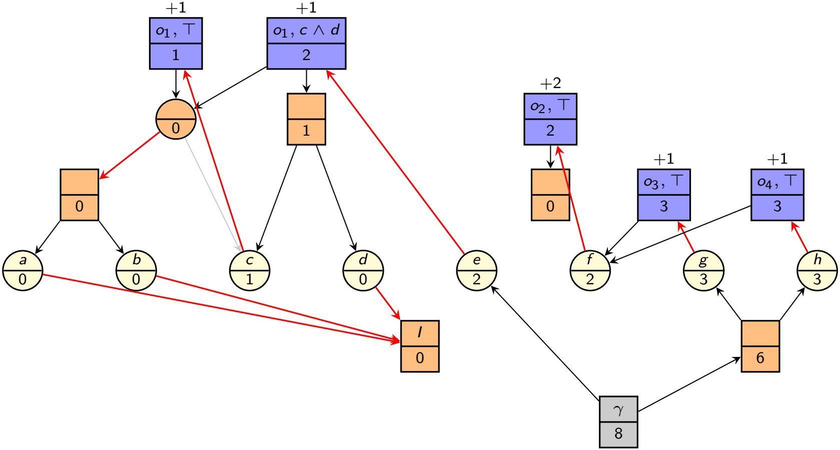
Example: Best Achievers (1)

best achievers for h^{add}



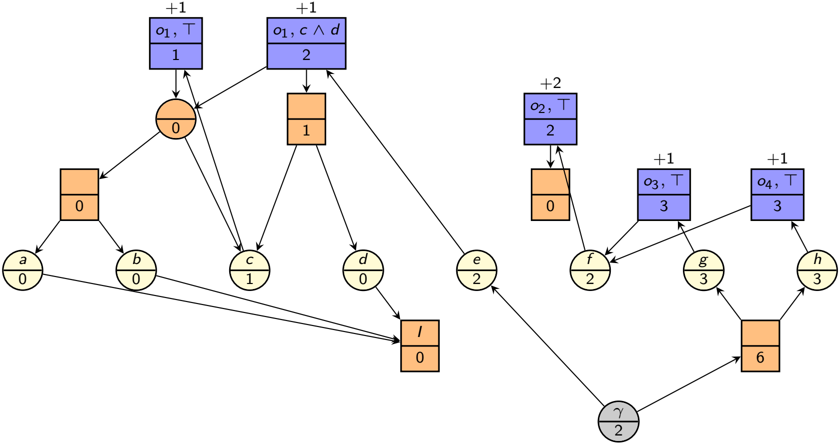
Example: Best Achievers (1)

best achievers for h^{add}



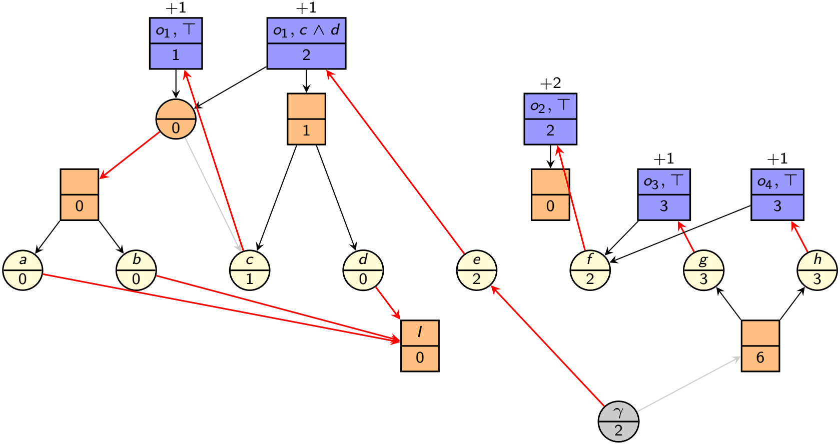
Example: Best Achievers (2)

best achievers for h^{add}; modified goal $e \vee (g \wedge h)$



Example: Best Achievers (2)

best achievers for h^{add}; modified goal $e \vee (g \wedge h)$



Best Achiever Graphs

- Observation: The $h^{\max}/h^{\text{add}}$ costs of nodes remain the same if we replace the RTG by the reduced graph for the respective best achiever function.
- The AND/OR graph that is obtained by removing all nodes with infinite cost from this reduced graph is called the best achiever graph for $h^{\max}/h^{\text{add}}$.
  - We write $G^{\max}$ and G^{add} for the best achiever graphs.
- $G^{\max}$ (G^{add}) is always acyclic: for all arcs $\langle n, n' \rangle$ it contains, n is processed by $h^{\max}$ (by h^{add}) after n'.

Paths in Best Achiever Graphs

Let n be a node of the best achiever graph.
Let N_{eff} be the set of effect nodes of the best achiever graph.
The cost of an effect node is the cost of the associated operator.
The cost of a path in the best achiever graph is the sum of costs of all effect nodes on the path.

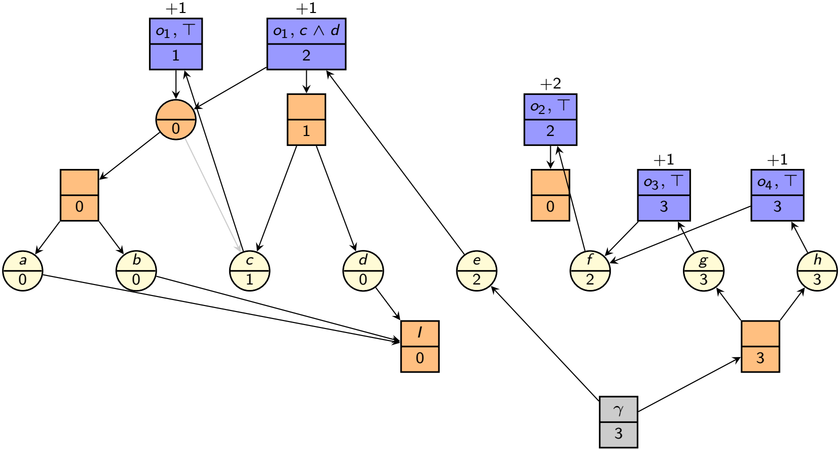
The following properties can be shown by induction:

- $h^{\max}(n)$ is the maximum cost of all paths originating from n in $G^{\max}$. A path achieving this maximum is called a critical path.
- $h^{\text{add}}(n)$ is the sum, over all effect nodes n', of the cost of n' multiplied by the number of paths from n to n' in G^{add}.

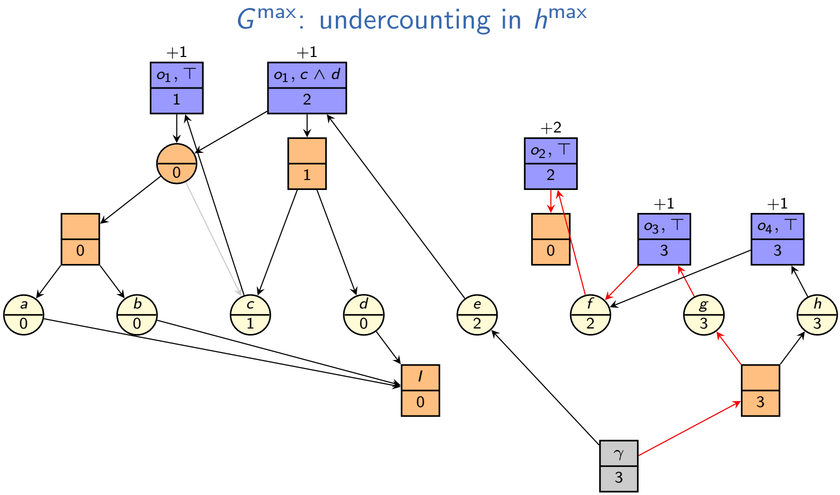
In particular, these properties hold for the goal node n_γ if it is reachable.

Example: Undercounting in $h^{\max}$

$G^{\max}$: undercounting in $h^{\max}$



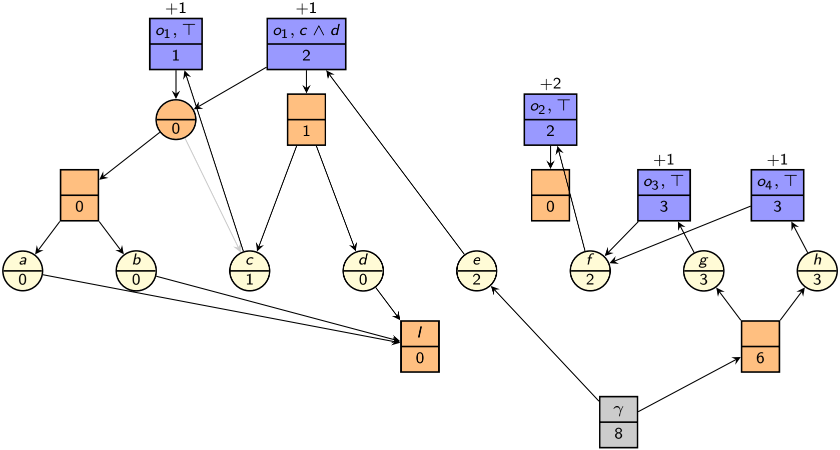
Example: Undercounting in $h^{\max}$



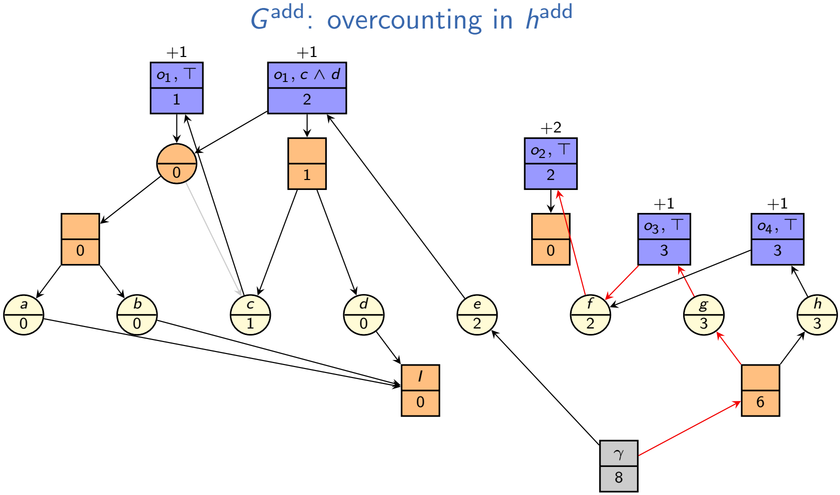
⇝ o_1 and o_4 not counted because they are off the critical path

Example: Overcounting in h^{add}

G^{add}: overcounting in h^{add}



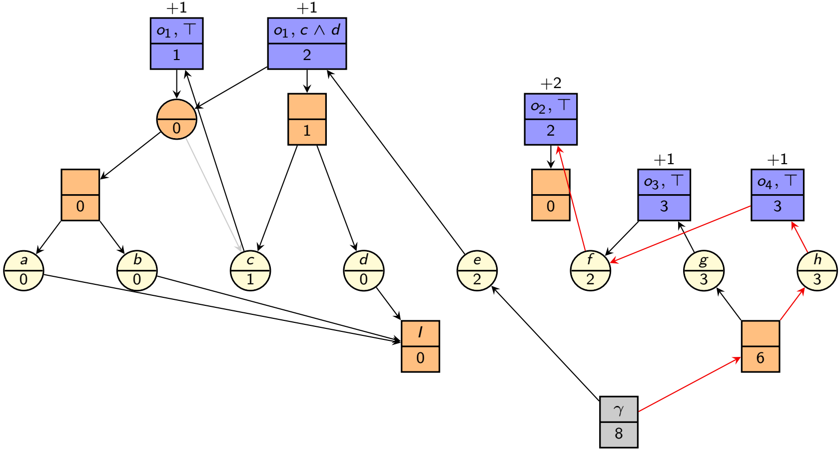
Example: Overcounting in h^{add}



$\rightsquigarrow$ o_2 counted twice because there are two paths to $n^{\top}_{o_2}$

Example: Overcounting in h^{add}

G^{add}: overcounting in h^{add}



⇝ o_2 counted twice because there are two paths to $n^{\top}_{o_2}$

Summary

Summary

- h^{max} and h^{add} can be used to decide how to achieve OR nodes in a relaxed task graph
  $\rightsquigarrow$ best achievers
- Best achiever graphs help identify shortcomings of h^{max} and h^{add} compared to the perfect delete relaxation heuristic h^+.
  - h^{max} underestimates h^+ because it only considers the cost of a critical path for the relaxed planning task.
  - h^{add} overestimates h^+ because it double-counts operators occurring on multiple paths in the best achiever graph.