

# Planning and Optimization

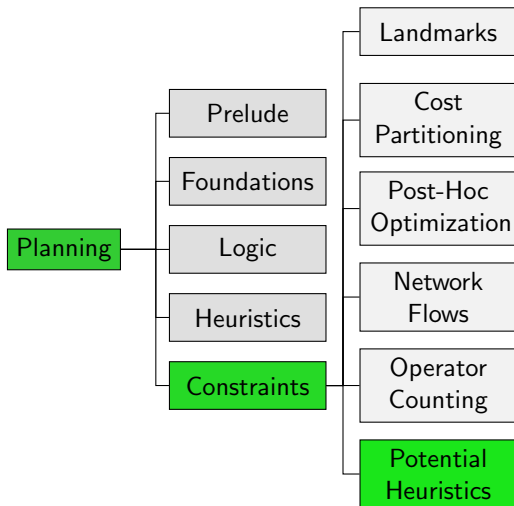
## G12. Potential Heuristics

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# Content of this Course



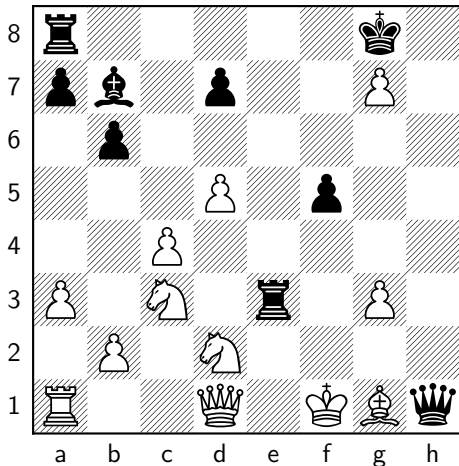
# Introduction

## Reminder: Transition Normal Form

In this chapter, we consider  $SAS^+$  tasks in transition normal form.

- A TNF operator mentions the **same variables** in the precondition and in the effect.
- A TNF goal specifies a value for **every** variable.

# Material Value of a Chess Position



Material value for white:

- + 1 · 6 (white pawns)
- 1 · 4 (black pawns)
- + 3 · 2 (white knights)
- 3 · 0 (black knights)
- + 3 · 1 (white bishops)
- 3 · 1 (black bishops)
- + 5 · 1 (white rooks)
- 5 · 2 (black rooks)
- + 9 · 1 (white queen)
- 9 · 1 (black queen)
- = 3

# Idea

- Define simple numerical **state features**  $f_1, \dots, f_n$ .
- Consider heuristics that are **linear combinations** of features:

$$h(s) = w_1 f_1(s) + \dots + w_n f_n(s)$$

with weights (**potentials**)  $w_i \in \mathbb{R}$

- heuristic **very fast to compute** if feature values are

# Potential Heuristics

# Definition

## Definition (Feature)

A (state) **feature** for a planning task is a numerical function defined on the states of the task:  $f : S \rightarrow \mathbb{R}$ .

## Definition (Potential Heuristic)

A **potential heuristic** for a set of features  $\mathcal{F} = \{f_1, \dots, f_n\}$  is a heuristic function  $h$  defined as a **linear combination** of the features:

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with weights (**potentials**)  $w_i \in \mathbb{R}$ .

Many possibilities  $\Rightarrow$  need some restrictions

# Features for SAS<sup>+</sup> Planning Tasks

Which features are good for planning?

**Atomic features** test if some atom is true in a state:

## Definition (Atomic Feature)

Let  $v = d$  be an atom of a FDR planning task.

The **atomic feature**  $f_{v=d}$  is defined as:

$$f_{v=d}(s) = [(v = d) \in s] = \begin{cases} 1 & \text{if variable } v \text{ has value } d \text{ in state } s \\ 0 & \text{otherwise} \end{cases}$$

Offer good **tradeoff** between computation time and guidance

## Example: Atomic Features

### Example

Consider a planning task  $\Pi$  with state variables  $v_1$  and  $v_2$  and  $\text{dom}(v_1) = \text{dom}(v_2) = \{d_1, d_2, d_3\}$ . The set

$$\mathcal{F} = \{f_{v_i=d_j} \mid i \in \{1, 2\}, j \in \{1, 2, 3\}\}$$

is the **set of atomic features** of  $\Pi$  and the function

$$h(s) = 3f_{v_1=d_1} + 0.5f_{v_1=d_2} - 2f_{v_1=d_3} + 2.5f_{v_2=d_1}$$

is a **potential heuristic** for  $\mathcal{F}$ .

The heuristic estimate for a state  $s = \{v_1 \mapsto d_2, v_2 \mapsto d_1\}$  is

$$h(s) = 3 \cdot 0 + 0.5 \cdot 1 - 2 \cdot 0 + 2.5 \cdot 1 = 3.$$

# Potentials for Optimal Planning

Which potentials are good for optimal planning and how can we compute them?

- We seek potentials for which  $h$  is admissible and well-informed  
⇒ **declarative approach** to heuristic design
- We derive potentials by solving an **optimization problem**

How to achieve this? **Linear programming to the rescue!**

# Admissible and Consistent Potential Heuristics

We achieve admissibility through goal-awareness and consistency

Goal-awareness

$$\sum_{a \in \gamma} w_a = 0$$

# Admissible and Consistent Potential Heuristics

We achieve admissibility through goal-awareness and consistency

## Goal-awareness

$$\sum_{a \in \gamma} w_a = 0$$

## Consistency

$$\sum_{a \in s} w_a - \sum_{a \in s'} w_a \leq \text{cost}(o) \quad \text{for all transitions } s \xrightarrow{o} s'$$

One constraint transition per transition.

Can we do this more compactly?

# Admissible and Consistent Potential Heuristics

Consistency for a transition  $s \xrightarrow{o} s'$

$$\begin{aligned} \text{cost}(o) &\geq \sum_{a \in s} w_a - \sum_{a \in s'} w_a \\ &= \sum_a w_a [a \in s] - \sum_a w_a [a \in s'] \\ &= \sum_a w_a ([a \in s] - [a \in s']) \\ &= \sum_a w_a [a \in s \text{ but } a \notin s'] - \sum_a w_a [a \notin s \text{ but } a \in s'] \\ &= \sum_{\substack{a \text{ consumed} \\ \text{by } o}} w_a - \sum_{\substack{a \text{ produced} \\ \text{by } o}} w_a \end{aligned}$$

# Admissible and Consistent Potential Heuristics

Goal-awareness and Consistency independent of  $s$

## Goal-awareness

$$\sum_{a \in \gamma} w_a = 0$$

## Consistency

$$\sum_{\substack{a \text{ consumed} \\ \text{by } o}} w_a - \sum_{\substack{a \text{ produced} \\ \text{by } o}} w_a \leq \text{cost}(o) \quad \text{for all operators } o$$



# Potential Heuristics

- All potential heuristics that satisfy these constraints are admissible and consistent
- Furthermore, all admissible and consistent potential heuristics satisfy these constraints

Constraints are a compact **characterization** of all admissible and consistent potential heuristics.

LP can be used to find **the best** admissible and consistent potential heuristics by encoding a **quality metric** in the **objective function**

# Well-Informed Potential Heuristics

What do we mean by **the best** potential heuristic?

Different possibilities, e.g., the potential heuristic that

- maximizes **heuristic value of a given state  $s$**  (e.g., initial state)
- maximizes average heuristic value of **all states** (including unreachable ones)
- maximizes average heuristic value of some **sample states**
- minimizes **estimated search effort**

# Potential and Flow Heuristic

## Theorem

For state  $s$ , let  $h^{\text{maxpot}}(s)$  denote the *maximal* heuristic value of all admissible and consistent atomic potential heuristics in  $s$ .

Then  $h^{\text{maxpot}}(s) = h^{\text{flow}}(s)$ .

**Proof idea:** compare dual of  $h^{\text{flow}}(s)$  LP to potential heuristic constraints optimized for state  $s$ .

If we optimize the potentials for a given state then for this state it equals the flow heuristic.

# Summary

# Summary

- Potential heuristics are computed as a **weighted sum of state features**
- Admissibility and consistency can be **encoded compactly** in constraints
- With linear programming, we can efficiently compute the **best potential heuristic** wrt some objective
- Potential heuristics can be used as **fast admissible approximations** of  $h^{\text{flow}}$ .