

Planning and Optimization

G9. Post-hoc Optimization

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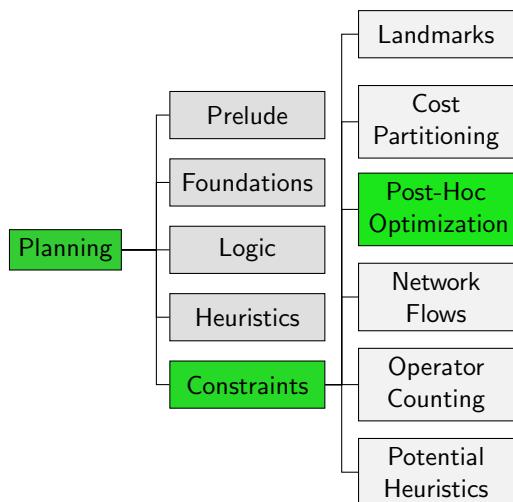
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Content of this Course



G9.1 Introduction

Example Task (1)

Example (Example Task)

SAS⁺ task $\Pi = \langle V, I, O, \gamma \rangle$ with

- ▶ $V = \{A, B, C\}$ with $\text{dom}(v) = \{0, 1, 2, 3, 4\}$ for all $v \in V$
- ▶ $I = \{A \mapsto 0, B \mapsto 0, C \mapsto 0\}$
- ▶ $O = \{inc_x^v \mid v \in V, x \in \{0, 1, 2\}\} \cup \{jump^v \mid v \in V\}$
 - ▶ $inc_x^v = \langle v = x, v := x + 1, 1 \rangle$
 - ▶ $jump^v = \langle \bigwedge_{v' \in V: v' \neq v} v' = 4, v := 3, 1 \rangle$
- ▶ $\gamma = A = 3 \wedge B = 3 \wedge C = 3$

- ▶ Each optimal plan consists of three increment operators for each variable $\rightsquigarrow h^*(I) = 9$
- ▶ Each operator affects only one variable.

Post-hoc Optimization Heuristic: Idea

Consider the example task:

- ▶ **type- v operator:** operator modifying variable v
- ▶ $h^{\{A,B\}} = 6$
 \Rightarrow in any plan **operators of type A or B incur at least cost 6.**
- ▶ $h^{\{A,C\}} = 6$
 \Rightarrow in any plan **operators of type A or C incur at least cost 6.**
- ▶ $h^{\{B,C\}} = 6$
 \Rightarrow in any plan **operators of type B or C incur at least cost 6.**
- ▶ \Rightarrow any plan **has at least cost ???.**
- ▶ (let's use linear programming...)
- ▶ \Rightarrow any plan **has at least cost 9.**

Can we generalize this kind of reasoning?

Example Task (2)

- ▶ In projections on single variables we can reach the goal with a *jump* operator: $h^{\{A\}}(I) = h^{\{B\}}(I) = h^{\{C\}}(I) = 1$.
- ▶ In projections on more variables, we need for each variable three applications of increment operators to reach the abstract goal from the abstract initial state:
 $h^{\{A,B\}}(I) = h^{\{A,C\}}(I) = h^{\{B,C\}}(I) = 6$

Example (Canonical Heuristic)

$\mathcal{C} = \{\{A\}, \{B\}, \{C\}, \{A, B\}, \{A, C\}, \{B, C\}\}$

$$h^{\mathcal{C}}(s) = \max\{h^{\{A\}}(s) + h^{\{B\}}(s) + h^{\{C\}}(s), h^{\{A\}}(s) + h^{\{B,C\}}(s), \\ h^{\{B\}}(s) + h^{\{A,C\}}(s), h^{\{C\}}(s) + h^{\{A,B\}}(s)\}$$

$$h^{\mathcal{C}}(I) = 7$$

G9.2 Post-hoc Optimization

Post-hoc Optimization

The heuristic that generalizes this kind of reasoning is the **Post-hoc Optimization Heuristic** (PhO)

- ▶ can be computed for any kind of heuristic ...
- ▶ ... as long as we are able to determine **relevance** of operators
- ▶ if in doubt, it's always safe to assume an operator is relevant for a heuristic
- ▶ but for PhO to work well, it's important that the set of relevant operators is as small as possible

Operator Relevance in Abstractions

Definition (Reminder: Affecting Transition Labels)

Let \mathcal{T} be a transition system, and let ℓ be one of its labels.

We say that ℓ **affects** \mathcal{T} if \mathcal{T} has a transition $s \xrightarrow{\ell} t$ with $s \neq t$.

Definition (Operator Relevance in Abstractions)

An operator o is **relevant** for an abstraction α if o **affects** \mathcal{T}^α .

We can efficiently determine operator relevance for abstractions.

Linear Program (1)

For a given set of abstractions $\{\alpha_1, \dots, \alpha_n\}$, we construct a **linear program**:

- ▶ variable X_o for each operator $o \in O$
- ▶ intuitively, X_o is **cost incurred** by operator o
- ▶ abstraction heuristics are admissible

$$\sum_{o \in O} X_o \geq h^\alpha(s) \quad \text{for } \alpha \in \{\alpha_1, \dots, \alpha_n\}$$

- ▶ can tighten these constraints to

$$\sum_{o \in O: o \text{ relevant for } \alpha} X_o \geq h^\alpha(s) \quad \text{for } \alpha \in \{\alpha_1, \dots, \alpha_n\}$$

Linear Program (2)

For set of abstractions $\{\alpha_1, \dots, \alpha_n\}$:

Variables

Non-negative variables X_o for all operators $o \in O$

Objective

Minimize $\sum_{o \in O} X_o$

Subject to

$$\begin{aligned} \sum_{o \in O: o \text{ relevant for } \alpha} X_o &\geq h^\alpha(s) & \text{for } \alpha \in \{\alpha_1, \dots, \alpha_n\} \\ X_o &\geq 0 & \text{for all } o \in O \end{aligned}$$

Simplifying the LP

- ▶ Reduce the size of the LP by aggregating variables which always occur together in constraints.
- ▶ Happens if several operators are relevant for exactly the same heuristics.
- ▶ Partitioning O/\sim induced by this equivalence relation
- ▶ One variable $X_{[o]}$ for each $[o] \in O/\sim$

Example

- ▶ only operators o_1, o_2, o_3 and o_4 are relevant for h_1 and $h_1(s_0) = 11$
 - ▶ only operators o_3, o_4, o_5 and o_6 are relevant for h_2 and $h_2(s_0) = 11$
 - ▶ only operators o_1, o_2 and o_6 are relevant for h_3 and $h_3(s_0) = 8$

Which operators are relevant for exactly the same heuristics?
What is the resulting partitioning?

Answer: $o_1 \sim o_2$ and $o_3 \sim o_4$
 $\Rightarrow O/\sim = \{[o_1], [o_3], [o_5], [o_6]\}$

Simplifying the LP: Example

LP before aggregation

Variables

Non-negative variable X_1, \dots, X_6
for operators o_1, \dots, o_6

Minimize $X_1 + X_2 + X_3 + X_4 + X_5 + X_6$ subject to

$$X_1 + X_2 + X_3 + X_4 \geq 11$$

$$X_3 + X_4 + X_5 + X_6 \geq 11$$

$$X_1 + X_2 + X_6 \geq 8$$

$$X_i \geq 0 \text{ for } i \in \{1, \dots, 6\}$$

Simplifying the LP: Example

LP after aggregation

Variables

Non-negative variable $X_{[1]}, X_{[3]}, X_{[5]}, X_{[6]}$
for equivalence classes $[o_1], [o_3], [o_5], [o_6]$

Minimize $X_{[1]} + X_{[3]} + X_{[5]} + X_{[6]}$ subject to

$$X_{[1]} + X_{[3]} \geq 11$$

$$X_{[3]} + X_{[5]} + X_{[6]} \geq 11$$

$$X_{[1]} + X_{[6]} \geq 8$$

$$X_i \geq 0 \text{ for } i \in \{[1], [3], [5], [6]\}$$

PhO Heuristic

Definition (Post-hoc Optimization Heuristic)

The post-hoc optimization heuristic $h_{\{\alpha_1, \dots, \alpha_n\}}^{\text{PhO}}$ for abstractions $\alpha_1, \dots, \alpha_n$ is the objective value of the following linear program:

$$\text{Minimize} \sum_{[o] \in O/\sim} X_{[o]} \text{ subject to}$$

$$\sum_{\substack{[o] \in O/\sim : o \text{ relevant for } \alpha}} X_{[o]} \geq h^\alpha(s) \quad \text{for all } \alpha \in \{\alpha_1, \dots, \alpha_n\}$$

$$X_{[o]} \geq 0 \quad \text{for all } [o] \in O/\sim,$$

where $o \sim o'$ iff o and o' are relevant for exactly the same abstractions in $\alpha_1, \dots, \alpha_n$.

PhO Heuristic

h^{PhO}

- ➊ Precompute all abstraction heuristics $h^{\alpha_1}, \dots, h^{\alpha_n}$.
- ➋ Create LP for initial state s_0 .
- ➌ For each new state s :
 - ▶ Look up $h^\alpha(s)$ for all $\alpha \in \{\alpha_1, \dots, \alpha_n\}$.
 - ▶ Adjust LP by replacing bounds with the $h^\alpha(s)$ values.

Post-hoc Optimization Heuristic: Admissibility

Theorem (Admissibility)

The post-hoc optimization heuristic is **admissible**.

Proof.

Let Π be a planning task and $\{\alpha_1, \dots, \alpha_n\}$ be a set of abstractions. We show that there is a feasible variable assignment with objective value equal to the cost of an optimal plan.

Let π be an optimal plan for state s and let $\text{cost}_\pi(O')$ be the cost incurred by operators from $O' \subseteq O$ in π .

Setting each $X_{[o]}$ to $\text{cost}_\pi([o])$ is a feasible variable assignment: Constraints $X_{[o]} \geq 0$ are satisfied. \dots

Post-hoc Optimization Heuristic: Admissibility

Theorem (Admissibility)

The post-hoc optimization heuristic is **admissible**.

Proof (continued).

For each $\alpha \in \{\alpha_1, \dots, \alpha_n\}$, π is a solution in the abstract transition system and the sum in the corresponding constraint equals the cost of the state-changing abstract state transitions (i.e.. not accounting for self-loops). As $h^\alpha(s)$ corresponds to the cost of an optimal solution in the abstraction, the inequality holds.

For this assignment, the objective function has value $h^*(s)$ (cost of π), so the objective value of the LP is admissible. \square

G9.3 Comparison

Combining Estimates from Abstraction Heuristics

- ▶ Post-Hoc optimization combines multiple admissible heuristic estimates into one.
- ▶ We have already heard of two other such approaches for abstraction heuristics,
 - ▶ the canonical heuristic (for PDBs), and
 - ▶ optimal cost partitioning (not covered in detail).
- ▶ How does PhO compare to these?

Reminder: The Canonical Heuristic Function

If for a set of patterns no operator affects more than one pattern, the sum of the heuristic estimates is admissible.

Definition (Canonical Heuristic Function)

Let \mathcal{C} be a pattern collection for an FDR planning task.

The **canonical heuristic** h^C for pattern collection \mathcal{C} is defined as

$$h^C(s) = \max_{\mathcal{D} \in \text{cliques}(\mathcal{C})} \sum_{P \in \mathcal{D}} h^P(s),$$

where $\text{cliques}(\mathcal{C})$ is the set of all maximal cliques in the compatibility graph for \mathcal{C} .

For a given pattern collection, the canonical heuristic is the best possible admissible heuristic not using cost partitioning.

What about Optimal Cost Partitioning for Abstractions?

Optimal cost partitioning for abstractions...

- ▶ ... uses a **state-specific LP** to find the **best possible cost partitioning**, and sums up the heuristic estimates.
- ▶ ... **dominates the canonical heuristic**, i.e. for the same pattern collection, it never gives lower estimates than h^C .
- ▶ ... is **very expensive** to compute (recomputing all abstract goal distances in every state).

PhO: Linear Program

For set of abstractions $\{\alpha_1, \dots, \alpha_n\}$:

Variables

$X_{[o]}$ for all equivalence classes $[o] \in O/\sim$

Objective

Minimize $\sum_{[o] \in O/\sim} X_{[o]}$

Subject to

$$\begin{aligned} \sum_{[o] \in O/\sim: o \text{ relevant for } \alpha} X_{[o]} &\geq h^\alpha(s) \quad \text{for all } \alpha \in \{\alpha_1, \dots, \alpha_n\} \\ X_{[o]} &\geq 0 \quad \text{for all } [o] \in O/\sim \end{aligned}$$

PhO: Dual Linear Program

For set of abstractions $\{\alpha_1, \dots, \alpha_n\}$:

Variables

Y_α for each abstraction $\alpha \in \{\alpha_1, \dots, \alpha_n\}$

Objective

Maximize $\sum_{\alpha \in \{\alpha_1, \dots, \alpha_n\}} h^\alpha(s) Y_\alpha$

Subject to

$$\begin{aligned} \sum_{\alpha \in \{\alpha_1, \dots, \alpha_n\}: o \text{ relevant for } \alpha} Y_\alpha &\leq 1 \quad \text{for all } [o] \in O/\sim \\ Y_\alpha &\geq 0 \quad \text{for all } \alpha \in \{\alpha_1, \dots, \alpha_n\} \end{aligned}$$

We compute a state-specific cost partitioning that can only scale the operator costs within each heuristic by a factor $0 \leq Y_\alpha \leq 1$.

Relation to Optimal Cost Partitioning

Theorem

Optimal cost partitioning dominates post-hoc optimization.

Proof Sketch.

Consider a feasible assignment $\langle Y_{\alpha_1}, \dots, Y_{\alpha_n} \rangle$ for the variables of the dual LP for PhO.

Its objective value is equivalent to the cost-partitioning heuristic for the same abstractions with cost partitioning

$\langle Y_{\alpha_1} \text{ cost}, \dots, Y_{\alpha_n} \text{ cost} \rangle$.

Relation to Canonical Heuristic

Theorem

Consider the **dual D** of the LP solved by the post-hoc optimization heuristic in state s for a given set of abstractions. If we **restrict the variables in D to integers**, the **objective value is the canonical heuristic value $h^c(s)$** .

Corollary

The post-hoc optimization heuristic **dominates the canonical heuristic** for the same set of abstractions.

h^{PhO} vs $h^{\mathcal{C}}$

- ▶ For the canonical heuristic, we need to find all maximal cliques, which is an **NP-hard** problem.
- ▶ The post-hoc optimization heuristic **dominates the canonical heuristic** and can be computed in **polynomial time**.
- ▶ The post-hoc optimization heuristic solves an LP in each state but does not require a preprocessing step
- ▶ With post-hoc optimization, a **large number of small patterns** works well.

G9.4 Summary

Summary

- ▶ **Post-hoc optimization heuristic** constraints express admissibility of heuristics
- ▶ exploits (ir-)relevance of operators for heuristics
- ▶ explores the middle ground between canonical heuristic and optimal cost partitioning.
- ▶ For the same set of abstractions, the post-hoc optimization heuristic **dominates the canonical heuristic**.
- ▶ The computation can be done in **polynomial time**.