

# Planning and Optimization

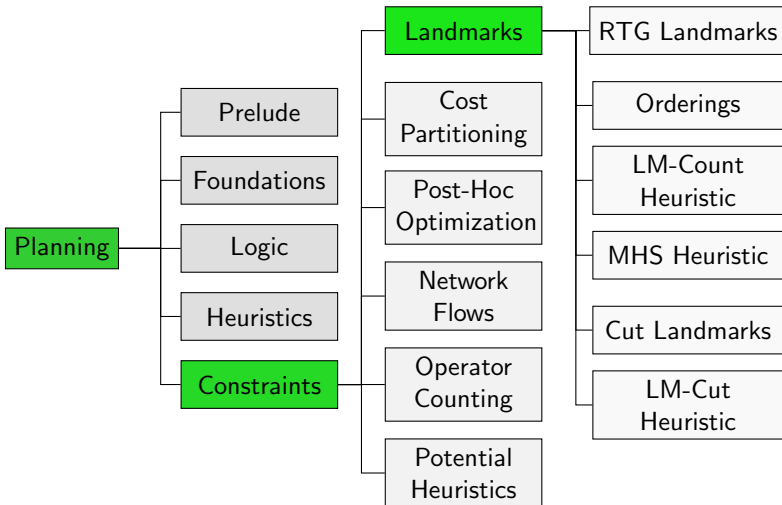
## G2. Landmarks: RTG Landmarks

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# Content of this Course



# Landmarks

# Landmarks

**Basic Idea:** Something that must happen **in every solution**

For example

- some operator must be applied (**action landmark**)
- some atomic proposition must hold (**fact landmark**)
- some formula must be true (**formula landmark**)

→ Derive heuristic estimate from this kind of information.

# Landmarks

**Basic Idea:** Something that must happen **in every solution**

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- some operator must be applied (**action landmark**)
- some atomic proposition must hold (**fact landmark**)
- some formula must be true (**formula landmark**)

→ Derive heuristic estimate from this kind of information.

We mostly consider **fact** and **disjunctive action landmarks**.

## Reminder: Terminology

Consider sequence of transitions  $s^0 \xrightarrow{\ell_1} s^1, \dots, s^{n-1} \xrightarrow{\ell_n} s^n$   
such that  $s^0 = s$  and  $s^n = s'$ .

- $s^0, \dots, s^n$  is called **(state) path** from  $s$  to  $s'$
- $\ell_1, \dots, \ell_n$  is called **(label) path** from  $s$  to  $s'$

# Disjunctive Action Landmarks

## Definition (Disjunctive Action Landmark)

Let  $s$  be a state of a propositional or FDR planning task  $\Pi = \langle V, I, O, \gamma \rangle$ .

A **disjunctive action landmark** for  $s$  is a set of operators  $L \subseteq O$  such that every label path from  $s$  to a goal state contains an operator from  $L$ .

The **cost** of landmark  $L$  is  $cost(L) = \min_{o \in L} cost(o)$ .

If we talk about landmarks for the initial state, we omit “for  $I$ ”.

## Fact and Formula Landmarks

### Definition (Formula and Fact Landmark)

Let  $s$  be a state of a propositional or FDR planning task  $\Pi = \langle V, I, O, \gamma \rangle$ .

A **formula landmark** for  $s$  is a formula  $\lambda$  over  $V$  such that every state path from  $s$  to a goal state contains a state  $s'$  with  $s' \models \lambda$ .

If  $\lambda$  is an atomic proposition then  $\lambda$  is a **fact landmark**.

If we talk about landmarks for the initial state, we omit “for  $I$ ”.










# Landmarks: Example

## Example

Consider a FDR planning task  $\langle V, I, O, \gamma \rangle$  with

- $V = \{robot-at, dishes-at\}$  with
  - $dom(robot-at) = \{A1, \dots, C3, B4, A5, \dots, B6\}$
  - $dom(dishes-at) = \{Table, Robot, Dishwasher\}$
- $I = \{robot-at \mapsto C1, dishes-at \mapsto Table\}$
- operators
  - move- $x$ - $y$  to move from cell  $x$  to adjacent cell  $y$
  - pickup dishes, and
  - load dishes into the dishwasher.
- $\gamma = (robot-at = B6) \wedge (dishes-at = Dishwasher)$

# Fact and Formula Landmarks: Example

	1	2	3	4	5	6
A						
B						
C						

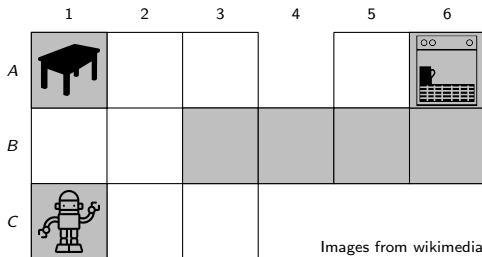
Images from wikimedia



Each fact in gray is a fact landmark:

- $\text{robot-at} = x$  for  $x \in \{A1, A6, B3, B4, B5, B6, C1\}$
- $\text{dishes-at} = x$  for  $x \in \{\text{Dishwasher, Robot, Table}\}$

# Fact and Formula Landmarks: Example



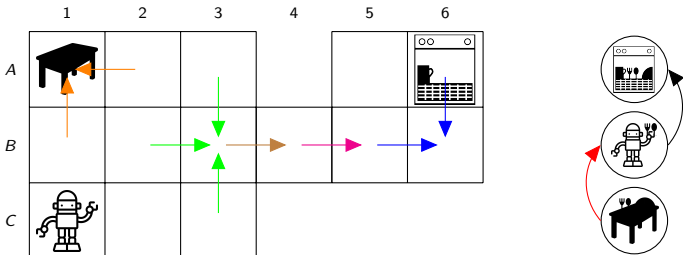
Each fact in gray is a fact landmark:

- $\text{robot-at} = x$  for  $x \in \{A1, A6, B3, B4, B5, B6, C1\}$
- $\text{dishes-at} = x$  for  $x \in \{\text{Dishwasher, Robot, Table}\}$

Formula landmarks:

- $\text{dishes-at} = \text{Robot} \wedge \text{robot-at} = B4$
- $\text{robot-at} = B1 \vee \text{robot-at} = A2$

# Disjunctive Action Landmarks: Example



Actions of same color form disjunctive action landmark:

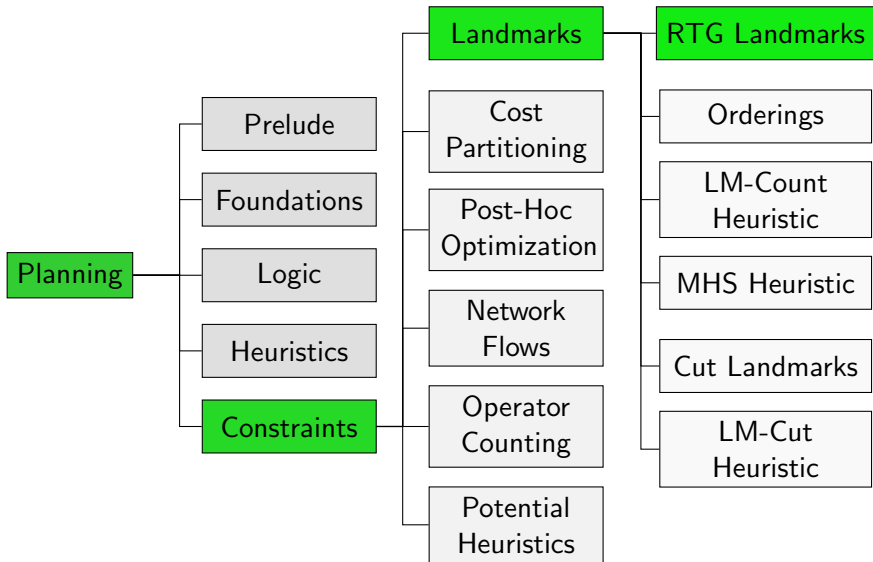
- {pickup}
- {load}
- {move-B3-B4}
- {move-B4-B5}
- {move-A6-B6, move-B5-B6}
- {move-A3-B3, move-B2-B3, move-C3-B3}
- {move-B1-A1, move-A2-A1}
- ...

## Remarks

- Not every landmark is informative. **Some examples:**
  - The set of all operators is a disjunctive action landmark unless the initial state is already a goal state.
  - Every variable that is initially true is a fact landmark.
  - The goal formula is a formula landmark.
- Deciding whether a given atomic proposition is a fact landmark is as hard as the plan existence problem.
- Deciding whether a given operator set is a disjunctive action landmark is as hard as the plan existence problem.
- Every fact landmark  $v$  that is initially false induces a disjunctive action landmark consisting of all operators that possibly make  $v$  true.

# Landmarks from RTGs

# Content of this Course



# Computing Landmarks

How can we come up with landmarks?

Most landmarks are derived from the **relaxed task graph**:

- RHW landmarks: Richter, Helmert & Westphal. Landmarks Revisited. (AAAI 2008)
- **LM-Cut**: Helmert & Domshlak. Landmarks, Critical Paths and Abstractions: What's the Difference Anyway? (ICAPS 2009)
- **$h^m$  landmarks**: Keyder, Richter & Helmert: Sound and Complete Landmarks for And/Or Graphs (ECAI 2010)

We will now discuss  **$h^m$  landmarks** restricted to STRIPS planning tasks, starting with  $m = 1$ .



# Incidental Landmarks: Example

## Example (Incidental Landmarks)

Consider a STRIPS planning task  $\langle V, I, \{o_1, o_2\}, G \rangle$  with

$$V = \{a, b, c, d, e, f\},$$

$$I = \{a \mapsto \mathbf{T}, b \mapsto \mathbf{T}, c \mapsto \mathbf{F}, d \mapsto \mathbf{F}, e \mapsto \mathbf{T}, f \mapsto \mathbf{F}\},$$

$$o_1 = \langle \{a\}, \{c, d, e\}, \{b\} \rangle,$$

$$o_2 = \langle \{d, e\}, \{f\}, \{a\} \rangle, \text{ and}$$

$$G = \{e, f\}.$$

Single solution:  $\langle o_1, o_2 \rangle$

- All variables are fact landmarks.
- Variable  $b$  is initially true but irrelevant for the plan.
- Variable  $c$  gets true as “side effect” of  $o_1$  but it is not necessary for the goal or to make an operator applicable.

# Causal Landmarks (1)

## Definition (Causal Formula Landmark)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a propositional or FDR planning task.

A formula  $\lambda$  over  $V$  is a **causal formula landmark** for  $I$  if  $\gamma \models \lambda$  or if for all plans  $\pi = \langle o_1, \dots, o_n \rangle$  there is an  $o_i$  with  $pre(o_i) \models \lambda$ .

## Causal Landmarks (2)

Special case: Fact Landmark for STRIPS task

### Definition (Causal Fact Landmark)

Let  $\Pi = \langle V, I, O, G \rangle$  be a STRIPS planning task (in set representation).

A variable  $v \in V$  is a **causal fact landmark** for  $I$

- if  $v \in G$  or
- if for all plans  $\pi = \langle o_1, \dots, o_n \rangle$  there is an  $o_i$  with  $v \in \text{pre}(o_i)$ .

# Causal Landmarks: Example

## Example (Causal Landmarks)

Consider a STRIPS planning task  $\langle V, I, \{o_1, o_2\}, G \rangle$  with

$$V = \{a, b, c, d, e, f\},$$

$$I = \{a \mapsto \mathbf{T}, b \mapsto \mathbf{T}, c \mapsto \mathbf{F}, d \mapsto \mathbf{F}, e \mapsto \mathbf{T}, f \mapsto \mathbf{F}\},$$

$$o_1 = \langle \{a\}, \{c, d, e\}, \{b\} \rangle,$$

$$o_2 = \langle \{d, e\}, \{f\}, \{a\} \rangle, \text{ and}$$

$$G = \{e, f\}.$$

Single solution:  $\langle o_1, o_2 \rangle$

- All variables are fact landmarks for the initial state.
- Only  $a, d, e$  and  $f$  are causal landmarks.

## What We Are Doing Next

- Causal landmarks are the desirable landmarks.
- We can use the simplified version of RTGs for STRIPS to compute causal landmarks for STRIPS planning tasks.
- We will define landmarks of AND/OR graphs, ...
- and show how they can be computed.
- Afterwards we establish that these are landmarks of the planning task.

# Reminder: Simplified Relaxed Task Graph

## Definition

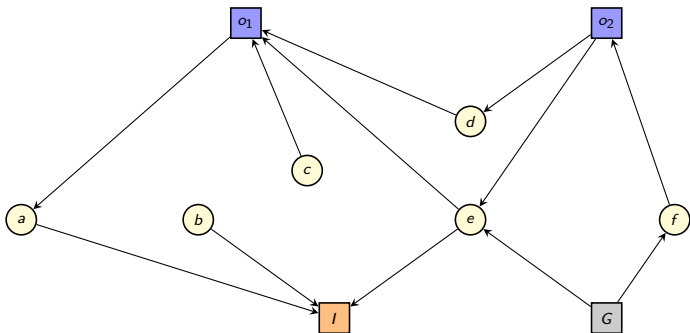
For a STRIPS planning task  $\Pi = \langle V, I, O, G \rangle$  (in set representation), the **simplified relaxed task graph**  $sRTG(\Pi^+)$  is the **AND/OR graph**  $\langle N_{\text{and}} \cup N_{\text{or}}, A, \text{type} \rangle$  with

- $N_{\text{and}} = \{n_o \mid o \in O\} \cup \{v_I, v_G\}$   
with  $\text{type}(n) = \wedge$  for all  $n \in N_{\text{and}}$ ,
- $N_{\text{or}} = \{n_v \mid v \in V\}$   
with  $\text{type}(n) = \vee$  for all  $n \in N_{\text{or}}$ , and
- $A = \{ \langle n_a, n_o \rangle \mid o \in O, a \in \text{add}(o) \} \cup$   
 $\{ \langle n_o, n_p \rangle \mid o \in O, p \in \text{pre}(o) \} \cup$   
 $\{ \langle n_v, n_I \rangle \mid v \in I \} \cup$   
 $\{ \langle n_G, n_v \rangle \mid v \in G \}$

Like RTG but without extra nodes to support arbitrary conditions.

## Simplified RTG: Example

The simplified RTG for our example task is:



# Justification

## Definition (Justification)

Let  $G = \langle N, A, type \rangle$  be an AND/OR graph.

A subgraph  $J = \langle N^J, A^J, type^J \rangle$  with  $N^J \subseteq N$  and  $A^J \subseteq A$  and  $type^J = type|_{N^J}$  **justifies**  $n_\star \in N$  iff

- $n_\star \in N^J$ ,
- $\forall n \in N^J$  with  $type(n) = \wedge$ :  
 $\forall \langle n, n' \rangle \in A : n' \in N^J$  and  $\langle n, n' \rangle \in A^J$
- $\forall n \in N^J$  with  $type(n) = \vee$ :  
 $\exists \langle n, n' \rangle \in A : n' \in N^J$  and  $\langle n, n' \rangle \in A^J$ , and
- $J$  is acyclic.

“Proves” that  $n_\star$  is forced true.



# Landmarks in AND/OR Graphs

## Definition (Landmarks in AND/OR Graphs)

Let  $G = \langle N, A, type \rangle$  be an AND/OR graph.

A node  $n \in N$  is a **landmark** for reaching  $n_\star \in N$  if  $n \in V^J$  for all justifications  $J$  for  $n_\star$ .

**But:** exponential number of possible justifications

# Characterizing Equation System

## Theorem

Let  $G = \langle N, A, \text{type} \rangle$  be an AND/OR graph. Consider the following system of equations:

$$LM(n) = \{n\} \cup \bigcap_{\langle n, n' \rangle \in A} LM(n') \quad \text{type}(n) = \vee$$

$$LM(n) = \{n\} \cup \bigcup_{\langle n, n' \rangle \in A} LM(n') \quad \text{type}(n) = \wedge$$

The equation system has a unique maximal solution (maximal with regard to set inclusion), and for this solution it holds that

$n' \in LM(n)$  iff  $n'$  is a landmark for reaching  $n$  in  $G$ .

# Computation of Maximal Solution

## Theorem

Let  $G = \langle N, A, \text{type} \rangle$  be an AND/OR graph. Consider the following system of equations:

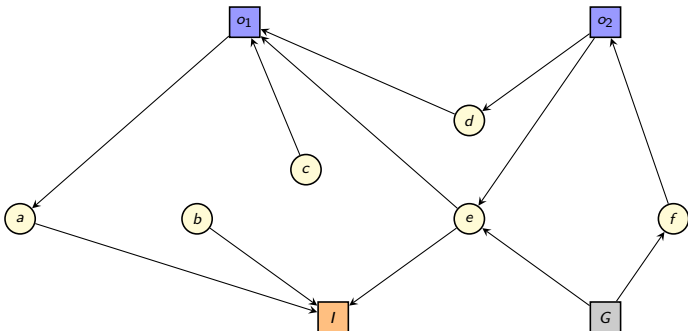
$$LM(n) = \{n\} \cup \bigcap_{\langle n, n' \rangle \in A} LM(n') \quad \text{type}(n) = \vee$$

$$LM(n) = \{n\} \cup \bigcup_{\langle n, n' \rangle \in A} LM(n') \quad \text{type}(n) = \wedge$$

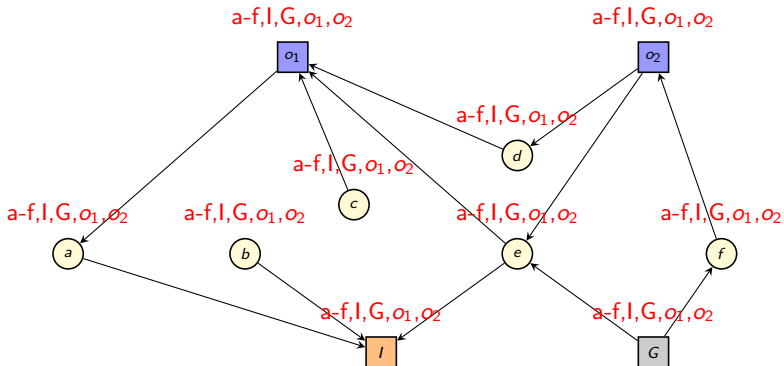
The equation system has a unique maximal solution (maximal with regard to set inclusion).

**Computation:** Initialize landmark sets as  $LM(n) = N$  and apply equations as update rules until fixpoint.

# Computation: Example

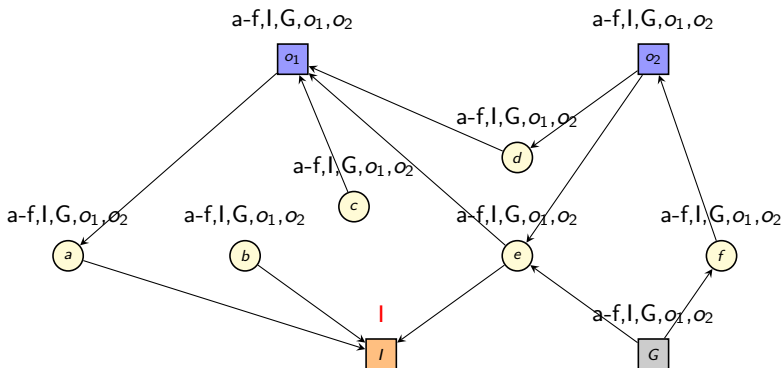


# Computation: Example



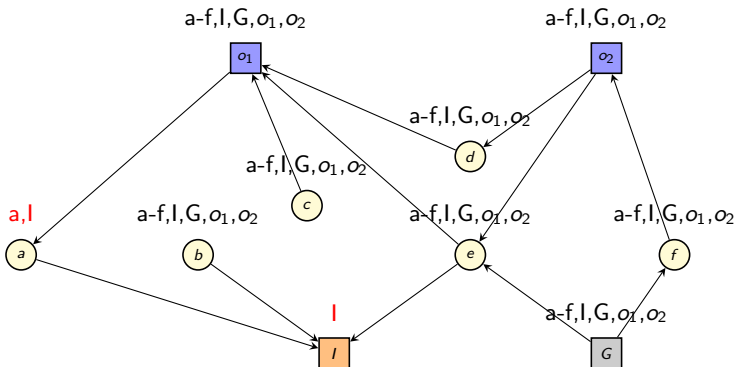
Initialize with all nodes

# Computation: Example



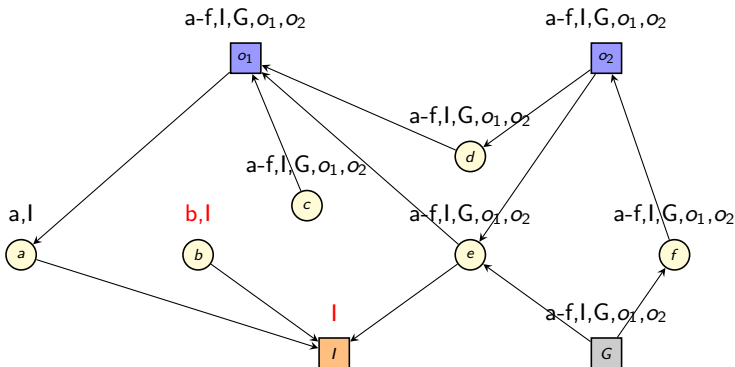
$$LM(I) = \{I\}$$

# Computation: Example



$$LM(a) = \{a\} \cup LM(I)$$

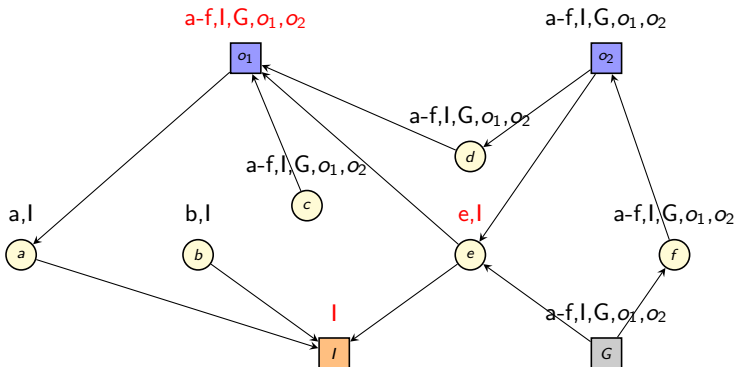
# Computation: Example



$$LM(b) = \{b\} \cup LM(I)$$

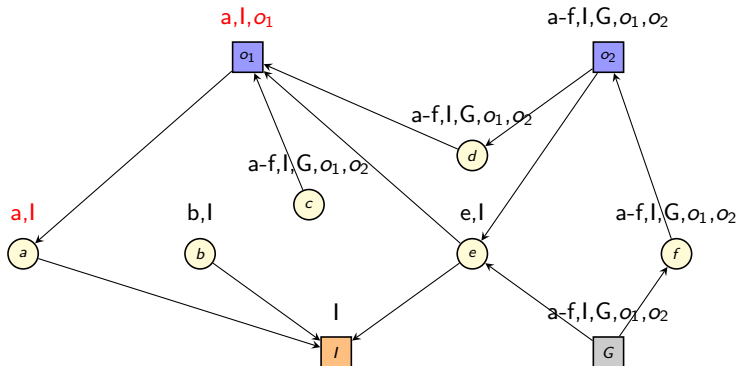


# Computation: Example



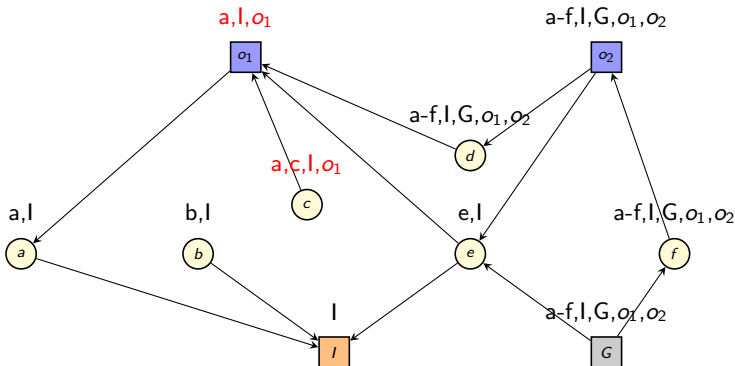
$$LM(e) = \{e\} \cup (LM(I) \cap LM(o_1))$$

# Computation: Example



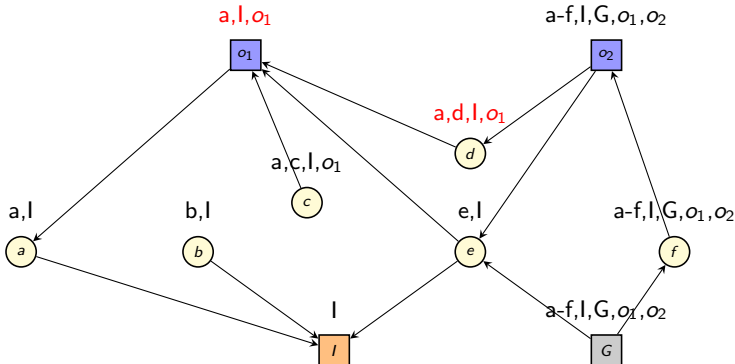
$$LM(o_1) = \{o_1\} \cup LM(a)$$

# Computation: Example



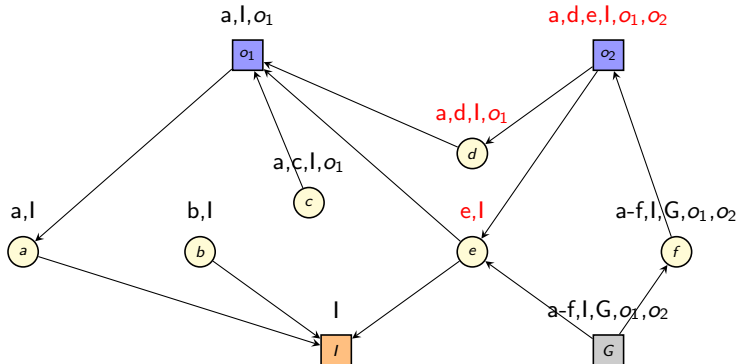
$$LM(c) = \{c\} \cup LM(o_1)$$

# Computation: Example



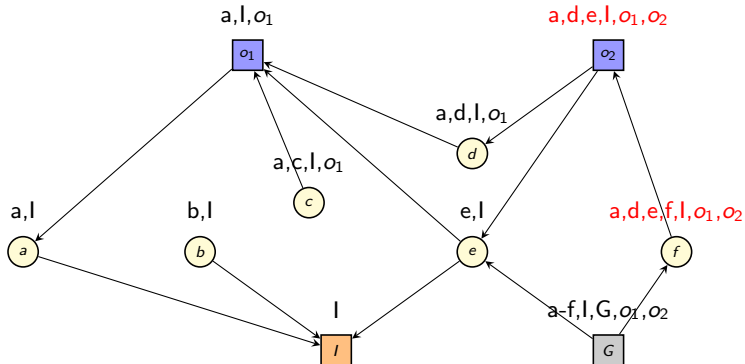
$$LM(d) = \{d\} \cup LM(o_1)$$

# Computation: Example



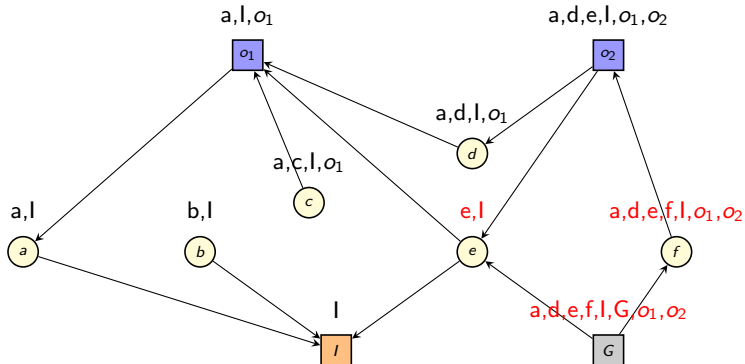
$$LM(o_2) = \{o_2\} \cup LM(d) \cup LM(e)$$

# Computation: Example



$$LM(f) = \{f\} \cup LM(o_2)$$

# Computation: Example



$$LM(G) = \{G\} \cup LM(e) \cup LM(f)$$

## Relation to Planning Task Landmarks

### Theorem

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a STRIPS planning task and let  $\mathcal{L}$  be the set of landmarks for reaching  $n_G$  in  $sRTG(\Pi^+)$ .

The set  $\{v \in V \mid n_v \in \mathcal{L}\}$  is exactly the set of *causal fact landmarks* in  $\Pi^+$ .

For operators  $o \in O$ , if  $n_o \in \mathcal{L}$  then  $\{o\}$  is a *disjunctive action landmark* in  $\Pi^+$ .

There are no other disjunctive action landmarks of size 1.

(Proofs omitted.)



# Computed RTG Landmarks: Example

## Example (Computed RTG Landmarks)

Consider a STRIPS planning task  $\langle V, I, \{o_1, o_2\}, G \rangle$  with

$$V = \{a, b, c, d, e, f\},$$

$$I = \{a \mapsto \mathbf{T}, b \mapsto \mathbf{T}, c \mapsto \mathbf{F}, d \mapsto \mathbf{F}, e \mapsto \mathbf{T}, f \mapsto \mathbf{F}\},$$

$$o_1 = \langle \{a\}, \{c, d, e\}, \{b\} \rangle,$$

$$o_2 = \langle \{d, e\}, \{f\}, \{a\} \rangle, \text{ and}$$

$$G = \{e, f\}.$$

- $LM(n_G) = \{a, d, e, f, I, G, o_1, o_2\}$
- $a, d, e,$  and  $f$  are causal fact landmarks of  $\Pi^+$ .
- $\{o_1\}$  and  $\{o_2\}$  are disjunctive action landmarks of  $\Pi^+$ .

# (Some) Landmarks of $\Pi^+$ Are Landmarks of $\Pi$

## Theorem

*Let  $\Pi$  be a STRIPS planning task.*

*All fact landmarks of  $\Pi^+$  are fact landmarks of  $\Pi$  and all disjunctive action landmarks of  $\Pi^+$  are disjunctive action landmarks of  $\Pi$ .*

## Proof.

Let  $L$  be a disjunctive action landmark of  $\Pi^+$  and  $\pi$  be a plan for  $\Pi$ . Then  $\pi$  is also a plan for  $\Pi^+$  and, thus,  $\pi$  contains an operator from  $L$ .

Let  $f$  be a fact landmark of  $\Pi^+$ . If  $f$  is already true in the initial state, then it is also a landmark of  $\Pi$ . Otherwise, every plan for  $\Pi^+$  contains an operator that adds  $f$  and the set of all these operators is a disjunctive action landmark of  $\Pi^+$ . Therefore, also each plan of  $\Pi$  contains such an operator, making  $f$  a fact landmark of  $\Pi$ .  $\square$

# Not All Landmarks of $\Pi^+$ are Landmarks of $\Pi$

## Example

Consider STRIPS task  $\langle \{a, b, c\}, \emptyset, \{o_1, o_2\}, \{c\} \rangle$  with  $o_1 = \langle \{\}, \{a\}, \{\}, 1 \rangle$  and  $o_2 = \langle \{a\}, \{c\}, \{a\}, 1 \rangle$ .

$a \wedge c$  is a formula landmark of  $\Pi^+$  but not of  $\Pi$ .

# Landmarks from $\Pi^m$

## Reminder: $\Pi^m$ Compilation

### Definition ( $\Pi^m$ )

Let  $\Pi = \langle V, I, O, G \rangle$  be a STRIPS planning task.

For  $m \in \mathbb{N}_1$ , the task  $\Pi^m$  is the STRIPS planning task  $\langle V^m, I^m, O^m, G^m \rangle$ , where

$O^m = \{a_{o,S} \mid o \in O, S \subseteq V, |S| < m, S \cap (\text{add}(o) \cup \text{del}(o)) = \emptyset\}$   
with

- $\text{pre}(a_{o,S}) = (\text{pre}(o) \cup S)^m$
- $\text{add}(a_{o,S}) = \{v_Y \mid Y \subseteq \text{add}(o) \cup S, |Y| \leq m, Y \cap \text{add}(o) \neq \emptyset\}$
- $\text{del}(a_{o,S}) = \emptyset$
- $\text{cost}(a_{o,S}) = \text{cost}(o)$

# Landmarks from the $\Pi^m$ Compilation (1)

Idea:

- $\Pi^m$  is delete-free, so we can compute all causal (meta-)fact landmarks from the AND/OR graph.
- These landmarks correspond to formula landmarks of the original problem.

## Landmarks from the $\Pi^m$ Compilation (2)

### Theorem

*Let  $\Pi = \langle V, I, O, G \rangle$  be a STRIPS planning task.  
If meta-variable  $v_S$  is a fact landmark for  $I^m$  in  $\Pi^m$  then  $\bigwedge_{v \in S} v$  is a formula landmark for  $I$  in  $\Pi$ .*

(Proof omitted.)

## $\Pi^m$ Landmarks: Example

Consider again our running example:

### Example

STRIPS planning task  $\Pi = \langle V, I, \{o_1, o_2\}, G \rangle$  with

$$V = \{a, b, c, d, e, f\},$$

$$I = \{a \mapsto \mathbf{T}, b \mapsto \mathbf{T}, c \mapsto \mathbf{F}, d \mapsto \mathbf{F}, e \mapsto \mathbf{T}, f \mapsto \mathbf{F}\},$$

$$o_1 = \langle \{a\}, \{c, d, e\}, \{b\} \rangle,$$

$$o_2 = \langle \{d, e\}, \{f\}, \{a\} \rangle, \text{ and}$$

$$G = \{e, f\}.$$

Meta-variable  $v_{\{d,e\}}$  is a causal fact landmark for  $I^2$  in  $\Pi^2$ ,  
so  $d \wedge e$  is a causal formula landmark for  $\Pi$ .



## Landmarks from the $\Pi^m$ Compilation (3)

### Theorem

Let  $\Pi = \langle V, I, O, G \rangle$  be a STRIPS planning task. For  $m \in \mathbb{N}_1$  let  $\mathcal{L}^m = \{ \bigwedge_{v \in C} v \mid C \subseteq V, v_C \text{ is a causal fact landmark of } \Pi^m \}$  be the set of formula landmarks derived from  $\Pi^m$ .

Let  $\lambda$  be a conjunction over  $V$  that is a causal formula landmark of  $\Pi$ . For sufficiently large  $m$ ,  $\mathcal{L}^m$  contains  $\lambda'$  with  $\lambda' \equiv \lambda$ .

(Proof omitted.)

$\rightsquigarrow$  can find all causal conjunctive formula landmarks

## $\Pi^m$ Landmarks: Discussion

- With the  $\Pi^m$  compilation, we can find causal fact landmarks of  $\Pi$  that are not causal fact landmarks of  $\Pi^+$ .
- In addition we can find conjunctive formula landmarks.
- The approach takes to some extent delete effects into account.
- However, the approach takes exponential time in  $m$ .
- Even for small  $m$ , the additional cost for computing the landmarks often outweighs the time saved from better heuristic guidance.

# Summary

# Summary

- **Fact landmark**: atomic proposition that is true in each state path to a goal
- **Disjunctive action landmark**: set  $L$  of operators such that every plan uses some operator from  $L$
- We can **efficiently compute all causal fact landmarks** of a delete-free STRIPS task from the (simplified) RTG.
- Fact landmarks of the delete relaxed task are also landmarks of the original task.
- We can use the  **$\Pi^m$  compilation** to find **more landmarks**.