# Planning and Optimization

## G4. Landmarks: Minimum Hitting Set Heuristic

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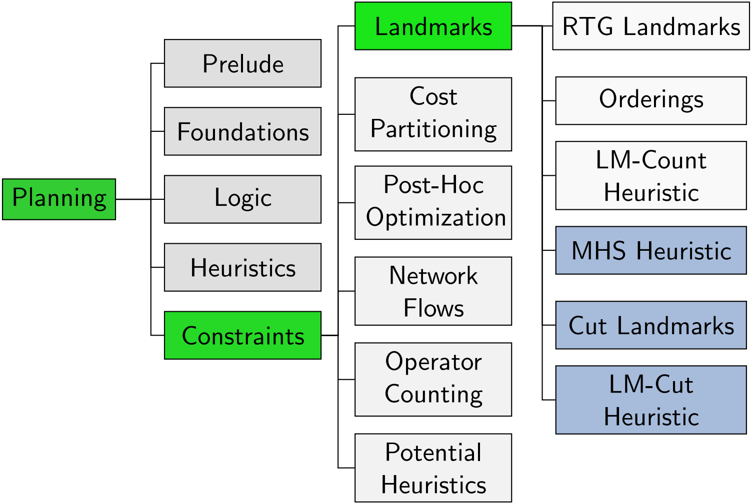
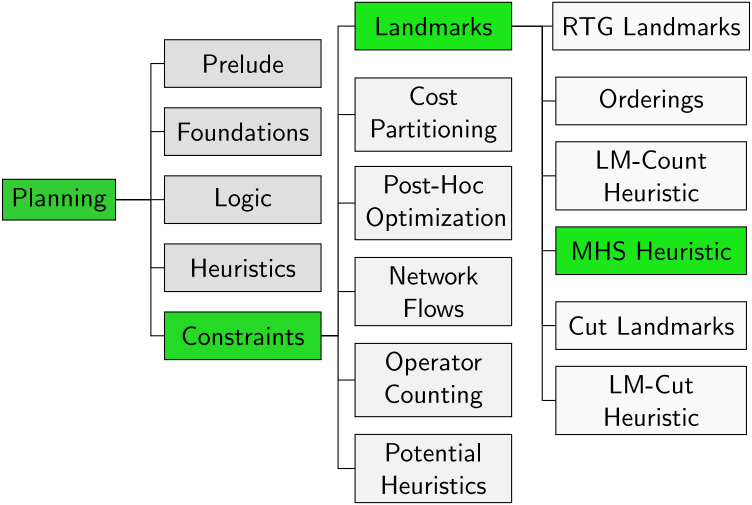
# Content of this Course




The remaining landmark topics focus on disjunctive action landmarks.

# Minimum Hitting Set Heuristic

# Content of this Course



# Exploiting Disjunctive Action Landmarks

- The cost $cost(L)$ of a disjunctive action landmark $L$ is an admissible heuristic, but it is usually not very informative.
- Landmark heuristics typically aim to combine multiple disjunctive action landmarks.

How can we exploit a given set $\mathcal{L}$ of disjunctive action landmarks?

- Sum of costs $\sum_{L \in \mathcal{L}} cost(L)$?
  $\rightsquigarrow$ not admissible!
- Maximize costs $\max_{L \in \mathcal{L}} cost(L)$?
  $\rightsquigarrow$ usually very weak heuristic
- better: Hitting sets

# Hitting Sets

## Definition (Hitting Set)

Let $X$ be a set, $\mathcal{F} = \{F_1, \ldots, F_n\} \subseteq 2^X$ be a family of subsets of $X$ and $c : X \to \mathbb{R}_0^+$ be a cost function for $X$.

A hitting set is a subset $H \subseteq X$ that "hits" all subsets in $\mathcal{F}$, i.e., $H \cap F \neq \emptyset$ for all $F \in \mathcal{F}$. The cost of $H$ is $\sum_{x \in H} c(x)$.

A minimum hitting set (MHS) is a hitting set with minimal cost.

MHS is a "classical" NP-complete problem (Karp, 1972)

# Example: Hitting Sets

**Example**

$X = \{o_1, o_2, o_3, o_4\}$

$\mathcal{F} = \{\{o_4\}, \{o_1, o_2\}, \{o_1, o_3\}, \{o_2, o_3\}\}$

$c(o_1) = 3, \;\; c(o_2) = 4, \;\; c(o_3) = 5, \;\; c(o_4) = 0$

Specify a minimum hitting set.

# Example: Hitting Sets

**Example**

$X = \{o_1, o_2, o_3, o_4\}$

$\mathcal{F} = \{\{o_4\}, \{o_1, o_2\}, \{o_1, o_3\}, \{o_2, o_3\}\}$

$c(o_1) = 3, \;\; c(o_2) = 4, \;\; c(o_3) = 5, \;\; c(o_4) = 0$

Specify a minimum hitting set.

Solution: $\{o_1, o_2, o_4\}$ with cost $3 + 4 + 0 = 7$

# Hitting Sets for Disjunctive Action Landmarks

Idea: disjunctive action landmarks are interpreted as instance of minimum hitting set

**Definition (Hitting Set Heuristic)**

Let $\mathcal{L}$ be a set of disjunctive action landmarks. The hitting set heuristic $h^{\mathrm{MHS}}(\mathcal{L})$ is defined as the cost of a minimum hitting set for $\mathcal{L}$ with $c(o) = cost(o)$.

**Proposition (Hitting Set Heuristic is Admissible)**

*Let $\mathcal{L}$ be a set of disjunctive action landmarks for state $s$. Then $h^{MHS}(\mathcal{L})$ is an admissible estimate for $s$.*

# Hitting Set Heuristic: Discussion

- The hitting set heuristic is the best possible heuristic that only uses the given information…
- …but is NP-hard to compute.
- $\rightsquigarrow$ Use approximations that can be efficiently computed.
  $\Rightarrow$ LP-relaxation, cost partitioning (both discussed later)

# Summary

# Summary

- Hitting sets yield the most accurate heuristic for a given set of disjunctive action landmarks.
- The computation of a minimal hitting set is NP-hard.