

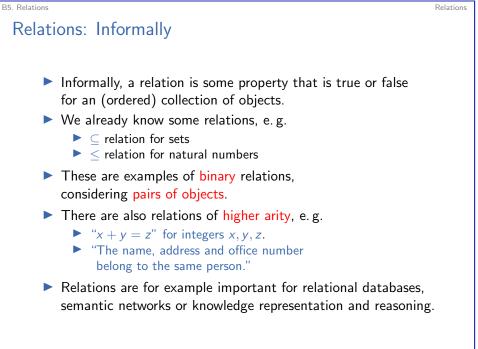
B5.1 Relations

Discrete Mathematics in Computer Science — B5. Relations

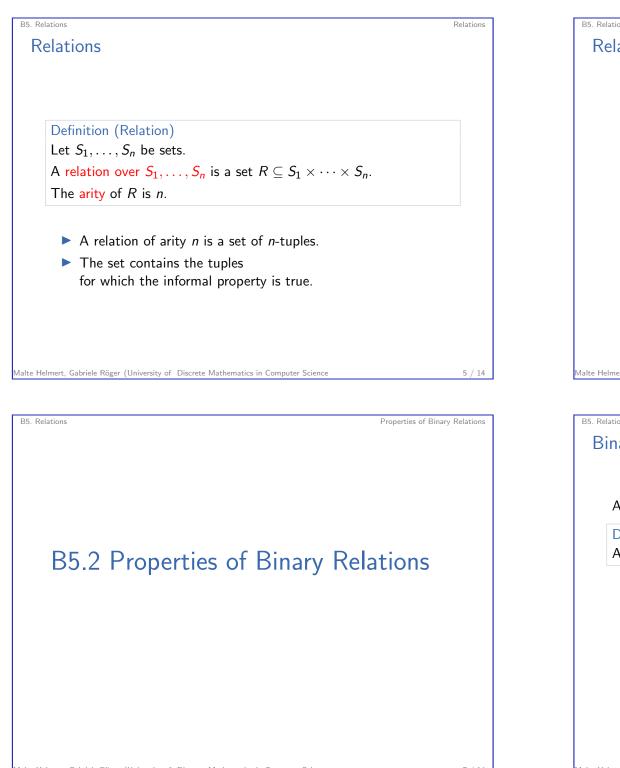
B5.1 Relations

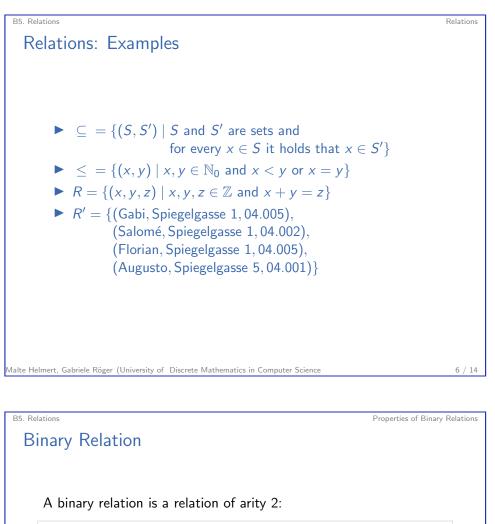
B5.2 Properties of Binary Relations

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Definition (binary relation)

A binary relation is a relation over two sets A and B.

- ▶ Instead of $(x, y) \in R$, we also write xRy, e.g. $x \leq y$ instead of $(x, y) \in d$
- If the sets are equal, we say "R is a binary relation over A" instead of "R is a binary relation over A and A".
- Such a relation over a set is also called a homogeneous relation or an endorelation.

B5. Relations

Properties of Binary Relations

Reflexivity

A reflexive relation relates every object to itself.

Definition (reflexive) A binary relation R over set A is reflexive if for all $a \in A$ it holds that $(a, a) \in R$.

Which of these relations are reflexive?

- $R = \{(a, a), (a, b), (a, c), (b, a), (b, c), (c, c)\}$ over $\{a, b, c\}$
- $R = \{(a, a), (a, b), (a, c), (b, b), (b, c), (c, c)\}$ over $\{a, b, c\}$
- equality relation = on natural numbers
- \blacktriangleright less-than relation \leq on natural numbers
- strictly-less-than relation < on natural numbers</p>

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Properties of Binary Relations

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B5. Relations Symmetry

Definition (symmetric)

A binary relation R over set A is symmetric if for all $a, b \in A$ it holds that $(a, b) \in R$ iff $(b, a) \in R$.

Which of these relations are symmetric?

- $R = \{(a, a), (a, b), (a, c), (b, a), (c, a), (c, c)\}$ over $\{a, b, c\}$
- ▶ $R = \{(a, a), (a, b), (a, c), (b, b), (b, c), (c, c)\}$ over $\{a, b, c\}$
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- \blacktriangleright less-than relation \leq on natural numbers
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Irreflexivity

A irreflexive relation never relates an object to itself.

Definition (irreflexive) A binary relation R over set A is irreflexive if for all $a \in A$ it holds that $(a, a) \notin R$.

Which of these relations are irreflexive?

- $R = \{(a, a), (a, b), (a, c), (b, a), (b, c), (c, c)\}$ over $\{a, b, c\}$
- ▶ $R = \{(a, a), (a, b), (a, c), (b, b), (b, c), (c, c)\}$ over $\{a, b, c\}$
- equality relation = on natural numbers
- ▶ less-than relation \leq on natural numbers
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Properties of Binary Relations

B5. Relations

Asymmetry and Antisymmetry

Definition (asymmetric and antisymmetric) Let R be a binary relation over set A. Relation R is asymmetric if for all $a, b \in A$ it holds that if $(a, b) \in R$ then $(b, a) \notin R$. Relation R is antisymmetric if for all $a, b \in A$ with $a \neq b$ it holds that if $(a, b) \in R$ then $(b, a) \notin R$.

Which of these relations are asymmetric/antisymmetric?

- $R = \{(a, a), (a, b), (a, c), (b, a), (c, a), (c, c)\}$ over $\{a, b, c\}$
- ▶ $R = \{(a, a), (a, b), (a, c), (b, b), (b, c), (c, c)\}$ over $\{a, b, c\}$
- equality relation = on natural numbers
- \blacktriangleright less-than relation \leq on natural numbers
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How do these properties relate to irreflexivity?

B5. Relations

Properties of Binary Relations

Transitivity

Definition

A binary relation R over set A is transitive if it holds for all $a, b, c \in A$ that if $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$.

Which of these relations are transitive?

- $R = \{(a, a), (a, b), (a, c), (b, a), (c, a), (c, c)\}$ over $\{a, b, c\}$
- $R = \{(a, a), (a, b), (a, c), (b, b), (b, c), (c, c)\}$ over $\{a, b, c\}$
- equality relation = on natural numbers
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