Planning and Optimization

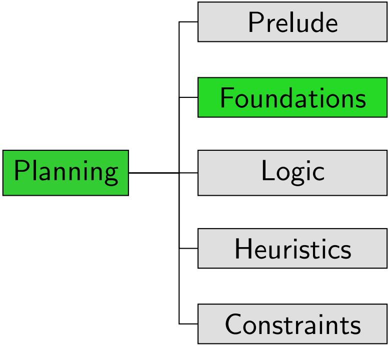
B2. State Variables, Operators and Effects

Malte Helmert and Gabriele Röger

Universität Basel

September 28, 2022

Content of this Course



State Variables

State Variables

How to specify huge transition systems
without enumerating the states?

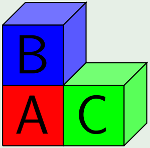
- represent different aspects of the world in terms of different (propositional) state variables
- individual state variables induce atomic propositions
  $\rightsquigarrow$ a state is a valuation of state variables
- n state variables induce 2^n states
  $\rightsquigarrow$ exponentially more compact than "flat" representations

Example: $O(n^2)$ variables suffice for blocks world with n blocks

Blocks World State with Propositional Variables

Example

$$s(A\text{-}on\text{-}B) = \mathbf{F}$$
$$s(A\text{-}on\text{-}C) = \mathbf{F}$$
$$s(A\text{-}on\text{-}table) = \mathbf{T}$$
$$s(B\text{-}on\text{-}A) = \mathbf{T}$$
$$s(B\text{-}on\text{-}C) = \mathbf{F}$$
$$s(B\text{-}on\text{-}table) = \mathbf{F}$$
$$s(C\text{-}on\text{-}A) = \mathbf{F}$$
$$s(C\text{-}on\text{-}B) = \mathbf{F}$$
$$s(C\text{-}on\text{-}table) = \mathbf{T}$$



Note: it may be useful to add auxiliary state variables like *A-clear*.

Propositional State Variables

Definition (Propositional State Variable)

A propositional state variable is a symbol X.

Let V be a finite set of propositional state variables.

A state s over V is a valuation for V, i.e.,
a truth assignment $s : V \to \{\mathbf{T}, \mathbf{F}\}$.

A formula over V is a propositional logic formula using V as the set of atomic propositions.

From State Variables to Succinct Transition Systems

State variables are the basis of compact descriptions of transition systems.

Problem:

- How to succinctly represent transitions and goal states?

Idea: Use formulas to describe sets of states

- states: all assignments to the state variables
- goal states: defined by a formula
- transitions: defined by operators (see following section)

Operators and Effects

Syntax of Operators

Definition (Operator)

An operator o over state variables V is an object with three properties:

- a precondition $pre(o)$, a formula over V
- an effect $\mathit{eff}(o)$ over V, defined on the following slides
- a cost $cost(o) \in \mathbb{R}_0^+$

Notes:

- Operators are also called actions.
- Operators are often written as triples $\langle pre(o), \mathit{eff}(o), cost(o) \rangle$.
- This can be abbreviated to pairs $\langle pre(o), \mathit{eff}(o) \rangle$ when the cost of the operator is irrelevant.

Operators: Intuition

Intuition for operators o:

- The operator precondition describes the set of states in which a transition labeled with o can be taken.
- The operator effect describes how taking such a transition changes the state.
- The operator cost describes the cost of taking a transition labeled with o.

Syntax of Effects

Definition (Effect)

Effects over propositional state variables V are inductively defined as follows:

- $\top$ is an effect (empty effect).
- If $v \in V$ is a propositional state variable, then v and $\neg v$ are effects (atomic effect).
- If e and e' are effects, then $(e \wedge e')$ is an effect (conjunctive effect).
- If χ is a formula over V and e is an effect, then $(\chi \rhd e)$ is an effect (conditional effect).

We may omit parentheses when this does not cause ambiguity.

Example: we will later see that $((e \wedge e') \wedge e'')$ behaves identically to $(e \wedge (e' \wedge e''))$ and will write this as $e \wedge e' \wedge e''$.

Applying Effects and Operators

Effects: Intuition

Intuition for effects:

- The empty effect $\top$ changes nothing.
- Atomic effects can be understood as assignments that update the value of a state variable.
  - v means "$v := \mathbf{T}$"
  - $\neg v$ means "$v := \mathbf{F}$"
- A conjunctive effect $e = (e' \wedge e'')$ means that both subeffects e and e' take place simultaneously.
- A conditional effect $e = (\chi \rhd e')$ means that subeffect e' takes place iff χ is true in the state where e takes place.

Semantics of Effects: Effect Conditions

Definition (Effect Condition for an Effect)

Let ℓ be an atomic effect, and let e be an effect.

The effect condition $\mathit{effcond}(\ell, e)$ under which ℓ triggers given the effect e is a propositional formula defined as follows:

- $\mathit{effcond}(\ell, \top) = \bot$
- $\mathit{effcond}(\ell, e) = \top$ for the atomic effect $e = \ell$
- $\mathit{effcond}(\ell, e) = \bot$ for all atomic effects $e = \ell' \neq \ell$
- $\mathit{effcond}(\ell, (e \land e')) = (\mathit{effcond}(\ell, e) \lor \mathit{effcond}(\ell, e'))$
- $\mathit{effcond}(\ell, (\chi \rhd e)) = (\chi \land \mathit{effcond}(\ell, e))$

Intuition: $\mathit{effcond}(\ell, e)$ represents the condition that must be true in the current state for the effect e to lead to the atomic effect ℓ

Semantics of Effects: Applying an Effect

first attempt:

Definition (Applying Effects)

Let V be a set of propositional state variables.
Let s be a state over V, and let e be an effect over V.

The resulting state of applying e in s, written $s[\![e]\!]$, is the state s' defined as follows for all $v \in V$:

$$s'(v) = \begin{cases} \mathbf{T} & \text{if } s \models \mathit{effcond}(v, e) \\ \mathbf{F} & \text{if } s \models \mathit{effcond}(\neg v, e) \\ s(v) & \text{otherwise} \end{cases}$$

What is the problem with this definition?

Semantics of Effects: Applying an Effect

correct definition:

Definition (Applying Effects)

Let V be a set of propositional state variables.
Let s be a state over V, and let e be an effect over V.

The resulting state of applying e in s, written $s[\![e]\!]$, is the state s' defined as follows for all $v \in V$:

$$s'(v) = \begin{cases} \mathbf{T} & \text{if } s \models \mathit{effcond}(v, e) \\ \mathbf{F} & \text{if } s \models \mathit{effcond}(\neg v, e) \wedge \neg \mathit{effcond}(v, e) \\ s(v) & \text{otherwise} \end{cases}$$

Add-after-Delete Semantics

Note:

- The definition implies that if a variable is simultaneously "added" (set to **T**) and "deleted" (set to **F**), the value **T** takes precedence.
- This is called add-after-delete semantics.
- This detail of effect semantics is somewhat arbitrary, but has proven useful in applications.

Semantics of Operators

Definition (Applicable, Applying Operators, Resulting State)

Let V be a set of propositional state variables.
Let s be a state over V, and let o be an operator over V.

Operator o is applicable in s if $s \models pre(o)$.

If o is applicable in s, the resulting state of applying o in s, written $s[\![o]\!]$, is the state $s[\![eff(o)]\!]$.

Summary

Summary

- Propositional state variables let us compactly describe properties of large transition systems.
- A state is an assignment to a set of state variables.
- Sets of states are represented as formulas over state variables.
- Operators describe when (precondition), how (effect) and at which cost the state of the world can be changed.
- Effects are structured objects including empty, atomic, conjunctive and conditional effects.
- We gave formal semantics for applying effects and operators.