Planning and Optimization

B2. State Variables, Operators and Effects

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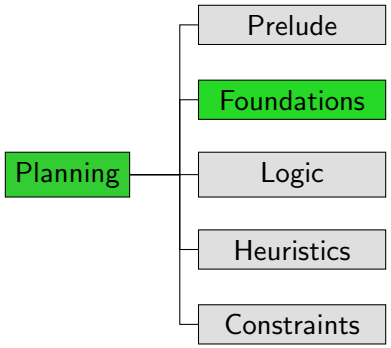
September 28, 2022

Content of this Course



B2.1 State Variables

State Variables

How to specify huge transition systems
without enumerating the states?

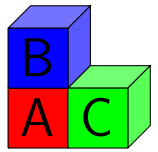
- represent different aspects of the world
  in terms of different (propositional) state variables
- individual state variables induce atomic propositions
  $\rightsquigarrow$ a state is a valuation of state variables
- n state variables induce 2^n states
  $\rightsquigarrow$ exponentially more compact than "flat" representations

Example: $O(n^2)$ variables suffice for blocks world with n blocks

Blocks World State with Propositional Variables

Example

$$
\begin{aligned}
s(\textit{A-on-B}) &= \mathbf{F} \\
s(\textit{A-on-C}) &= \mathbf{F} \\
s(\textit{A-on-table}) &= \mathbf{T} \\
s(\textit{B-on-A}) &= \mathbf{T} \\
s(\textit{B-on-C}) &= \mathbf{F} \\
s(\textit{B-on-table}) &= \mathbf{F} \\
s(\textit{C-on-A}) &= \mathbf{F} \\
s(\textit{C-on-B}) &= \mathbf{F} \\
s(\textit{C-on-table}) &= \mathbf{T}
\end{aligned}
$$



Note: it may be useful to add auxiliary state variables like *A-clear*.

Propositional State Variables

Definition (Propositional State Variable)

A propositional state variable is a symbol X.

Let V be a finite set of propositional state variables.

A state s over V is a valuation for V, i.e.,
a truth assignment $s : V \to \{\mathbf{T}, \mathbf{F}\}$.

A formula over V is a propositional logic formula using V
as the set of atomic propositions.

From State Variables to Succinct Transition Systems

State variables are the basis of compact descriptions
of transition systems.

Problem:

- How to succinctly represent transitions and goal states?

Idea: Use formulas to describe sets of states

- states: all assignments to the state variables
- goal states: defined by a formula
- transitions: defined by operators (see following section)

B2.2 Operators and Effects

Syntax of Operators

Definition (Operator)

An operator o over state variables V is an object with three properties:

- a precondition $pre(o)$, a formula over V
- an effect $eff(o)$ over V, defined on the following slides
- a cost $cost(o) \in \mathbb{R}_0^+$

Notes:

- Operators are also called actions.
- Operators are often written as triples $\langle pre(o), eff(o), cost(o) \rangle$.
- This can be abbreviated to pairs $\langle pre(o), eff(o) \rangle$ when the cost of the operator is irrelevant.

Operators: Intuition

Intuition for operators o:

- The operator precondition describes the set of states in which a transition labeled with o can be taken.
- The operator effect describes how taking such a transition changes the state.
- The operator cost describes the cost of taking a transition labeled with o.

Syntax of Effects

Definition (Effect)

Effects over propositional state variables V are inductively defined as follows:

- $\top$ is an effect (empty effect).
- If $v \in V$ is a propositional state variable, then v and $\neg v$ are effects (atomic effect).
- If e and e' are effects, then $(e \wedge e')$ is an effect (conjunctive effect).
- If χ is a formula over V and e is an effect, then $(\chi \rhd e)$ is an effect (conditional effect).

We may omit parentheses when this does not cause ambiguity.

Example: we will later see that $((e \wedge e') \wedge e'')$ behaves identically to $(e \wedge (e' \wedge e''))$ and will write this as $e \wedge e' \wedge e''$.

B2.3 Applying Effects and Operators

Effects: Intuition

Intuition for effects:

- The empty effect $\top$ changes nothing.
- Atomic effects can be understood as assignments that update the value of a state variable.
  - v means "$v := \mathbf{T}$"
  - $\neg v$ means "$v := \mathbf{F}$"
- A conjunctive effect $e = (e' \wedge e'')$ means that both subeffects e and e' take place simultaneously.
- A conditional effect $e = (\chi \rhd e')$ means that subeffect e' takes place iff χ is true in the state where e takes place.

Semantics of Effects: Effect Conditions

Definition (Effect Condition for an Effect)

Let ℓ be an atomic effect, and let e be an effect.

The effect condition $\mathit{effcond}(\ell, e)$ under which ℓ triggers given the effect e is a propositional formula defined as follows:

- $\mathit{effcond}(\ell, \top) = \bot$
- $\mathit{effcond}(\ell, e) = \top$ for the atomic effect $e = \ell$
- $\mathit{effcond}(\ell, e) = \bot$ for all atomic effects $e = \ell' \neq \ell$
- $\mathit{effcond}(\ell, (e \wedge e')) = (\mathit{effcond}(\ell, e) \vee \mathit{effcond}(\ell, e'))$
- $\mathit{effcond}(\ell, (\chi \rhd e)) = (\chi \wedge \mathit{effcond}(\ell, e))$

Intuition: $\mathit{effcond}(\ell, e)$ represents the condition that must be true in the current state for the effect e to lead to the atomic effect ℓ

Semantics of Effects: Applying an Effect

first attempt:

Definition (Applying Effects)

Let V be a set of propositional state variables.
Let s be a state over V, and let e be an effect over V.

The resulting state of applying e in s, written $s[\![e]\!]$, is the state s' defined as follows for all $v \in V$:

$$s'(v) = \begin{cases} \mathbf{T} & \text{if } s \models \mathit{effcond}(v, e) \\ \mathbf{F} & \text{if } s \models \mathit{effcond}(\neg v, e) \wedge \neg \mathit{effcond}(v, e) \\ s(v) & \text{otherwise} \end{cases}$$

What is the problem with this definition?

Semantics of Effects: Applying an Effect

correct definition:

Definition (Applying Effects)

Let V be a set of propositional state variables.
Let s be a state over V, and let e be an effect over V.

The resulting state of applying e in s, written $s[\![e]\!]$,
is the state s' defined as follows for all $v \in V$:

$$s'(v) = \begin{cases} \mathbf{T} & \text{if } s \models \mathit{effcond}(v, e) \\ \mathbf{F} & \text{if } s \models \mathit{effcond}(\neg v, e) \wedge \neg \mathit{effcond}(v, e) \\ s(v) & \text{otherwise} \end{cases}$$

Add-after-Delete Semantics

Note:

- The definition implies that if a variable is simultaneously "added" (set to **T**) and "deleted" (set to **F**), the value **T** takes precedence.
- This is called add-after-delete semantics.
- This detail of effect semantics is somewhat arbitrary, but has proven useful in applications.

Semantics of Operators

Definition (Applicable, Applying Operators, Resulting State)

Let V be a set of propositional state variables.
Let s be a state over V, and let o be an operator over V.

Operator o is applicable in s if $s \models \mathit{pre}(o)$.

If o is applicable in s, the resulting state of applying o in s, written $s[\![o]\!]$, is the state $s[\![\mathit{eff}(o)]\!]$.

B2.4 Summary

Summary

- Propositional state variables let us compactly describe properties of large transition systems.
- A state is an assignment to a set of state variables.
- Sets of states are represented as formulas over state variables.
- Operators describe when (precondition), how (effect) and at which cost the state of the world can be changed.
- Effects are structured objects including empty, atomic, conjunctive and conditional effects.
- We gave formal semantics for applying effects and operators.