

Planning and Optimization

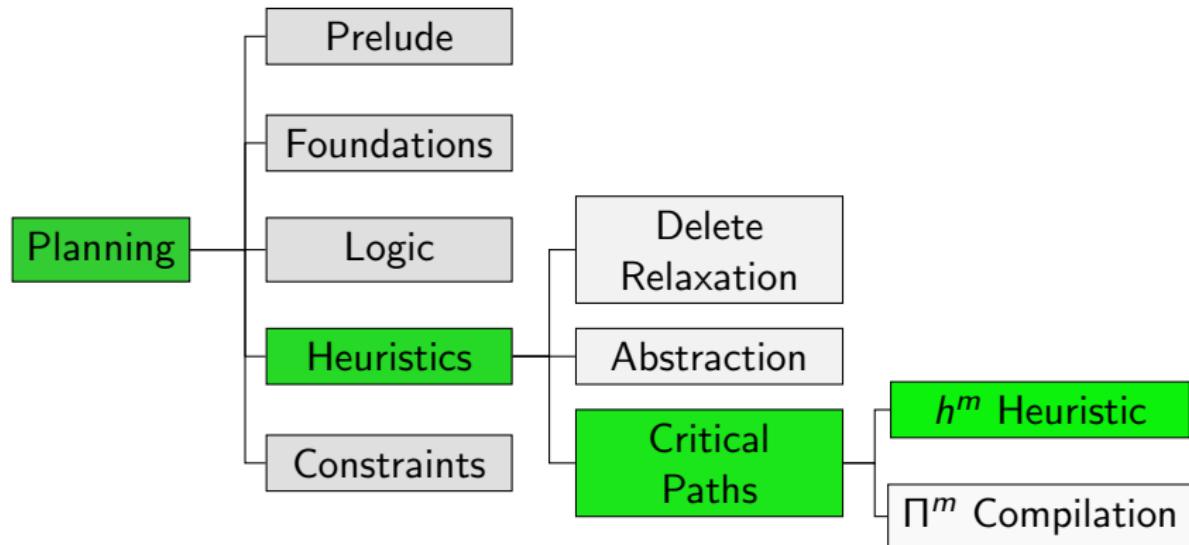
F2. Critical Path Heuristics: Properties and Π^m Compilation

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Content of this Course



Heuristic Properties

Heuristic for Forward or Backward Search? (1)

Any heuristic can be used for both, forward and backward search:

- Let h_f be a forward search heuristic (as in earlier chapters). We can use it to get estimate for state S in backward search on task (V, I, O, G) , computing $h_f(I)$ on task $(V, I, O, \textcolor{red}{S})$.

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- We also can use a backward search heuristic h_b in forward search on task (V, I, O, G) , determining estimate for state s as $h_b(G)$ on task $(V, \textcolor{red}{s}, O, G)$.

Heuristic for Forward or Backward Search? (2)

We defined h^m so that it can directly be used for both directions on task (V, I, O, G) as

- $h_f^m(s) := h^m(s, G)$ for forward search, or
- $h_b^m(S) := h^m(I, S)$ for backward search.

Precomputation determines $h^m(s, B)$ for all $B \subseteq V$ with $|B| \leq m$.

- For h_f^m , we can only use these values for a single heuristic evaluation, because the state s changes.
- For h_b^m , we can re-use these values and all subsequent heuristic evaluations are quite cheap.

→ h^m better suited for backward search

→ We examine it in the following in this context.

Heuristic Properties (1)

Theorem

Let $\Pi = \langle V, I, O, G \rangle$ be a STRIPS planning tasks and $S \subseteq V$ be a backward search state. Then $h_b^m(S) := h^m(I, S)$ is a **safe, goal-aware, consistent, and admissible** heuristic for Π .

Proof.

We prove goal-awareness and consistency, the other properties follow from these two.

Goal-awareness: S is a goal state iff $S \subseteq I$. Then
 $h_b^m(S) = h^m(I, S) = 0$.

...

Heuristic Properties (2)

Proof (continued).

Consistency: Assume h_b^m is not consistent, i.e., there is a state S and an operator o , where $R := \text{sregr}(S, o) \neq \perp$ such that $h_b^m(S) > \text{cost}(o) + h_b^m(R)$.

Then $h_b^m(S) = h^m(I, S)$ and there is $S' \subseteq S$ with $|S'| \leq m$ and $h^m(I, S') = h^m(I, S)$: if $|S| \leq m$, choose $S' = S$, otherwise choose any maximizing subset from the last h^m equation.

As $S' \subseteq S$ and $\text{sregr}(S, o) \neq \perp$, also $R' := \text{sregr}(S', o) \neq \perp$ and $(R', o) \in R(S', O)$. This gives $h^m(I, S') \leq \text{cost}(o) + h^m(I, R')$.

As $S' \subseteq S$, it holds that $R' \subseteq R$ and $h^m(I, R') \leq h^m(I, R)$.

Overall, we get $h_b^m(S) = h^m(I, S) = h^m(I, S') \leq \text{cost}(o) + h^m(I, R') \leq \text{cost}(o) + h^m(I, R) = \text{cost}(o) + h_b^m(R)$. \checkmark \square

Heuristic Properties (3)

Theorem

For $m, m' \in \mathbb{N}_1$ with $m < m'$ it holds that $h^m \leq h^{m'}$.

(Proof omitted.)

Heuristic Properties (4)

Theorem

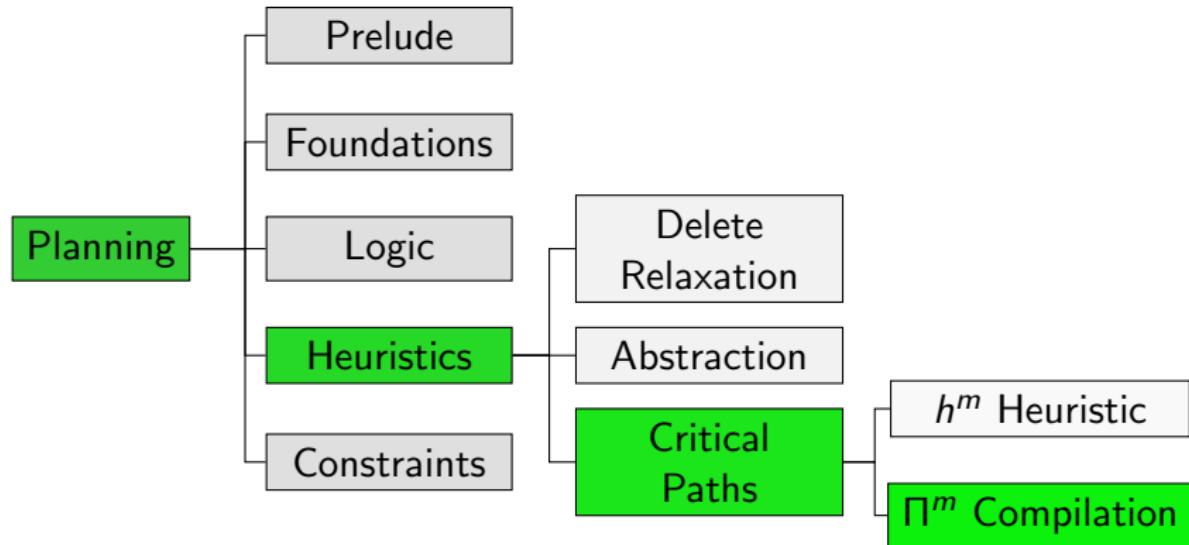
Let $\Pi = \langle V, I, O, G \rangle$ be a STRIPS planning task.
For a **sufficiently large m** , it holds that $h^m = r^*$ on Π .

Proof Sketch.

It is easy to check that for $m = |V|$ the heuristic definition of h^m can be simplified so that it becomes the definition of r^* .

Π^m Compilation

Content of this Course



Π^m Compilation: Motivation

- We have seen that $h^1 = h^{\max}$ and that h^{\max} corresponds to the cost of a critical path in the relaxed task graph.
- What about $m > 1$?
- Π^m compilation derives for a given m a task Π^m from the original task Π .
- h^m corresponds to cost of critical path in the relaxed task graph of Π^m .

→ Better understanding of h^m

→ Also interesting in the context of landmark heuristics

Idea of Π^m Compilation

- h^{\max} only considers variables individually.
- For example, it cannot detect that a goal $\{a, b\}$ is unreachable from the empty set if every action that adds a deletes b and vice versa.
- Idea: Use **meta-variable** $v_{\{a,b\}}$ to capture such interactions.
- Intuitively $v_{\{a,b\}}$ is reachable in Π^m if a state where a and b are both true would be reachable in Π when only capturing interactions of at most m variables.

Some Notation

- For a set X of **variables** and $m \in \mathbb{N}_1$ we define
$$X^m := \{v_Y \mid Y \subseteq X, |Y| \leq m\}.$$
- Example: $\{a, b, c\}^2 = \{v_\emptyset, v_{\{a\}}, v_{\{b\}}, v_{\{c\}}, v_{\{a,b\}}, v_{\{a,c\}}, v_{\{b,c\}}\}$

Π^m Compilation

Definition (Π^m)

Let $\Pi = \langle V, I, O, G \rangle$ be a STRIPS planning task.

For $m \in \mathbb{N}_1$, the task Π^m is the STRIPS planning task

$\langle V^m, I^m, O^m, G^m \rangle$, where

$O^m = \{a_{o,S} \mid o \in O, S \subseteq V, |S| < m, S \cap (add(o) \cup del(o)) = \emptyset\}$

with

- $pre(a_{o,S}) = (pre(o) \cup S)^m$
- $add(a_{o,S}) = \{v_Y \mid Y \subseteq add(o) \cup S, |Y| \leq m, Y \cap add(o) \neq \emptyset\}$
- $del(a_{o,S}) = \emptyset$
- $cost(a_{o,S}) = cost(o)$

Π^m for Running Example with $m = 2$

For running example Π we get $\Pi^2 = \langle V', I', O', G' \rangle$, where

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$$V = \{a, b, c\}$$

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$$I' = \{v_\emptyset, v_{\{a\}}\}$$

$$I = \{a\}$$

$$I' = I^2 = \{v_Y \mid Y \subseteq I, |Y| \leq 2\}$$

Π^m for Running Example with $m = 2$

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$$G' = \{v_\emptyset, v_{\{a\}}, v_{\{b\}}, v_{\{c\}}, v_{\{a,b\}}, v_{\{a,c\}}, v_{\{b,c\}}\}$$

$$G = \{a, b, c\}$$

$$G' = G^2 = \{v_Y \mid Y \subseteq G, |Y| \leq 2\}$$

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$$G' = \{v_\emptyset, v_{\{a\}}, v_{\{b\}}, v_{\{c\}}, v_{\{a,b\}}, v_{\{a,c\}}, v_{\{b,c\}}\}$$

$$O' = \{a_{o_1, \emptyset}, a_{o_1, \{a\}}, a_{o_2, \emptyset}, a_{o_2, \{c\}}, a_{o_3, \emptyset}, a_{o_3, \{b\}}, a_{o_3, \{c\}}\}$$

$$o_1 = \langle \{a, b\}, \{c\}, \{b\}, 1 \rangle$$

$$o_2 = \langle \{a\}, \{b\}, \{a\}, 2 \rangle$$

$$o_3 = \langle \{b\}, \{a\}, \emptyset, 2 \rangle$$

$$O' = \{a_{o, S} \mid o \in O, S \subseteq V, |S| < m, S \cap (add(o) \cup del(o)) = \emptyset\}$$

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with (for example)

$$a_{o_3, \{c\}} = \langle \{v_\emptyset, v_{\{b\}}, v_{\{c\}}, v_{\{b,c\}}\}, \dots, \dots, \dots \rangle$$

$$o_3 = \langle \{b\}, \{a\}, \emptyset, 2 \rangle$$

$$pre(a_{o, S}) = (pre(o) \cup S)^2$$

Π^m for Running Example with $m = 2$

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$$add(a_{o,S}) = \{v_Y \mid Y \subseteq add(o) \cup S, |Y| \leq m, Y \cap add(o) \neq \emptyset\}$$

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$$del(a_{o_3}) = \emptyset$$

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with (for example)

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$$o_3 = \langle \{b\}, \{a\}, \emptyset, 2 \rangle$$

$$\text{cost}(a_{o_3}) = \text{cost}(o)$$

Π^m : Properties

Theorem ($h_{\Pi}^m = h_{\Pi^m}^{\max}$)

Let Π be a STRIPS planning task and $m \in \mathbb{N}_1$.

Then for each state s of Π it holds that $h_{\Pi}^m(s) = h_{\Pi^m}^{\max}(s^m)$, where the subscript denotes on which task the heuristic is computed.

(Proof omitted.)

Can we in general compute an admissible heuristic on Π^m and get admissible estimates for Π ? \leadsto No!

Theorem

There are STRIPS planning tasks Π , $m \in \mathbb{N}_1$ and admissible heuristics h such that $h_{\Pi}^(s) < h_{\Pi^m}(s^m)$ for some state s of Π .*

(Proof omitted.)

Intuition: we may need separate copies of the same action to achieve different meta-fluents

Π^C Compilation

Outlook: Π^C and Π_{ce}^C Compilation

- Π^m (and h^m) must consider **all** subsets up to size m .
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- The compilation Π^C is defined for a set C of atom sets.
 - C can contain arbitrary subsets of arbitrary size.
 - Task Π^C is again delete-free.
 - $h_{\Pi^C}^+ = h_{\Pi^C}^*$ is admissible for Π .
 - The task representation is exponential in $|C|$ (one action copy for every set of meta-variables the action can make true).

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 - The task representation is exponential in $|C|$ (one action copy for every set of meta-variables the action can make true).
- Π_{ce}^C is an alternative to Π^C using conditional effects
 - Π_{ce}^C can be exponentially smaller (in $|C|$) than Π^C .
 - $h_{\Pi_{ce}^C}^+$ dominates $h_{\Pi^C}^+$ for set C of non-unit sets.

Heuristic Properties
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Π^m Compilation
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Π^C Compilation
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Summary
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Literature
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Summary

Summary

- h^m heuristics are **best suited for backward search**.
- h^m heuristics are **safe, goal aware, consistent and admissible**.
- The Π^m compilation explicitly represents sets
($\hat{=}$ conjunctions) of variables as **meta-variables**.
- $h_{\Pi}^m(s) = h_{\Pi^m}^{\max}(s^m)$
- The ideas underlying the Π^m compilation have been generalized to the Π^C and Π_{ce}^C compilation.

Heuristic Properties
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Π^m Compilation
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Π^C Compilation
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Summary
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Literature
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Literature

Literature (1)

References on critical path heuristics:

-  **Patrik Haslum and Hector Geffner.**
Admissible Heuristics for Optimal Planning.
Proc. AIPS 2000, pp. 140–149, 2000.
Introduces h^m heuristics.
-  **Patrik Haslum.**
 $h^m(P) = h^1(P^m)$: Alternative Characterisations of the Generalisation From h^{\max} to h^m .
Proc. ICAPS 2009, pp. 354–357, 2009.
Introduces Π^m compilation.

Literature (2)



Patrik Haslum.

Incremental Lower Bounds for Additive Cost Planning Problems.

Proc. ICAPS 2012, pp. 74–82, 2012.

Introduces Π^C compilation.



Emil Keyder, Jörg Hoffmann and Patrik Haslum.

Improving Delete Relaxation Heuristics Through Explicitly Represented Conjunctions.

JAIR 50, pp. 487–533, 2014.

Introduces Π_{ce}^C compilation.