Planning and Optimization

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Exercise Sheet 7

Due: November 20, 2023

Important: For submission, consult the rules at the end of the exercise. Non-adherence to these rules might lead to a penalty in the form of a deduction of marks or, in the worst case, that your submission will not be marked at all.

Exercise 7.1 (1+2+1+1 marks)

Consider the SAS$^+$ planning task $\Pi = \langle V, I, O, \gamma \rangle$ with

- $V = \{x\} \cup \{y_i \mid 1 \leq i \leq 4\} \cup \{z_1, z_2\}$ with $dom(x) = dom(y_i) = \{0, 1\}$ for $1 \leq i \leq 4$ and $dom(z_1) = dom(z_2) = \{A, B, C\}$;
- $I = \{x \mapsto 0\} \cup \{y_i \mapsto 0 \mid 1 \leq i \leq 4\} \cup \{z_1 \mapsto A, z_2 \mapsto A\}$;
- $O = \{o_B, o'_B, o_C, o'_C\} \cup \{o_i \mid 1 \leq i \leq 4\}$ where
  - $o_B = \langle x = 0, x := 1 \wedge z_1 := B, 2 \rangle$,
  - $o'_B = \langle \top, z_2 := B, 1 \rangle$,
  - $o_C = \langle x = 1 \wedge y_2 = 1 \wedge z_1 = B, z_1 := C, 2 \rangle$,
  - $o'_C = \langle y_4 = 1 \wedge z_2 = B, z_2 := C, 3 \rangle$,
  - $o_1 = \langle x = 0, y_1 := 1, 1 \rangle$, and
  - $o_i = \langle y_{i-1} = 1, y_i := 1, i \rangle$ for $2 \leq i \leq 4$; and
- $\gamma = (z_1 = C \wedge z_2 = C)$.

Furthermore, consider the pattern collection $\mathcal{C} = \{P_1, P_2, P_3, P_4\}$ for Π where

$$
\begin{aligned}
P_1 &= \{x, y_1, z_1\}, \\
P_2 &= \{x, y_2, z_1\}, \\
P_3 &= \{y_1, y_2, y_3, y_4\}, \text{ and} \\
P_4 &= \{y_1, y_2, y_4, z_1, z_2\}.
\end{aligned}
$$

(a) Provide the causal graph of Π.

(b) Derive a simplified pattern collection $\mathcal{C}'$ from $\mathcal{C}$. To do so, remove causally irrelevant variables, decompose causally disconnected patterns, and remove non-goal patterns as much as possible.

(c) Construct the compatibility graph for your collection $\mathcal{C}'$ from part (b) and determine all maximal cliques.

(d) Use your insights from part (c) to provide the canonical heuristic $h^{\mathcal{C}'}$ for your pattern collection from part (b) and simplify it with the help of the dominated sum theorem.

Exercise 7.2 (0.5+0.5+0.5+0.5+1.5+1.5 marks)

In this exercise, you will prove the equivalence theorem for syntactic projections from the lecture:

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a SAS$^+$ planning task, and let $P \subseteq V$ be a pattern for Π.
Then $\mathcal{T}(\Pi)^{\pi_P} \sim \mathcal{T}(\Pi|_P)$.

We refer to the components of the transition systems as $\mathcal{T}(\Pi)^{\pi_P} = \langle S^{abs}, L^{abs}, c^{abs}, T^{abs}, s_0^{abs}, S_\star^{abs} \rangle$ and $\mathcal{T}(\Pi|_P) = \langle S^{syn}, L^{syn}, c^{syn}, T^{syn}, s_0^{syn}, S_\star^{syn} \rangle$.

For establishing that $\mathcal{T}(\Pi)^{\pi_P} \sim \mathcal{T}(\Pi|_P)$ we will use functions $\phi : S^{abs} \to S^{syn}$ and $\lambda : L^{abs} \to L^{syn}$ defined by $\phi = \mathbf{id}$ (i.e. $\phi(s) = s$ for all $s \in S^{abs}$) and $\lambda(o) = o|_P$ for $o \in L^{abs}$.

The individual parts (except d) of the exercise will establish the different requirements from the definition of isomorphic transition systems. Part (d) establishes an insight that helps proving parts (e) and (f).

(a) Show that ϕ and λ are indeed bijections from their domain to their codomain.

Hint: Recall that two operators o_1 and o_2 can be distinct objects even if $pre(o_1) = pre(o_2)$, $eff(o_1) = eff(o_2)$ and $cost(o_1) = cost(o_2)$ (definition of operators, slide set B2).

(b) Show that $\phi(s_0^{abs}) = s_0^{syn}$.

(c) Show that for all $l \in L^{abs}$ it holds that $c^{syn}(\lambda(l)) = c^{abs}(l)$.

(d) Explain why for every formula ψ occurring as an operator precondition or goal condition in Π it holds that $\psi \models \psi|_P$.

(e) Show that $s \in S_\star^{abs}$ iff $\phi(s) \in S_\star^{syn}$.

(f) Show that $s \xrightarrow{o} t \in T^{abs}$ iff $\phi(s) \xrightarrow{\lambda(o)} \phi(t) \in T^{syn}$.

Submission rules:

- Exercise sheets must be submitted in groups of 2–3 students. Create a team on ADAM including all members of your group and submit a single copy of the exercises per group.
- Create a single PDF file (ending .pdf) for all non-programming exercises. Use a file name that does not contain any spaces or special characters other than the underscore "_". If you want to submit handwritten solutions, include their scans in the single PDF. Make sure it is in a reasonable resolution so that it is readable, but ensure at the same time that the PDF size is not astronomically large. Put the names of all group members on top of the first page. Either use page numbers on all pages or put your names on each page. Make sure your PDF has size A4 (fits the page size if printed on A4).
- For programming exercises, only create those code text file(s) required by the exercise. Put your names in a comment on top of each file. Make sure your code compiles and test it. Code that does not compile or which we cannot successfully execute will not be graded.
- For the submission: if the exercise sheet does not include programming exercises, simply upload the single PDF. If the exercise sheet includes programming exercises, upload a ZIP file (ending .zip, .tar.gz or .tgz; *not* .rar or anything else) containing the single PDF and the code text file(s) and nothing else. Do not use directories within the ZIP, i.e., zip the files directly. After creating your zip file and before submitting it, open the file and verify that it complies with these requirements.
- Do not upload several versions to ADAM, i.e., if you need to resubmit, use the same file name again so that the previous submission is overwritten.