

# Planning and Optimization

## C3. General Regression

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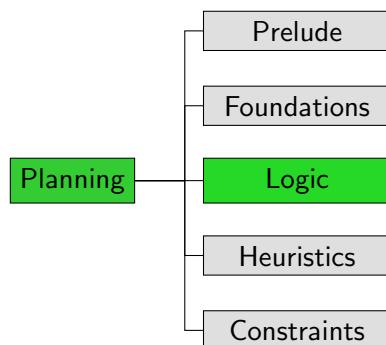
## C3.1 Regressing State Variables

## C3.2 Regressing Formulas Through Effects

## C3.3 Regressing Formulas Through Operators

## C3.4 Summary

## Content of this Course



## Regression for General Planning Tasks

- ▶ With disjunctions and conditional effects, things become more tricky. How to regress  $a \vee (b \wedge c)$  with respect to  $\langle q, d \triangleright b \rangle$ ?
- ▶ In this chapter, we show how to regress **general sets of states** through **general operators**.
- ▶ We extensively use the idea of representing sets of states as formulas.

## C3.1 Regressing State Variables

## Regressing State Variables: Motivation

Key question for general regression:

- ▶ Assume we are applying an operator with effect  $e$ .
- ▶ What must be true in the **predecessor state** for propositional state variable  $v$  to be true in the **successor state**?

If we can answer this question, a general definition of regression is only a small additional step.

## Regressing State Variables: Key Idea

Assume we are in state  $s$  and apply effect  $e$  to obtain successor state  $s'$ .

Propositional state variable  $v$  is true in  $s'$  iff

- ▶ effect  $e$  **makes it true**, or
- ▶ it **remains true**, i.e., it is true in  $s$  and not made false by  $e$ .

## Regressing a State Variable Through an Effect

**Definition (Regressing a State Variable Through an Effect)**

Let  $e$  be an effect of a propositional planning task, and let  $v$  be a propositional state variable.

The **regression of  $v$  through  $e$** , written  $regr(v, e)$ , is defined as the following logical formula:

$$regr(v, e) = \text{effcond}(v, e) \vee (v \wedge \neg \text{effcond}(\neg v, e)).$$

Does this capture add-after-delete semantics correctly?

## Regressing State Variables: Example

### Example

Let  $e = (b \triangleright a) \wedge (c \triangleright \neg a) \wedge b \wedge \neg d$ .

$v$	$\text{regr}(v, e)$
$a$	$b \vee (a \wedge \neg c)$
$b$	$\top \vee (b \wedge \neg \perp) \equiv \top$
$c$	$\perp \vee (c \wedge \neg \perp) \equiv c$
$d$	$\perp \vee (d \wedge \neg \top) \equiv \perp$

Reminder:  $\text{regr}(v, e) = \text{effcond}(v, e) \vee (v \wedge \neg \text{effcond}(\neg v, e))$

## Regressing State Variables: Correctness (1)

### Lemma (Correctness of $\text{regr}(v, e)$ )

Let  $s$  be a state,  $e$  be an effect and  $v$  be a state variable of a propositional planning task.

Then  $s \models \text{regr}(v, e)$  iff  $s[e] \models v$ .

## Regressing State Variables: Correctness (2)

### Proof.

( $\Rightarrow$ ): We know  $s \models \text{regr}(v, e)$ , and hence  $s \models \text{effcond}(v, e) \vee (v \wedge \neg \text{effcond}(\neg v, e))$ .

Do a case analysis on the two disjuncts.

**Case 1:**  $s \models \text{effcond}(v, e)$ .

Then  $s[e] \models v$  by the first case in the definition of  $s[e]$  (Ch. B2).

**Case 2:**  $s \models (v \wedge \neg \text{effcond}(\neg v, e))$ .

Then  $s \models v$  and  $s \not\models \text{effcond}(\neg v, e)$ .

We may additionally assume  $s \not\models \text{effcond}(v, e)$  because otherwise we can apply Case 1 of this proof.

Then  $s[e] \models v$  by the third case in the definition of  $s[e]$ . . .

## Regressing State Variables: Correctness (3)

### Proof (continued).

( $\Leftarrow$ ): Proof by contraposition.

We show that if  $\text{regr}(v, e)$  is false in  $s$ , then  $v$  is false in  $s[e]$ .

- ▶ By prerequisite,  $s \not\models \text{effcond}(v, e) \vee (v \wedge \neg \text{effcond}(\neg v, e))$ .
- ▶ Hence  $s \models \neg \text{effcond}(v, e) \wedge (\neg v \vee \text{effcond}(\neg v, e))$ .
- ▶ From the first conjunct, we get  $s \models \neg \text{effcond}(v, e)$  and hence  $s \not\models \text{effcond}(v, e)$ .
- ▶ From the second conjunct, we get  $s \models \neg v \vee \text{effcond}(\neg v, e)$ .
- ▶ **Case 1:**  $s \models \neg v$ . Then  $v$  is false before applying  $e$  and remains false, so  $s[e] \not\models v$ .
- ▶ **Case 2:**  $s \models \text{effcond}(\neg v, e)$ . Then  $v$  is deleted by  $e$  and not simultaneously added, so  $s[e] \not\models v$ .



## C3.2 Regressing Formulas Through Effects

### Regressing Formulas Through Effects: Idea

- ▶ We can now generalize regression from state variables to general formulas over state variables.
- ▶ The basic idea is to replace **every occurrence** of every state variable  $v$  by  $regr(v, e)$  as defined in the previous section.
- ▶ The following definition makes this more formal.

### Regressing Formulas Through Effects: Definition

#### Definition (Regressing a Formula Through an Effect)

In a propositional planning task, let  $e$  be an effect, and let  $\varphi$  be a formula over propositional state variables.

The **regression of  $\varphi$  through  $e$** , written  $regr(\varphi, e)$ , is defined as the following logical formula:

$$regr(\top, e) = \top$$

$$regr(\perp, e) = \perp$$

$$regr(v, e) = effcond(v, e) \vee (v \wedge \neg effcond(\neg v, e))$$

$$regr(\neg v, e) = \neg regr(v, e)$$

$$regr(\psi \vee \chi, e) = regr(\psi, e) \vee regr(\chi, e)$$

$$regr(\psi \wedge \chi, e) = regr(\psi, e) \wedge regr(\chi, e).$$

### Regressing Formulas Through Effects: Example

#### Example

Let  $e = (b \triangleright a) \wedge (c \triangleright \neg a) \wedge b \wedge \neg d$ .

Recall:

- ▶  $regr(a, e) \equiv b \vee (a \wedge \neg c)$
- ▶  $regr(b, e) \equiv \top$
- ▶  $regr(c, e) \equiv c$
- ▶  $regr(\neg d, e) \equiv \perp$

We get:

$$\begin{aligned} regr((a \vee d) \wedge (c \vee d), e) &\equiv ((b \vee (a \wedge \neg c)) \vee \perp) \wedge (c \vee \perp) \\ &\equiv (b \vee (a \wedge \neg c)) \wedge c \\ &\equiv b \wedge c \end{aligned}$$

## Regressing Formulas Through Effects: Correctness (1)

### Lemma (Correctness of $regr(\varphi, e)$ )

Let  $\varphi$  be a logical formula,  $e$  an effect and  $s$  a state of a propositional planning task.

Then  $s \models regr(\varphi, e)$  iff  $s[e] \models \varphi$ .

## Regressing Formulas Through Effects: Correctness (3)

Proof (continued).

Inductive case  $\varphi = \neg\psi$ :

$$\begin{aligned} s \models regr(\neg\psi, e) &\text{ iff } s \models \neg regr(\psi, e) \\ &\text{ iff } s \not\models regr(\psi, e) \\ &\text{ iff } s[e] \not\models \psi \\ &\text{ iff } s[e] \models \neg\psi \end{aligned}$$

Inductive case  $\varphi = \psi \vee \chi$ :

$$\begin{aligned} s \models regr(\psi \vee \chi, e) &\text{ iff } s \models regr(\psi, e) \vee regr(\chi, e) \\ &\text{ iff } s \models regr(\psi, e) \text{ or } s \models regr(\chi, e) \\ &\text{ iff } s[e] \models \psi \text{ or } s[e] \models \chi \\ &\text{ iff } s[e] \models \psi \vee \chi \end{aligned}$$

Inductive case  $\varphi = \psi \wedge \chi$ :

Like previous case, replacing “ $\vee$ ” by “ $\wedge$ ”  
and replacing “or” by “and”. □

## Regressing Formulas Through Effects: Correctness (2)

Proof.

The proof is by structural induction on  $\varphi$ .

Induction hypothesis:  $s \models regr(\psi, e)$  iff  $s[e] \models \psi$   
for all proper subformulas  $\psi$  of  $\varphi$ .

Base case  $\varphi = \top$ :

We have  $regr(\top, e) = \top$ , and  $s \models \top$  iff  $s[e] \models \top$  is correct.

Base case  $\varphi = \perp$ :

We have  $regr(\perp, e) = \perp$ , and  $s \models \perp$  iff  $s[e] \models \perp$  is correct.

Base case  $\varphi = v$ :

We have  $s \models regr(v, e)$  iff  $s[e] \models v$  from the previous lemma. ...

## C3.3 Regressing Formulas Through Operators

## Regressing Formulas Through Operators: Idea

- ▶ We can now regress arbitrary formulas through arbitrary effects.
- ▶ The last missing piece is a definition of regression through **operators**, describing exactly in which states  $s$  applying a given operator  $o$  leads to a state satisfying a given formula  $\varphi$ .
- ▶ There are two requirements:
  - ▶ The operator  $o$  must be **applicable** in the state  $s$ .
  - ▶ The **resulting state  $s[o]$**  must **satisfy**  $\varphi$ .

## Regressing Formulas Through Operators: Correctness (1)

### Theorem (Correctness of $\text{regr}(\varphi, o)$ )

Let  $\varphi$  be a logical formula,  $o$  an operator and  $s$  a state of a propositional planning task.

Then  $s \models \text{regr}(\varphi, o)$  iff  $o$  is applicable in  $s$  and  $s[o] \models \varphi$ .

## Regressing Formulas Through Operators: Definition

### Definition (Regressing a Formula Through an Operator)

In a propositional planning task, let  $o$  be an operator, and let  $\varphi$  be a formula over state variables.

The **regression of  $\varphi$  through  $o$** , written  $\text{regr}(\varphi, o)$ , is defined as the following logical formula:

$$\text{regr}(\varphi, o) = \text{pre}(o) \wedge \text{regr}(\varphi, \text{eff}(o)).$$

## Regressing Formulas Through Operators: Correctness (2)

Reminder:  $\text{regr}(\varphi, o) = \text{pre}(o) \wedge \text{regr}(\varphi, \text{eff}(o))$

Proof.

**Case 1:**  $s \models \text{pre}(o)$ .

Then  $o$  is applicable in  $s$  and the statement we must prove simplifies to:  $s \models \text{regr}(\varphi, e)$  iff  $s[e] \models \varphi$ , where  $e = \text{eff}(o)$ . This was proved in the previous lemma.

**Case 2:**  $s \not\models \text{pre}(o)$ .

Then  $s \not\models \text{regr}(\varphi, o)$  and  $o$  is not applicable in  $s$ .

Hence both statements are false and therefore equivalent. □

## Regression Examples (1)

Examples: compute regression and simplify to DNF

- ▶  $\text{regr}(b, \langle a, b \rangle)$   
 $\equiv a \wedge (\top \vee (b \wedge \neg\perp))$   
 $\equiv a$
- ▶  $\text{regr}(b \wedge c \wedge d, \langle a, b \rangle)$   
 $\equiv a \wedge (\top \vee (b \wedge \neg\perp)) \wedge (\perp \vee (c \wedge \neg\perp)) \wedge (\perp \vee (d \wedge \neg\perp))$   
 $\equiv a \wedge c \wedge d$
- ▶  $\text{regr}(b \wedge \neg c, \langle a, b \wedge c \rangle)$   
 $\equiv a \wedge (\top \vee (b \wedge \neg\perp)) \wedge \neg(\top \vee (c \wedge \neg\perp))$   
 $\equiv a \wedge \top \wedge \perp$   
 $\equiv \perp$

## Regression Examples (2)

Examples: compute regression and simplify to DNF

- ▶  $\text{regr}(b, \langle a, c \triangleright b \rangle)$   
 $\equiv a \wedge (c \vee (b \wedge \neg\perp))$   
 $\equiv a \wedge (c \vee b)$   
 $\equiv (a \wedge c) \vee (a \wedge b)$
- ▶  $\text{regr}(b, \langle a, (c \triangleright b) \wedge ((d \wedge \neg c) \triangleright \neg b) \rangle)$   
 $\equiv a \wedge (c \vee (b \wedge \neg(d \wedge \neg c)))$   
 $\equiv a \wedge (c \vee (b \wedge (\neg d \vee c)))$   
 $\equiv a \wedge (c \vee (b \wedge \neg d) \vee (b \wedge c))$   
 $\equiv a \wedge (c \vee (b \wedge \neg d))$   
 $\equiv (a \wedge c) \vee (a \wedge b \wedge \neg d)$

## C3.4 Summary

## Summary

- ▶ Regressing a **propositional state variable** through an (arbitrary) operator must consider two cases:
  - ▶ state variables **made true** (by add effects)
  - ▶ state variables **remaining true** (by absence of delete effects)
- ▶ Regression of propositional state variables can be generalized to arbitrary formulas  $\varphi$  by replacing each occurrence of a state variable in  $\varphi$  by its regression.
- ▶ **Regressing a formula  $\varphi$  through an operator** involves regressing  $\varphi$  through the effect and enforcing the precondition.