

**B5.1** Relations

Discrete Mathematics in Computer Science October 16, 2023 — B5. Relations

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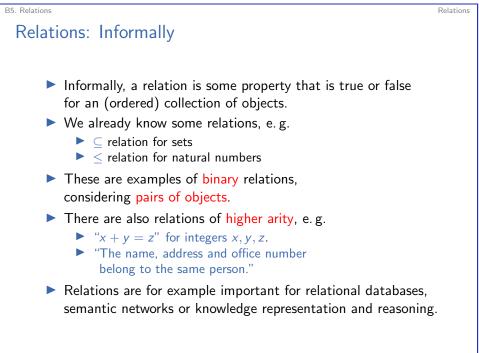
**B5.2** Properties of Binary Relations

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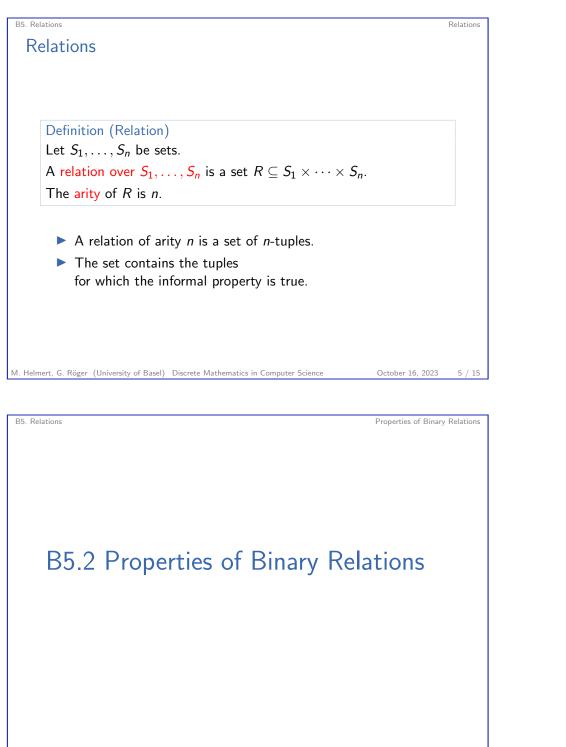
October 16, 2023 2 / 15

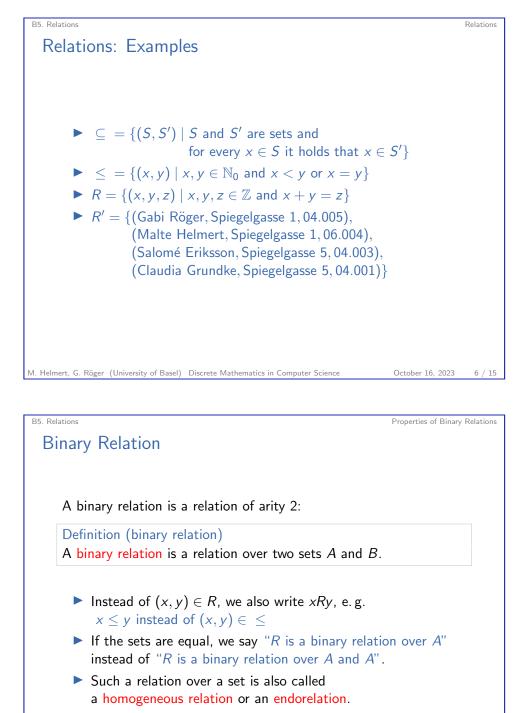
October 16, 2023

4 / 15



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Properties of Binary Relations

### Reflexivity

A reflexive relation relates every object to itself.

Definition (reflexive) A binary relation R over set A is reflexive if for all  $a \in A$  it holds that  $(a, a) \in R$ .

Which of these relations are reflexive?

- ▶  $R = \{(a, a), (a, b), (a, c), (b, a), (b, c), (c, c)\}$  over  $\{a, b, c\}$
- ▶  $R = \{(a, a), (a, b), (a, c), (b, b), (b, c), (c, c)\}$  over  $\{a, b, c\}$
- equality relation = on natural numbers
- ▶ less-than relation  $\leq$  on natural numbers
- $\blacktriangleright$  strictly-less-than relation < on natural numbers

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October 16, 2023 9 / 15

Properties of Binary Relations

B5. Relations

Symmetry

Definition (symmetric) A binary relation R over set A is symmetric if for all  $a, b \in A$  it holds that  $(a, b) \in R$  iff  $(b, a) \in R$ .

Which of these relations are symmetric?

- ▶  $R = \{(a, a), (a, b), (a, c), (b, a), (c, a), (c, c)\}$  over  $\{a, b, c\}$
- ▶  $R = \{(a, a), (a, b), (a, c), (b, b), (b, c), (c, c)\}$  over  $\{a, b, c\}$
- equality relation = on natural numbers
- $\blacktriangleright$  less-than relation  $\leq$  on natural numbers
- $\blacktriangleright$  strictly-less-than relation < on natural numbers

# Irreflexivity

A irreflexive relation never relates an object to itself.

Definition (irreflexive) A binary relation R over set A is irreflexive if for all  $a \in A$  it holds that  $(a, a) \notin R$ .

Which of these relations are irreflexive?

- $R = \{(a, a), (a, b), (a, c), (b, a), (b, c), (c, c)\}$  over  $\{a, b, c\}$
- ▶  $R = \{(a, a), (a, b), (a, c), (b, b), (b, c), (c, c)\}$  over  $\{a, b, c\}$
- equality relation = on natural numbers
- $\blacktriangleright$  less-than relation  $\leq$  on natural numbers
- strictly-less-than relation < on natural numbers</p>

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October 16, 2023 10 / 15

Properties of Binary Relations

#### B5. Relations

# Asymmetry and Antisymmetry

Definition (asymmetric and antisymmetric) Let R be a binary relation over set A. Relation R is asymmetric if for all  $a, b \in A$  it holds that if  $(a, b) \in R$  then  $(b, a) \notin R$ . Relation R is antisymmetric if for all  $a, b \in A$  with  $a \neq b$  it holds that if  $(a, b) \in R$  then  $(b, a) \notin R$ .

Which of these relations are asymmetric/antisymmetric?

- $R = \{(a, a), (a, b), (a, c), (b, a), (c, a), (c, c)\}$  over  $\{a, b, c\}$
- ▶  $R = \{(a, a), (a, b), (a, c), (b, b), (b, c), (c, c)\}$  over  $\{a, b, c\}$
- equality relation = on natural numbers
- $\blacktriangleright$  less-than relation  $\leq$  on natural numbers
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How do these properties relate to irreflexivity?

Properties of Binary Relations

October 16, 2023

13 / 15

#### Transitivity

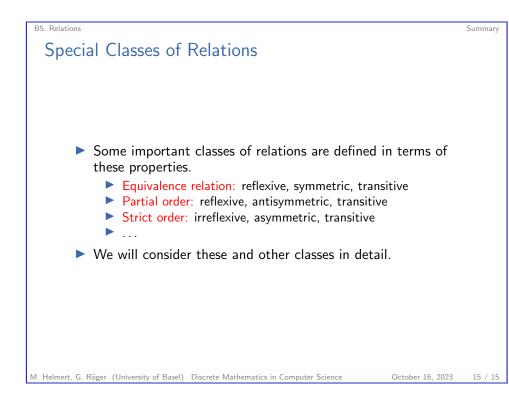
#### Definition

A binary relation R over set A is transitive if it holds for all  $a, b, c \in A$  that if  $(a, b) \in R$  and  $(b, c) \in R$  then  $(a, c) \in R$ .

Which of these relations are transitive?

- $R = \{(a, a), (a, b), (a, c), (b, a), (c, a), (c, c)\}$  over  $\{a, b, c\}$
- $R = \{(a, a), (a, b), (a, c), (b, b), (b, c), (c, c)\}$  over  $\{a, b, c\}$
- equality relation = on natural numbers
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### Summary

- ▶ A relation over sets  $S_1, \ldots, S_n$  is a set  $R \subseteq S_1 \times \cdots \times S_n$ .
- ► A binary relation is a relation over two sets.
- A binary relation over set S is a relation R ⊆ S × S and also called a homogeneous relation.
- ► A binary relation *R* over *A* is
  - reflexive if  $(a, a) \in R$  for all  $a \in A$ ,
  - ▶ irreflexive if  $(a, a) \notin R$  for all  $a \in A$ ,
  - symmetric if for all a, b ∈ A it holds that (a, b) ∈ R iff (b, a) ∈ R,
  - asymmetric if for all a, b ∈ A it holds that if (a, b) ∈ R then (b, a) ∉ R,
  - ▶ antisymmetric if for all  $a, b \in A$  with  $a \neq b$  it holds that if  $(a, b) \in R$  then  $(b, a) \notin R$ ,
  - ▶ transitive if for all  $a, b, c \in A$  it holds that if  $(a, b) \in R$  and  $(b, c) \in R$  then  $(a, c) \in R$ .

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October 16, 2023 14 / 15