

# Planning and Optimization

## E1. Planning Tasks in Finite-Domain Representation

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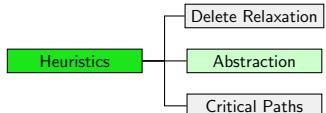
## E1.1 Finite-Domain Representation

## E1.2 Equivalence and Normal Forms

## E1.3 Summary

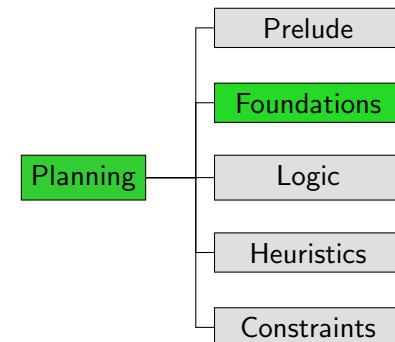
## How we Continue

- ▶ The next class of heuristics we will consider are **abstraction heuristics**.



- ▶ Abstraction heuristics benefit from a more compact task representation, called **finite-domain representation**.
- ▶ To understand the relationship to the propositional task representation, we need to know a special kind of **invariants**, namely **mutexes**.
- ~~ We first get to know finite-domain representation (this chapter) and then speak about invariants and transformations between the representations (next chapter).
- ~~ not specific to abstraction heuristics, but general foundations

## Content of this Course



## E1.1 Finite-Domain Representation

## Finite-Domain State Variables

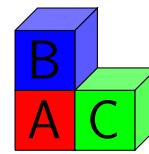
- ▶ So far, we used propositional (Boolean) state variables.  
~~ possible values **T** and **F**
- ▶ We now consider **finite-domain variables**.  
~~ every variable has a **finite set of possible values**
- ▶ A state is still a valuation of state variables.

**Example:**  $O(n^2)$  Boolean variables or  $O(n)$  finite-domain variables with domain size  $O(n)$  suffice for blocks world with  $n$  blocks.

### Blocks World State with Propositional Variables

#### Example

$$\begin{aligned}
 s(A\text{-on-}B) &= \mathbf{F} \\
 s(A\text{-on-}C) &= \mathbf{F} \\
 s(A\text{-on-table}) &= \mathbf{T} \\
 s(B\text{-on-}A) &= \mathbf{T} \\
 s(B\text{-on-}C) &= \mathbf{F} \\
 s(B\text{-on-table}) &= \mathbf{F} \\
 s(C\text{-on-}A) &= \mathbf{F} \\
 s(C\text{-on-}B) &= \mathbf{F} \\
 s(C\text{-on-table}) &= \mathbf{T}
 \end{aligned}$$



$$\rightsquigarrow 2^9 = 512 \text{ states}$$

**Note:** it may be useful to add auxiliary state variables like *A-clear*.

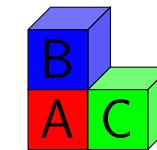
**Note:** it may be useful to add auxiliary state variables like *A-clear*.

#### Example

Use three finite-domain state variables:

- ▶ *below-a*: {b, c, table}
- ▶ *below-b*: {a, c, table}
- ▶ *below-c*: {a, b, table}

$$\begin{aligned}
 s(\text{below-}a) &= \text{table} \\
 s(\text{below-}b) &= a \\
 s(\text{below-}c) &= \text{table}
 \end{aligned}$$



$$\rightsquigarrow 3^3 = 27 \text{ states}$$

## Advantage of Finite-Domain Representation

How many “useless” (physically impossible) states are there with these blocks world state representations?

- ▶ There are 13 physically possible states with three blocks:
  - ▶ all blocks on table: 1 state
  - ▶ all blocks in one stack:  $3! = 6$  states
  - ▶ two block stacked, the other separate:  $\binom{3}{2}2! = 6$
- ▶ With propositional variables,  $2^9 - 13 = 499$  states are useless.
- ▶ With finite-domain variables, only  $27 - 13 = 14$  are useless.

Although useless states are unreachable, they can introduce “shortcuts” in some heuristics and thus lead to lower heuristic estimates.

## Finite-Domain State Variables

### Definition (Finite-Domain State Variable)

A **finite-domain state variable** is a symbol  $v$  with an associated **domain**  $\text{dom}(v)$ , which is a finite non-empty set of values.

Let  $V$  be a finite set of finite-domain state variables.

A **state**  $s$  over  $V$  is an assignment  $s : V \rightarrow \bigcup_{v \in V} \text{dom}(v)$  such that  $s(v) \in \text{dom}(v)$  for all  $v \in V$ .

A **formula** over  $V$  is a propositional logic formula whose atomic propositions are of the form  $v = d$  where  $v \in V$  and  $d \in \text{dom}(v)$ .

Slightly extending propositional logic, we treat states  $s$  over finite-domain variables as **logical valuations** where  $s \models v = d$  iff  $s(v) = d$ .

## Example: Finite-Domain State Variables

### Example

Consider finite-domain variables  $V = \{\text{location}, \text{bike}\}$  with  $\text{dom}(\text{location}) = \{\text{at-home}, \text{in-front-of-uni}, \text{in-lecture}\}$  and  $\text{dom}(\text{bike}) = \{\text{locked}, \text{unlocked}, \text{stolen}\}$ .

Consider state  $s = \{\text{location} \mapsto \text{at-home}, \text{bike} \mapsto \text{locked}\}$ .

Does  $s \models (\text{location} = \text{at-home} \wedge \neg \text{bike} = \text{stolen})$  hold?

## Reminder: Syntax of Operators

### Definition (Operator)

An **operator**  $o$  over state variables  $V$  is an object with three properties:

- ▶ a **precondition**  $\text{pre}(o)$ , a formula over  $V$
- ▶ an **effect**  $\text{eff}(o)$  over  $V$
- ▶ a **cost**  $\text{cost}(o) \in \mathbb{R}_0^+$

Only necessary adaptation: What is an effect?

### Example

$\langle \text{location} = \text{in-front-of-uni},$   
 $\text{location} := \text{in-lecture} \wedge (\text{bike} = \text{unlocked} \triangleright \text{bike} := \text{stolen}), 1 \rangle$

## Syntax of Effects

### Definition (Effect over Finite-Domain State Variables)

Effects over finite-domain state variables  $V$

are inductively defined as follows:

- ▶  $\top$  is an effect (empty effect).
- ▶ If  $v \in V$  is a finite-domain state variable and  $d \in \text{dom}(v)$ , then  $v := d$  is an effect (atomic effect).
- ▶ If  $e$  and  $e'$  are effects, then  $(e \wedge e')$  is an effect (conjunctive effect).
- ▶ If  $\chi$  is a formula over  $V$  and  $e$  is an effect, then  $(\chi \triangleright e)$  is an effect (conditional effect).

Parentheses can be omitted when this does not cause ambiguity.

only change compared to propositional case: atomic effects

## Conflicting Effects and Consistency Condition

- ▶ What should an effect of the form  $v := a \wedge v := b$  mean?
- ▶ For finite-domain representations, the accepted semantics is to make this illegal, i.e., to make an operator inapplicable if it would lead to conflicting effects.

### Definition (Consistency Condition)

Let  $e$  be an effect over finite-domain state variables  $V$ .

The consistency condition for  $e$ ,  $\text{consist}(e)$  is defined as

$$\bigwedge_{v \in V} \bigwedge_{d, d' \in \text{dom}(v), d \neq d'} \neg(\text{effcond}(v := d, e) \wedge \text{effcond}(v := d', e)).$$

How did we handle conflicting effects  
in propositional planning tasks?

## Semantics of Effects: Effect Conditions

### Definition (Effect Condition with Finite-Domain Representation)

Let  $v := d$  be an atomic effect, and let  $e$  be an effect.

The effect condition  $\text{effcond}(v := d, e)$  under which  $v := d$  triggers given the effect  $e$  is a propositional formula defined as follows:

- ▶  $\text{effcond}(v := d, \top) = \perp$
- ▶  $\text{effcond}(v := d, v := d) = \top$
- ▶  $\text{effcond}(v := d, v' := d') = \perp$   
for atomic effects with  $v' \neq v$  or  $d' \neq d$
- ▶  $\text{effcond}(v := d, (e \wedge e')) = (\text{effcond}(v := d, e) \vee \text{effcond}(v := d, e'))$
- ▶  $\text{effcond}(v := d, (\chi \triangleright e)) = (\chi \wedge \text{effcond}(v := d, e))$

Same definition as for propositional tasks,  
we just use the adapted definition of atomic effects.

## Semantics of Operators: Finite-Domain Case

### Definition (Applicable, Resulting State)

Let  $V$  be a set of finite-domain state variables and  $e$  be an effect over  $V$ .

If  $s \models \text{consist}(e)$ , the resulting state of applying  $e$  in  $s$ , written  $s[e]$ , is the state  $s'$  defined as follows for all  $v \in V$ :

$$s'(v) = \begin{cases} d & \text{if } s \models \text{effcond}(v := d, e) \text{ for some } d \in \text{dom}(v) \\ s(v) & \text{otherwise} \end{cases}$$

Let  $o$  be an operator over  $V$ .

Operator  $o$  is applicable in  $s$  if  $s \models \text{pre}(o) \wedge \text{consist}(o)$ .

If  $o$  is applicable in  $s$ , the resulting state of applying  $o$  in  $s$ , written  $s[o]$ , is the state  $s[\text{eff}(o)]$ .

## Applying Operators: Example

### Example

$V = \{location, bike\}$  with  
 $\text{dom}(location) = \{\text{at-home}, \text{in-front-of-uni}, \text{in-lecture}\}$  and  
 $\text{dom}(bike) = \{\text{locked}, \text{unlocked}, \text{stolen}\}$ .

State  $s = \{location \mapsto \text{in-front-of-uni}, bike \mapsto \text{unlocked}\}$

$o = \langle \text{location} = \text{in-front-of-uni}, \text{location} := \text{at-home}, 1 \rangle$

$o' = \langle \text{location} = \text{in-front-of-uni},$   
 $\quad \text{location} := \text{in-lecture} \wedge (\text{bike} = \text{unlocked} \triangleright \text{bike} := \text{stolen}), 1 \rangle$

What is  $s[o]$ ? What is  $s[o']$ ?

## FDR Planning Tasks

### Definition (Planning Task)

An **FDR planning task** (or planning task in finite-domain representation) is a 4-tuple  $\Pi = \langle V, I, O, \gamma \rangle$  where

- ▶  $V$  is a finite set of **finite-domain state variables**,
- ▶  $I$  is a valuation over  $V$  called the **initial state**,
- ▶  $O$  is a finite set of **operators** over  $V$ , and
- ▶  $\gamma$  is a formula over  $V$  called the **goal**.

Apart from the variables, this is the same definition as for propositional planning tasks, but the underlying concepts have been adapted.

## Mapping FDR Planning Tasks to Transition Systems

### Definition (Transition System Induced by an FDR Planning Task)

The FDR planning task  $\Pi = \langle V, I, O, \gamma \rangle$  **induces** the transition system  $\mathcal{T}(\Pi) = \langle S, L, c, T, s_0, S_* \rangle$ , where

- ▶  $S$  is the set of all states over  $V$ ,
- ▶  $L$  is the set of operators  $O$ ,
- ▶  $c(o) = \text{cost}(o)$  for all operators  $o \in O$ ,
- ▶  $T = \{\langle s, o, s' \rangle \mid s \in S, o \text{ applicable in } s, s' = s[o]\}$ ,
- ▶  $s_0 = I$ , and
- ▶  $S_* = \{s \in S \mid s \models \gamma\}$ .

Exactly the same definition as for propositional planning tasks, but the underlying concepts have been adapted.

## E1.2 Equivalence and Normal Forms

## Equivalence and Flat Operators

- ▶ The definitions of equivalent effects/operators and flat effects/operators apply equally to finite-domain representation.
- ▶ The same is true for the equivalence transformations.

You find the definitions and transformations in Chapter B4.

## Conflict-Free Operators

### Definition (Conflict-Free)

An **effect**  $e$  over **finite-domain** state variables  $V$  is called **conflict-free** if  $\text{effcond}(v := d, e) \wedge \text{effcond}(v := d', e)$  is unsatisfiable for all  $v \in V$  and  $d, d' \in \text{dom}(v)$  with  $d \neq d'$ .  
An **operator**  $o$  is called **conflict-free** if  $\text{eff}(o)$  is conflict-free.

**Note:**  $\text{consist}(e) \equiv \top$  for conflict-free  $e$ .

**Algorithm to make given operator  $o$  conflict-free:**

- ▶ replace all atomic effects  $v := d$  by  $(\text{consist}(\text{eff}(o)) \rhd v := d)$
- ▶ replace  $\text{pre}(o)$  with  $\text{pre}(o) \wedge \text{consist}(\text{eff}(o))$

The resulting operator  $o'$  is conflict-free and  $o \equiv o'$ .

## SAS<sup>+</sup> Operators and Planning Tasks

### Definition (SAS<sup>+</sup> Operator)

An operator  $o$  of an FDR planning task is a **SAS<sup>+</sup> operator** if

- ▶  $\text{pre}(o)$  is a satisfiable conjunction of atoms, and
- ▶  $\text{eff}(o)$  is a conflict-free conjunction of atomic effects.

### Definition (SAS<sup>+</sup> Planning Task)

An FDR planning task  $\langle V, O, I, \gamma \rangle$  is a **SAS<sup>+</sup> planning task** if all operators  $o \in O$  are SAS<sup>+</sup> operators and  $\gamma$  is a satisfiable conjunction of atoms.

**Note:** SAS<sup>+</sup> operators are conflict-free and flat.

## SAS<sup>+</sup> Operators: Remarks

- ▶ Every SAS<sup>+</sup> operator is of the form

$$\langle v_1 = d_1 \wedge \dots \wedge v_n = d_n, \quad v'_1 := d'_1 \wedge \dots \wedge v'_m := d'_m \rangle$$

where all  $v_i$  are distinct and all  $v'_j$  are distinct.

- ▶ Often, SAS<sup>+</sup> operators  $o$  are described via two **sets of partial assignments**:
  - ▶ the **preconditions**  $\{v_1 \mapsto d_1, \dots, v_n \mapsto d_n\}$
  - ▶ the **effects**  $\{v'_1 \mapsto d'_1, \dots, v'_m \mapsto d'_m\}$

## SAS<sup>+</sup> vs. STRIPS

- ▶ SAS<sup>+</sup> is an analogue of STRIPS planning tasks for FDR, but there is no special role of “positive” conditions.
- ▶ Apart from this difference, all comments for STRIPS apply analogously.
- ▶ If all variable domains are binary, SAS<sup>+</sup> is essentially STRIPS with negation.

### SAS<sup>+</sup>

Derives from SAS = Simplified Action Structures  
(Bäckström & Klein, 1991)

## E1.3 Summary

## Summary

- ▶ Planning tasks in **finite-domain representation (FDR)** are an alternative to propositional planning tasks.
- ▶ FDR tasks are often more compact (have fewer states).
- ▶ This makes many planning algorithms more efficient when working with a finite-domain representation.
- ▶ **SAS<sup>+</sup>** tasks are a restricted form of FDR tasks where only conjunctions of atoms are allowed in the preconditions, effects and goal.  
No conditional effects are allowed.