Planning and Optimization

F8. Monte-Carlo Tree Search Algorithms (Part I)

Malte Helmert and Gabriele Röger

Universität Basel

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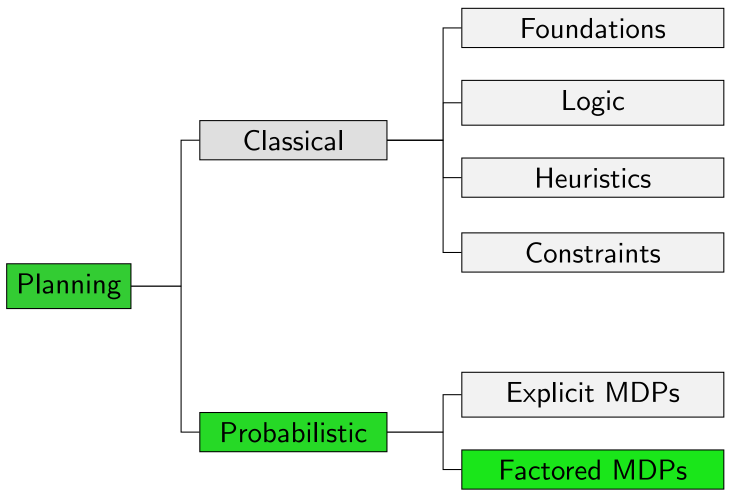
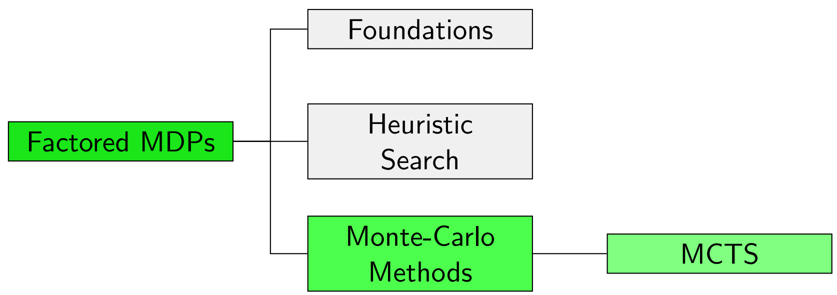
Content of this Course



Content of this Course: Factored MDPs

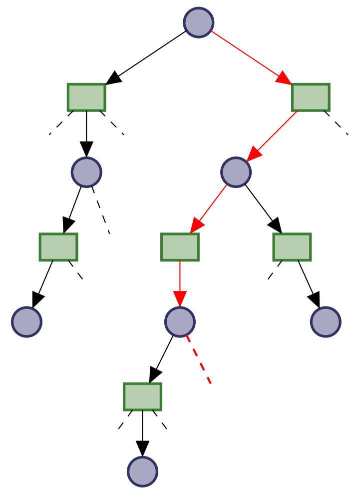


F8.1 Introduction

Monte-Carlo Tree Search: Reminder

Performs iterations with 4 phases:

- selection: use given tree policy to traverse explicated tree
- expansion: add node(s) to the tree
- simulation: use given default policy to simulate run
- backpropagation: update visited nodes with Monte-Carlo backups



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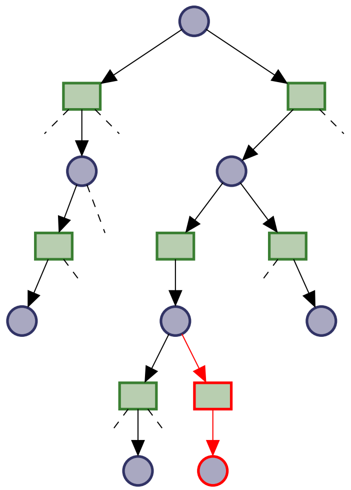

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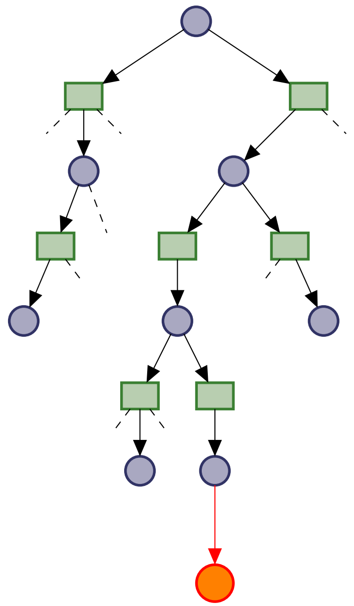

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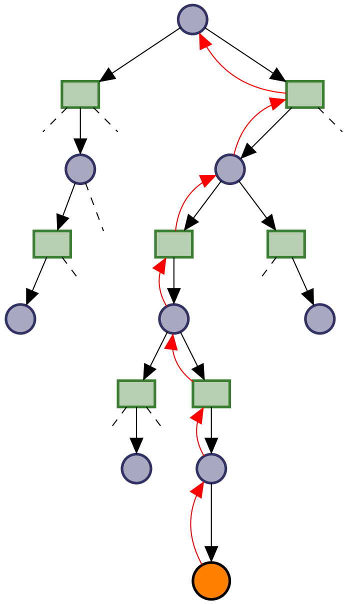


Motivation

- Monte-Carlo Tree Search is a framework of algorithms
- concrete MCTS algorithms are specified in terms of
  - a tree policy;
  - and a default policy
- for most tasks, a well-suited MCTS configuration exists
- but for each task, many MCTS configurations perform poorly
- and every MCTS configuration that works well in one problem performs poorly in another problem

⇒ There is no "Swiss army knife" configuration for MCTS

Role of Tree Policy

- used to traverse explicated tree from root node to a leaf
- maps decision nodes to a probability distribution over actions (usually as a function over a decision node and its children)
- exploits information from search tree
  - able to learn over time
  - requires MCTS tree to memorize collected information

Role of Default Policy

- used to simulate run from some state to a goal
- maps states to a probability distribution over actions
- independent from MCTS tree
  - does not improve over time
  - can be computed quickly
  - constant memory requirements
- accumulated cost of simulated run used to initialize state-value estimate of decision node

F8.2 Default Policy

MCTS Simulation

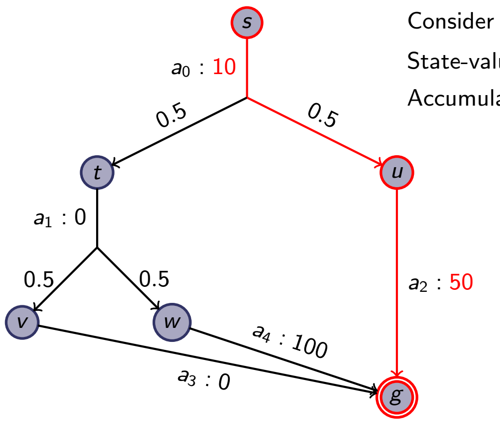
MCTS simulation with default policy π from state s

cost := 0
while $s \notin S_\star$:
    $a :\sim \pi(s)$
    cost := cost + $c(a)$
    $s :\sim \text{succ}(s, a)$
return cost

Default policy must be proper

- to guarantee termination of the procedure
- and a finite cost

Default Policy: Example



Consider deterministic default policy π

State-value of s under π: 60

Accumulated cost of run: 60

Default Policy Realizations

- Early MCTS implementations used random default policy:

$$\pi(a \mid s) = \begin{cases} \frac{1}{|A(s)|} & \text{if } a \in A(s) \\ 0 & \text{otherwise} \end{cases}$$

- only proper if goal can be reached from each state
- poor guidance, and due to high variance even misguidance

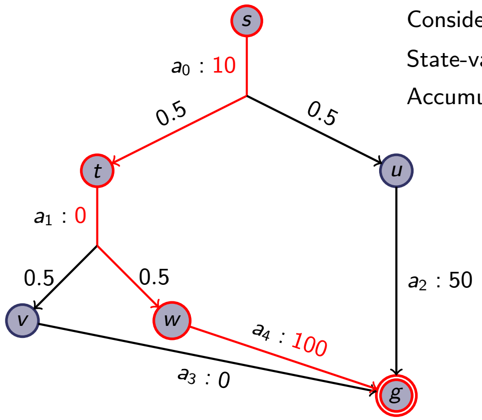
Default Policy Realizations

There are only few alternatives to random default policy, e.g.,

- heuristic-based policy
- domain-specific policy

Reason: No matter how good the policy, result of simulation can be arbitrarily poor

Default Policy: Example (2)



Consider deterministic default policy π

State-value of s under π: 60

Accumulated cost of run: 110

Default Policy Realizations

Possible solution to overcome this weakness:

- average over multiple random walks
- converges to true action-values of policy
- computationally often very expensive

Cheaper and more successful alternative:

- skip simulation step of MCTS
- use heuristic directly for initialization of state-value estimates
- instead of simulating execution of heuristic-guided policy
- much more successful (e.g. neural networks of AlphaGo)

F8.3 Asymptotic Optimality

Optimal Search

Heuristic search algorithms (like RTDP) achieve optimality by combining
- greedy search
- admissible heuristic
- Bellman backups

In Monte-Carlo Tree Search
- search behavior defined by a tree policy
- admissibility of default policy / heuristic irrelevant (and usually not given)
- Monte-Carlo backups

MCTS requires a different idea for optimal behavior in the limit.

Asymptotic Optimality

Asymptotic Optimality

Let an MCTS algorithm build an MCTS tree $\mathcal{G} = \langle d_0, D, C, E \rangle$.
The MCTS algorithm is asymptotically optimal if

$$\lim_{k \to \infty} \hat{Q}^k(c) = Q_\star(s(c), a(c)) \text{ for all } c \in C^k,$$

where k is the number of trials.

- this is just one special form of asymptotic optimality
- some optimal MCTS algorithms are not asymptotically optimal by this definition (e.g., $\lim_{k \to \infty} \hat{Q}^k(c) = \ell \cdot Q_\star(s(c), a(c))$ for some $\ell \in \mathbb{R}^+$)
- all practically relevant optimal MCTS algorithms are asymptotically optimal by this definition

Asymptotically Optimal Tree Policy

An MCTS algorithm is asymptotically optimal if
1. its tree policy explores forever:
   - the (infinite) sum of the probabilities that a decision node is visited must diverge
   - $\Rightarrow$ every search node is explicated eventually and visited infinitely often
2. its tree policy is greedy in the limit:
   - probability that optimal action is selected converges to 1
   - $\Rightarrow$ in the limit, backups based on iterations where only an optimal policy is followed dominate suboptimal backups
3. its default policy initializes decision nodes with finite values

Example: Random Tree Policy

Example

Consider the random tree policy for decision node d where:

$$\pi(a \mid d) = \begin{cases} \frac{1}{|A(s(d))|} & \text{if } a \in A(s(d)) \\ 0 & \text{otherwise} \end{cases}$$

The random tree policy explores forever:

Let $\langle d_0, c_0, \ldots, d_n, c_n, d \rangle$ be a sequence of connected nodes in $\mathcal{G}^k$ and let $p := \min_{0 < i < n-1} T(s(d_i), a(c_i), s(d_{i+1}))$.

Let $\mathbb{P}^k$ be the probability that d is visited in trial k. With $\mathbb{P}^k \geq (\frac{1}{|A|} \cdot p)^n$, we have that

$$lim_{k \to \infty} \sum_{i=1}^{k} \mathbb{P}^k \geq k \cdot (\frac{1}{|A|} \cdot p)^n = \infty$$

Example: Random Tree Policy

Example

Consider the random tree policy for decision node d where:

$$\pi(a \mid d) = \begin{cases} \frac{1}{|A(s(d))|} & \text{if } a \in A(s(d)) \\ 0 & \text{otherwise} \end{cases}$$

The random tree policy is not greedy in the limit unless all actions are always optimal:

The probability that an optimal action a is selected in decision node d is

$$\lim_{k\to\infty} 1 - \sum_{\{a' \not\in \pi_{V^\star}(s)\}} \frac{1}{|A(s(d))|} < 1.$$

⇝ MCTS with random tree policy not asymptotically optimal

Example: Greedy Tree Policy

Example

Consider the greedy tree policy for decision node d where:

$$\pi(a \mid d) = \begin{cases} \frac{1}{|A^k_\star(d)|} & \text{if } a \in A^k_\star(d)) \\ 0 & \text{otherwise,} \end{cases}$$

with $A^k_\star(d) = \{a(c) \in A(s(d)) \mid c \in \arg\min_{c' \in \text{children}(d)} \hat{Q}^k(c')\}$.

- Greedy tree policy is greedy in the limit
- Greedy tree policy does not explore forever

⇝ MCTS with greedy tree policy not asymptotically optimal

Tree Policy: Objective

To satisfy both requirements, MCTS tree policies have two contradictory objectives:

- explore parts of the search space that have not been investigated thoroughly
- exploit knowledge about good actions to focus search on promising areas of the search space

central challenge: balance exploration and exploitation

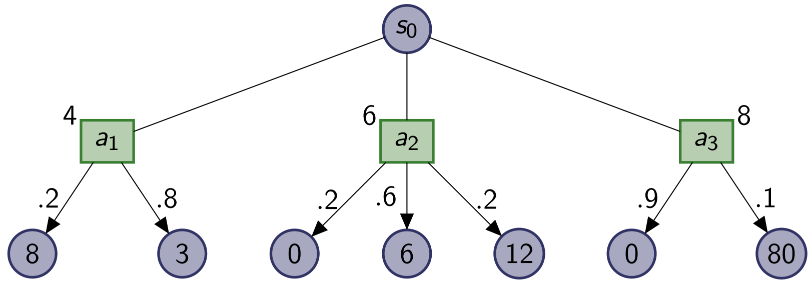
⇒ borrow ideas from related multi-armed bandit problem

F8.4 Multi-armed Bandit Problem

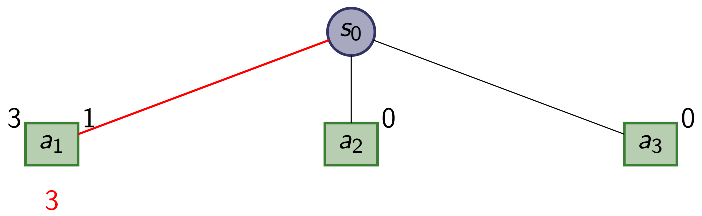
Multi-armed Bandit Problem

- most commonly used tree policies are inspired from research on the multi-armed bandit problem (MAB)
- MAB is a learning scenario (model not revealed to agent)
- agent repeatedly faces the same decision: to pull one of several arms of a slot machine
- pulling an arm yields stochastic reward
  ⇒ in MABs, we have rewards rather than costs
- can be modeled as an MDP

Multi-armed Bandit Problem: Planning Scenario

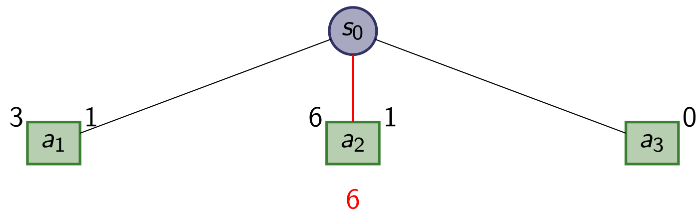


Multi-armed Bandit Problem: Learning Scenario



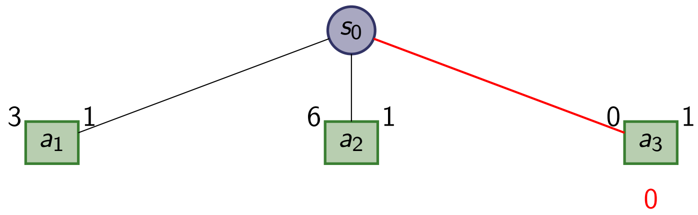
- Pull arms following policy to explore or exploit
- Update $\hat{Q}$ and N based on observations
- Accumulated reward after 1 trial is 3

Multi-armed Bandit Problem: Learning Scenario



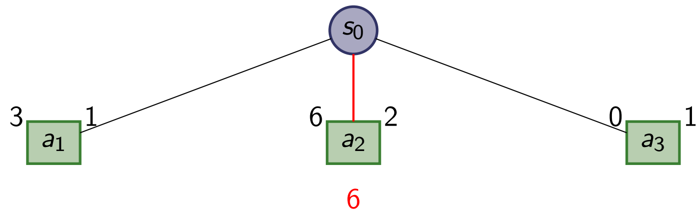
- Pull arms following policy to explore or exploit
- Update $\hat{Q}$ and N based on observations
- Accumulated reward after 2 trials is $3 + 6 = 9$

Multi-armed Bandit Problem: Learning Scenario



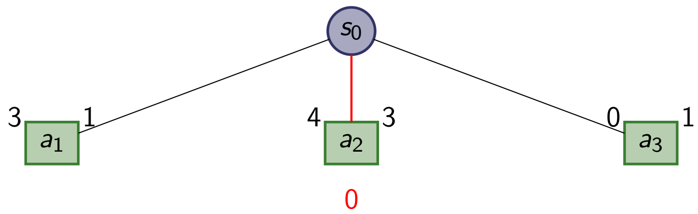
- Pull arms following policy to explore or exploit
- Update $\hat{Q}$ and N based on observations
- Accumulated reward after 3 trials is $3 + 6 + 0 = 9$

Multi-armed Bandit Problem: Learning Scenario



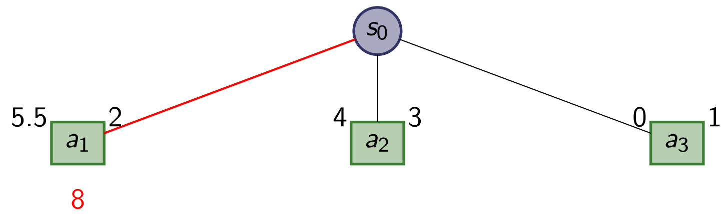
- Pull arms following policy to explore or exploit
- Update $\hat{Q}$ and N based on observations
- Accumulated reward after 4 trials is $3 + 6 + 0 + 6 = 15$

Multi-armed Bandit Problem: Learning Scenario



- Pull arms following policy to explore or exploit
- Update $\hat{Q}$ and N based on observations
- Accumulated reward after 5 trials is $3 + 6 + 0 + 6 + 0 = 15$

Multi-armed Bandit Problem: Learning Scenario



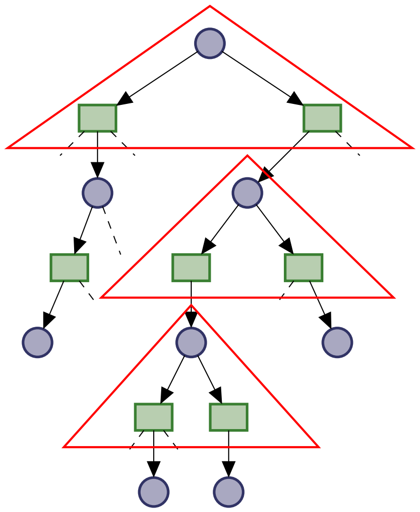
- Pull arms following policy to explore or exploit
- Update $\hat{Q}$ and N based on observations
- Accumulated reward after 6 trials is $3 + 6 + 0 + 6 + 0 + 8 = 23$

Policy Quality

- Since model unknown to MAB agent, it cannot achieve accumulated reward of $k \cdot V_\star$ with $V_\star := \max_a Q_\star(a)$ in k trials
- Quality of MAB policy π measured in terms of regret, i.e., the difference between $k \cdot V_\star$ and expected reward of π in k trials
- Regret cannot grow slower than logarithmically in the number of trials

MABs in MCTS Tree

- many tree policies treat each decision node as MAB
- where each action yields a stochastic reward
- dependence of reward on future decision is ignored
- MCTS planner uses simulations to learn reasonable behavior
- SSP model is not considered



F8.5 Summary

Summary

- The simulation phase simulates the execution of the default policy
- MCTS algorithms are optimal in the limit if
  - the tree policy is greedy in the limit,
  - the tree policy explores forever, and
  - the default policy initializes with finite value
- Central challenge of most tree policies: balance exploration and exploitation
- each decision of an MCTS tree policy can be viewed as an multi-armed bandit problem.