Planning and Optimization

C3. General Regression

Malte Helmert and Gabriele Röger

Universität Basel

October 9, 2023

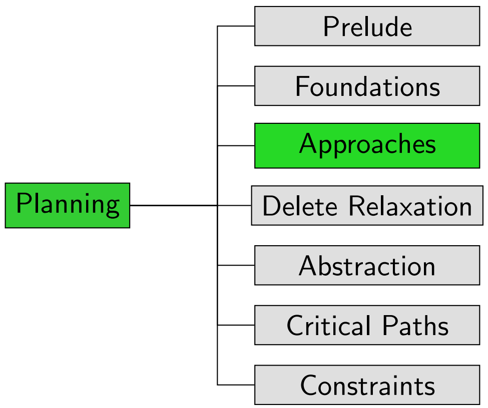
C3.1 Regressing State Variables

C3.2 Regressing Formulas Through Effects

C3.3 Regressing Formulas Through Operators

C3.4 Summary

Content of this Course



Regression for General Planning Tasks

- With disjunctions and conditional effects, things become more tricky. How to regress $a \lor (b \land c)$ with respect to $\langle q, d \rhd b \rangle$?
- In this chapter, we show how to regress general sets of states through general operators.
- We extensively use the idea of representing sets of states as formulas.

C3.1 Regressing State Variables

Regressing State Variables: Motivation

Key question for general regression:

- Assume we are applying an operator with effect e.
- What must be true in the predecessor state for propositional state variable v to be true in the successor state?

If we can answer this question, a general definition of regression is only a small additional step.

Regressing State Variables: Key Idea

Assume we are in state s and apply effect e to obtain successor state s'.

Propositional state variable v is true in s' iff

- effect e makes it true, or
- it remains true, i.e., it is true in s and not made false by e.

Regressing a State Variable Through an Effect

Definition (Regressing a State Variable Through an Effect)

Let e be an effect of a propositional planning task, and let v be a propositional state variable.

The regression of v through e, written $regr(v, e)$, is defined as the following logical formula:

$$regr(v, e) = effcond(v, e) \lor (v \land \neg effcond(\neg v, e)).$$

Does this capture add-after-delete semantics correctly?

Regressing State Variables: Example

Example

Let $e = (b \rhd a) \land (c \rhd \neg a) \land b \land \neg d$.

v	$regr(v, e)$
a	$b \lor (a \land \neg c)$
b	$\top \lor (b \land \neg\bot) \equiv \top$
c	$\bot \lor (c \land \neg\bot) \equiv c$
d	$\bot \lor (d \land \neg\top) \equiv \bot$

Reminder: $regr(v, e) = effcond(v, e) \lor (v \land \neg effcond(\neg v, e))$

Regressing State Variables: Correctness (1)

Lemma (Correctness of $regr(v, e)$)

Let s be a state, e be an effect and v be a state variable of a propositional planning task.

Then $s \models regr(v, e)$ iff $s[\![e]\!] \models v$.

Regressing State Variables: Correctness (2)

Proof.

($\Rightarrow$): We know $s \models regr(v, e)$, and hence $s \models effcond(v, e) \lor (v \land \neg effcond(\neg v, e))$.

Do a case analysis on the two disjuncts.

Case 1: $s \models effcond(v, e)$.
Then $s[\![e]\!] \models v$ by the first case in the definition of $s[\![e]\!]$ (Ch. B3).

Case 2: $s \models (v \land \neg effcond(\neg v, e))$.
Then $s \models v$ and $s \not\models effcond(\neg v, e)$.
We may additionally assume $s \not\models effcond(v, e)$ because otherwise we can apply Case 1 of this proof.
Then $s[\![e]\!] \models v$ by the third case in the definition of $s[\![e]\!]$. ...

Regressing State Variables: Correctness (3)

Proof (continued).

($\Leftarrow$): Proof by contraposition.
We show that if $regr(v, e)$ is false in s, then v is false in $s[\![e]\!]$.

- By prerequisite, $s \not\models effcond(v, e) \lor (v \land \neg effcond(\neg v, e))$.
- Hence $s \models \neg effcond(v, e) \land (\neg v \lor effcond(\neg v, e))$.
- From the first conjunct, we get $s \models \neg effcond(v, e)$ and hence $s \not\models effcond(v, e)$.
- From the second conjunct, we get $s \models \neg v \lor effcond(\neg v, e)$.
- Case 1: $s \models \neg v$. Then v is false before applying e and remains false, so $s[\![e]\!] \not\models v$.
- Case 2: $s \models effcond(\neg v, e)$. Then v is deleted by e and not simultaneously added, so $s[\![e]\!] \not\models v$.

□

C3.2 Regressing Formulas Through Effects

Regressing Formulas Through Effects: Idea

- We can now generalize regression from state variables to general formulas over state variables.
- The basic idea is to replace every occurrence of every state variable v by $regr(v, e)$ as defined in the previous section.
- The following definition makes this more formal.

Regressing Formulas Through Effects: Definition

Definition (Regressing a Formula Through an Effect)

In a propositional planning task, let e be an effect, and let φ be a formula over propositional state variables.

The regression of φ through e, written $regr(\varphi, e)$, is defined as the following logical formula:

$$\begin{aligned}
regr(\top, e) &= \top \\
regr(\bot, e) &= \bot \\
regr(v, e) &= effcond(v, e) \lor (v \land \neg effcond(\neg v, e)) \\
regr(\neg\psi, e) &= \neg regr(\psi, e) \\
regr(\psi \lor \chi, e) &= regr(\psi, e) \lor regr(\chi, e) \\
regr(\psi \land \chi, e) &= regr(\psi, e) \land regr(\chi, e).
\end{aligned}$$

Regressing Formulas Through Effects: Example

Example

Let $e = (b \rhd a) \land (c \rhd \neg a) \land b \land \neg d$.

Recall:

- $regr(a, e) \equiv b \lor (a \land \neg c)$
- $regr(b, e) \equiv \top$
- $regr(c, e) \equiv c$
- $regr(d, e) \equiv \bot$

We get:

$$\begin{aligned}
regr((a \lor d) \land (c \lor d), e) &\equiv ((b \lor (a \land \neg c)) \lor \bot) \land (c \lor \bot) \\
&\equiv (b \lor (a \land \neg c)) \land c \\
&\equiv b \land c
\end{aligned}$$

Regressing Formulas Through Effects: Correctness (1)

Lemma (Correctness of $regr(\varphi, e)$)

Let φ be a logical formula, e an effect and s a state of a propositional planning task.

Then $s \models regr(\varphi, e)$ iff $s[\![e]\!] \models \varphi$.

Regressing Formulas Through Effects: Correctness (2)

Proof.

The proof is by structural induction on φ.

Induction hypothesis: $s \models regr(\psi, e)$ iff $s[\![e]\!] \models \psi$ for all proper subformulas ψ of φ.

Base case $\varphi = \top$:

We have $regr(\top, e) = \top$, and $s \models \top$ iff $s[\![e]\!] \models \top$ is correct.

Base case $\varphi = \bot$:

We have $regr(\bot, e) = \bot$, and $s \models \bot$ iff $s[\![e]\!] \models \bot$ is correct.

Base case $\varphi = v$:

We have $s \models regr(v, e)$ iff $s[\![e]\!] \models v$ from the previous lemma. ...

Regressing Formulas Through Effects: Correctness (3)

Proof (continued).

Inductive case $\varphi = \neg\psi$:

$$\begin{aligned} s \models regr(\neg\psi, e) &\text{ iff } s \models \neg regr(\psi, e) \\ &\text{ iff } s \not\models regr(\psi, e) \\ &\text{ iff } s[\![e]\!] \not\models \psi \\ &\text{ iff } s[\![e]\!] \models \neg\psi \end{aligned}$$

Inductive case $\varphi = \psi \lor \chi$:

$$\begin{aligned} s \models regr(\psi \lor \chi, e) &\text{ iff } s \models regr(\psi, e) \lor regr(\chi, e) \\ &\text{ iff } s \models regr(\psi, e) \text{ or } s \models regr(\chi, e) \\ &\text{ iff } s[\![e]\!] \models \psi \text{ or } s[\![e]\!] \models \chi \\ &\text{ iff } s[\![e]\!] \models \psi \lor \chi \end{aligned}$$

Inductive case $\varphi = \psi \land \chi$:

Like previous case, replacing "$\lor$" by "$\land$" and replacing "or" by "and". □

C3.3 Regressing Formulas Through Operators

Regressing Formulas Through Operators: Idea

- We can now regress arbitrary formulas through arbitrary effects.
- The last missing piece is a definition of regression through operators, describing exactly in which states s applying a given operator o leads to a state satisfying a given formula φ.
- There are two requirements:
  - The operator o must be applicable in the state s.
  - The resulting state $s[\![o]\!]$ must satisfy φ.

Regressing Formulas Through Operators: Definition

Definition (Regressing a Formula Through an Operator)

In a propositional planning task, let o be an operator, and let φ be a formula over state variables.

The regression of φ through o, written $regr(\varphi, o)$, is defined as the following logical formula:

$$regr(\varphi, o) = pre(o) \land regr(\varphi, eff(o)).$$

Regressing Formulas Through Operators: Correctness (1)

Theorem (Correctness of $regr(\varphi, o)$)

Let φ be a logical formula, o an operator and s a state of a propositional planning task.

Then $s \models regr(\varphi, o)$ iff o is applicable in s and $s[\![o]\!] \models \varphi$.

Regressing Formulas Through Operators: Correctness (2)

Reminder: $regr(\varphi, o) = pre(o) \land regr(\varphi, eff(o))$

Proof.

Case 1: $s \models pre(o)$.

Then o is applicable in s and the statement we must prove simplifies to: $s \models regr(\varphi, e)$ iff $s[\![e]\!] \models \varphi$, where $e = eff(o)$. This was proved in the previous lemma.

Case 2: $s \not\models pre(o)$.

Then $s \not\models regr(\varphi, o)$ and o is not applicable in s. Hence both statements are false and therefore equivalent. □

Regression Examples (1)

Examples: compute regression and simplify to DNF

- $regr(b, \langle a, b \rangle)$
  $\equiv a \land (\top \lor (b \land \neg\bot))$
  $\equiv a$
- $regr(b \land c \land d, \langle a, b \rangle)$
  $\equiv a \land (\top \lor (b \land \neg\bot)) \land (\bot \lor (c \land \neg\bot)) \land (\bot \lor (d \land \neg\bot))$
  $\equiv a \land c \land d$
- $regr(b \land \neg c, \langle a, b \land c \rangle)$
  $\equiv a \land (\top \lor (b \land \neg\bot)) \land \neg(\top \lor (c \land \neg\bot))$
  $\equiv a \land \top \land \bot$
  $\equiv \bot$

Regression Examples (2)

Examples: compute regression and simplify to DNF

- $regr(b, \langle a, c \rhd b \rangle)$
  $\equiv a \land (c \lor (b \land \neg\bot))$
  $\equiv a \land (c \lor b)$
  $\equiv (a \land c) \lor (a \land b)$
- $regr(b, \langle a, (c \rhd b) \land ((d \land \neg c) \rhd \neg b) \rangle)$
  $\equiv a \land (c \lor (b \land \neg(d \land \neg c)))$
  $\equiv a \land (c \lor (b \land (\neg d \lor c)))$
  $\equiv a \land (c \lor (b \land \neg d) \lor (b \land c))$
  $\equiv a \land (c \lor (b \land \neg d))$
  $\equiv (a \land c) \lor (a \land b \land \neg d)$

C3.4 Summary

Summary

- Regressing a propositional state variable through an (arbitrary) operator must consider two cases:
  - state variables made true (by add effects)
  - state variables remaining true (by absence of delete effects)
- Regression of propositional state variables can be generalized to arbitrary formulas φ by replacing each occurrence of a state variable in φ by its regression.
- Regressing a formula φ through an operator involves regressing φ through the effect and enforcing the precondition.