Discrete Mathematics in Computer Science

Tuples and the Cartesian Product

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Sets vs. Tuples

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Tuples

- *k*-tuple: ordered sequence of k objects ($k \in \mathbb{N}_0$)
- written $(o_1, \ldots, o_k)$ or $\langle o_1, \ldots, o_k \rangle$
- unlike sets, order matters ($\langle 1, 2 \rangle \neq \langle 2, 1 \rangle$)
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- unlike sets, order matters ($\langle 1, 2 \rangle \neq \langle 2, 1 \rangle$)
- objects may occur multiple times in a tuple
- objects contained in tuples are called components
- terminology:
  - $k = 2$: (ordered) pair
  - $k = 3$: triple
  - more rarely: quadruple, quintuple, sextuple, septuple, ...
- if k is clear from context (or does not matter), often just called tuple

Equality of Tuples

Definition (Equality of Tuples)

Two n-tuples $t = \langle o_1, \dots, o_n \rangle$ and $t' = \langle o'_1, \dots, o'_n \rangle$ are equal ($t = t'$) if for $i \in \{1, \dots, n\}$ it holds that $o_i = o'_i$.

Cartesian Product

Definition (Cartesian Product and Cartesian Power)

Let $S_1, \ldots, S_n$ be sets. The Cartesian product $S_1 \times \cdots \times S_n$ is the following set of n-tuples:

$$S_1 \times \cdots \times S_n = \{\langle x_1, \ldots, x_n \rangle \mid x_1 \in S_1, x_2 \in S_2, \ldots, x_n \in S_n\}.$$

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The k-ary Cartesian power of a set S (with $k \in \mathbb{N}_1$) is the set $S^k = \{\langle o_1, \ldots, o_k \rangle \mid o_i \in S \text{ for all } i \in \{1, \ldots, k\}\} = \underbrace{S \times \cdots \times S}_{k \text{ times}}$.

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Example: $A = \{a, b\}$, $B = \{1, 2, 3\}$

$A^2 =$

(Non-)properties of the Cartesian Product

The Cartesian product is

- not commutative, in most cases $A \times B \neq B \times A$.
- not associative, in most cases $(A \times B) \times C \neq A \times (B \times C)$

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Why? Exceptions?