

# Planning and Optimization

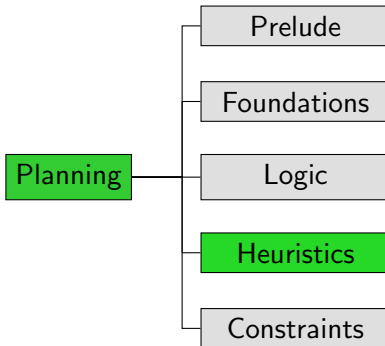
## E3. Abstractions: Introduction

Malte Helmert and Gabriele Röger

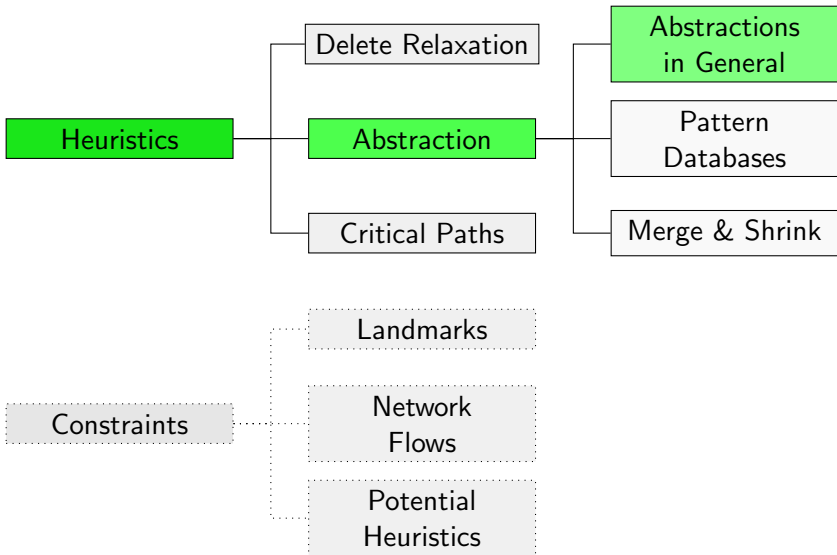
Universität Basel

November 7, 2022

# Content of this Course



# Content of this Course: Heuristics



# Introduction

# Coming Up with Heuristics in a Principled Way

## General Procedure for Obtaining a Heuristic

Solve a simplified version of the problem.

Major ideas for heuristics in the planning literature:

- delete relaxation
- **abstraction**
- landmarks
- critical paths
- network flows
- potential heuristics

Heuristics based on **abstraction** are among the most prominent techniques for **optimal planning**.

# Abstracting a Transition System

Abstracting a transition system means **dropping some distinctions** between states, while **preserving the transition behaviour** as much as possible.

- An abstraction of a transition system  $\mathcal{T}$  is defined by an **abstraction mapping**  $\alpha$  that defines which states of  $\mathcal{T}$  should be distinguished and which ones should not.
- From  $\mathcal{T}$  and  $\alpha$ , we compute an **abstract transition system**  $\mathcal{T}^\alpha$  which is similar to  $\mathcal{T}$ , but smaller.
- The **abstract goal distances** (goal distances in  $\mathcal{T}^\alpha$ ) are used as heuristic estimates for goal distances in  $\mathcal{T}$ .

# Abstracting a Transition System: Example

example from domain-specific heuristic search:

## Example (15-Puzzle)

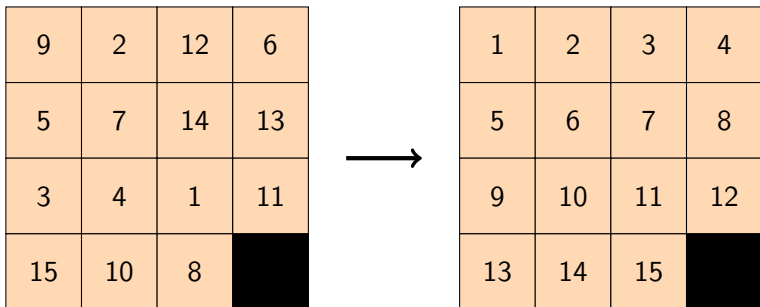
A **15-puzzle** state is given by a permutation  $\langle b, t_1, \dots, t_{15} \rangle$  of  $\{1, \dots, 16\}$ , where  $b$  denotes the blank position and the other components denote the positions of the 15 tiles.

One possible **abstraction mapping** ignores the precise location of tiles 8–15, i.e., two states are distinguished iff they differ in the position of the blank or one of the tiles 1–7:

$$\alpha(\langle b, t_1, \dots, t_{15} \rangle) = \langle b, t_1, \dots, t_7 \rangle$$

The heuristic values for this abstraction correspond to the cost of moving tiles 1–7 to their goal positions.

# Abstraction Example: 15-Puzzle

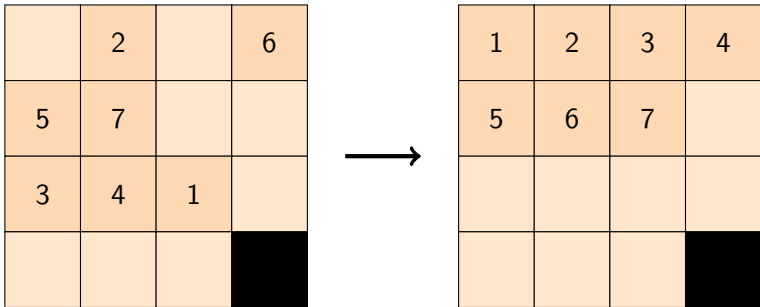


real state space:

- $16! = 20922789888000 \approx 2 \cdot 10^{13}$  states
- $\frac{16!}{2} = 10461394944000 \approx 10^{13}$  reachable states



# Abstraction Example: 15-Puzzle



abstract state space:

- $16 \cdot 15 \cdot \dots \cdot 9 = 518918400 \approx 5 \cdot 10^8$  states
- $16 \cdot 15 \cdot \dots \cdot 9 = 518918400 \approx 5 \cdot 10^8$  reachable states

# Computing the Abstract Transition System

Given  $\mathcal{T}$  and  $\alpha$ , how do we compute  $\mathcal{T}^\alpha$ ?

## Requirement

We want to obtain an **admissible heuristic**.

Hence,  $h^*(\alpha(s))$  (in the abstract state space  $\mathcal{T}^\alpha$ ) should never overestimate  $h^*(s)$  (in the concrete state space  $\mathcal{T}$ ).

An easy way to achieve this is to ensure that **all solutions in  $\mathcal{T}$  are also present in  $\mathcal{T}^\alpha$** :

- If  $s$  is a goal state in  $\mathcal{T}$ , then  $\alpha(s)$  is a goal state in  $\mathcal{T}^\alpha$ .
- If  $\mathcal{T}$  has a transition from  $s$  to  $t$ , then  $\mathcal{T}^\alpha$  has a transition from  $\alpha(s)$  to  $\alpha(t)$  with the same cost.

# Computing the Abstract Transition System: Example

## Example (15-Puzzle)

In the running example:

- $\mathcal{T}$  has the unique goal state  $\langle 16, 1, 2, \dots, 15 \rangle$ .
  - ↪  $\mathcal{T}^\alpha$  has the unique goal state  $\langle 16, 1, 2, \dots, 7 \rangle$ .
- Let  $x$  and  $y$  be neighbouring positions in the  $4 \times 4$  grid.  
 $\mathcal{T}$  has a transition from  $\langle x, t_1, \dots, t_{i-1}, y, t_{i+1}, \dots, t_{15} \rangle$   
to  $\langle y, t_1, \dots, t_{i-1}, x, t_{i+1}, \dots, t_{15} \rangle$  for all  $i \in \{1, \dots, 15\}$ .
  - ↪  $\mathcal{T}^\alpha$  has a transition from  $\langle x, t_1, \dots, t_{i-1}, y, t_{i+1}, \dots, t_7 \rangle$   
to  $\langle y, t_1, \dots, t_{i-1}, x, t_{i+1}, \dots, t_7 \rangle$  for all  $i \in \{1, \dots, 7\}$ .
  - ↪ Moreover,  $\mathcal{T}^\alpha$  has a transition from  $\langle x, t_1, \dots, t_7 \rangle$   
to  $\langle y, t_1, \dots, t_7 \rangle$  if  $y \notin \{t_1, \dots, t_7\}$ .

# Practical Requirements

# Practical Requirements for Abstractions

To be useful in practice, an abstraction heuristic must be efficiently computable. This gives us two requirements for  $\alpha$ :

- For a given state  $s$ , the **abstract state**  $\alpha(s)$  must be efficiently computable.
- For a given abstract state  $\alpha(s)$ , the **abstract goal distance**  $h^*(\alpha(s))$  must be efficiently computable.

There are a number of ways of achieving these requirements:

- **pattern database heuristics** (Culberson & Schaeffer, 1996)
- **merge-and-shrink abstractions** (Dräger, Finkbeiner & Podelski, 2006)
- Cartesian abstractions (Ball, Podelski & Rajamani, 2001)
- structural patterns (Katz & Domshlak, 2008)

# Practical Requirements for Abstractions: Example

## Example (15-Puzzle)

In our running example,  $\alpha$  can be very efficiently computed: just project the given 16-tuple to its first 8 components.

To compute abstract goal distances efficiently during search, the most common approach is to precompute **all abstract goal distances** prior to search by performing a backward uniform-cost search from the abstract goal state(s). These distances are then stored in a table (requires  $\approx 495$  MiB RAM).

During search, computing  $h^*(\alpha(s))$  is just a table lookup.

This heuristic is an example of a **pattern database heuristic**.

# Multiple Abstractions

# Multiple Abstractions

- One important practical question is how to come up with a suitable abstraction mapping  $\alpha$ .
- Indeed, there is usually a **huge number of possibilities**, and it is important to pick good abstractions (i.e., ones that lead to informative heuristics).
- However, it is generally **not necessary to commit to a single abstraction**.



# Combining Multiple Abstractions

**Maximizing** several abstractions:

- Each abstraction mapping gives rise to an admissible heuristic.
- By computing the **maximum** of several admissible heuristics, we obtain another admissible heuristic which **dominates** the component heuristics.
- Thus, we can always compute several abstractions and maximize over the individual abstract goal distances.

**Adding** several abstractions:

- In some cases, we can even compute the **sum** of individual estimates and still stay admissible.
- Summation often leads to **much higher estimates** than maximization, so it is important to understand **under which conditions** summation of heuristics is **admissible**.

# Maximizing Several Abstractions: Example

## Example (15-Puzzle)

- mapping to tiles 1–7 was arbitrary  
     $\rightsquigarrow$  can use **any subset** of tiles
- with the same amount of memory required for the tables for the mapping to tiles 1–7, we could store the tables for **nine different abstractions** to six tiles and the blank
- use **maximum** of individual estimates

## Adding Several Abstractions: Example

9	2	12	6
5	7	14	13
3	4	1	11
15	10	8	

9	2	12	6
5	7	14	13
3	4	1	11
15	10	8	

- **1st abstraction:** ignore precise location of 8–15
  - **2nd abstraction:** ignore precise location of 1–7
- ↪ Is the **sum** of the abstraction heuristics **admissible**?

## Adding Several Abstractions: Example

	2		6
5	7		
3	4	1	

9		12	
		14	13
			11
15	10	8	

- **1st abstraction:** ignore precise location of 8–15
  - **2nd abstraction:** ignore precise location of 1–7
- ↪ The **sum** of the abstraction heuristics is **not admissible**.

## Adding Several Abstractions: Example

		2			6	
5	7					
3	4	1				

9			12			
			14		13	
					11	
15	10	8				

- **1st abstraction:** ignore precise location of 8–15 and blank
  - **2nd abstraction:** ignore precise location of 1–7 and blank
- ↪ The **sum** of the abstraction heuristics is **admissible**.

# Outlook

# Our Plan for the Next Lectures

In the following, we take a deeper look at abstractions and their use for admissible heuristics.

In the next two chapters, we **formally introduce** abstractions and abstraction heuristics and study some of their most important properties.

Afterwards, we discuss some particular classes of abstraction heuristics in detail, namely

- **pattern database heuristics** and
- **merge-and-shrink abstractions**.

# Summary



# Summary

- **Abstraction** is one of the principled ways of deriving heuristics for planning tasks and transition systems in general.
- The key idea is to map states to a smaller **abstract transition system**  $\mathcal{T}^\alpha$  by means of an **abstraction function**  $\alpha$ .
- **Goal distances in  $\mathcal{T}^\alpha$**  are then used as **admissible estimates** for goal distances in the original transition system.
- To be **practical**, we must be able to **compute abstraction functions** and **determine abstract goal distances efficiently**.
- Often, **multiple abstractions** are used.  
They can always be **maximized** admissibly.
- **Adding** abstraction heuristics is not always admissible.  
When it is, it leads to a stronger heuristic than maximizing.