

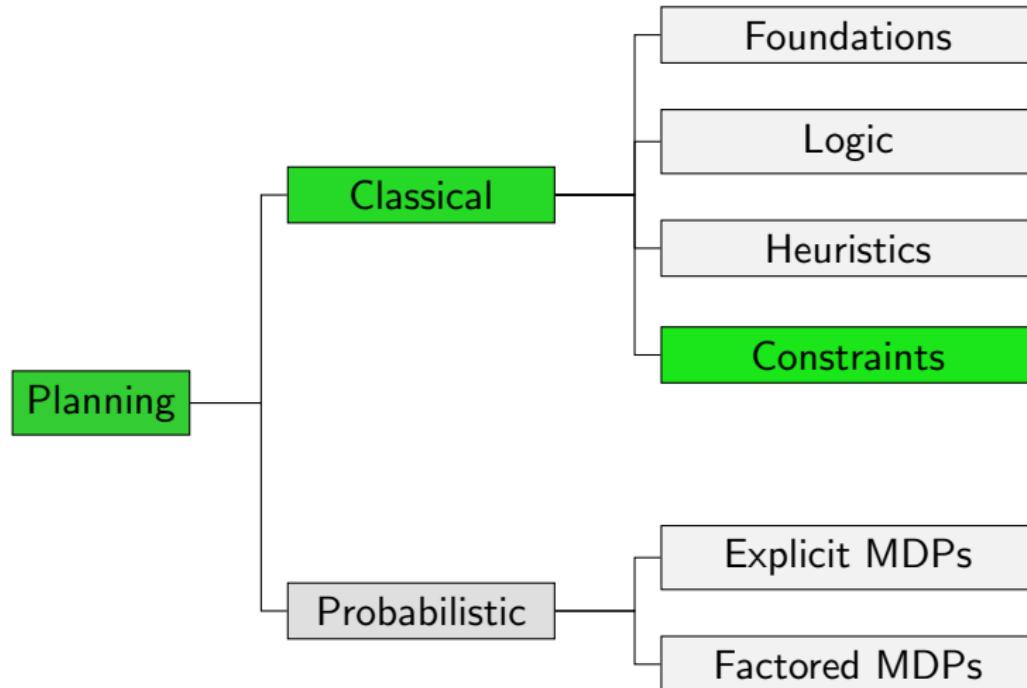
Planning and Optimization

E5. Cost Partitioning

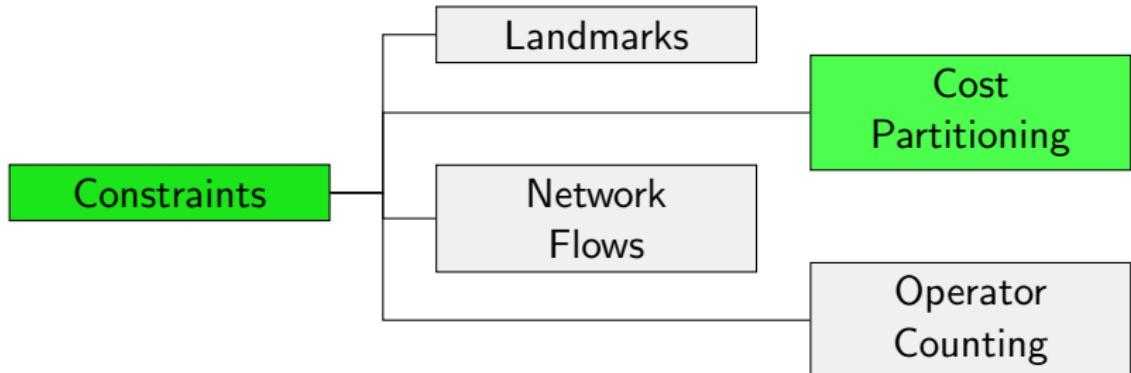
Malte Helmert and Gabriele Röger

Universität Basel

Content of this Course



Content of this Course: Constraints



Introduction

Exploiting Additivity

- Additivity allows to add up heuristic estimates admissibly. This gives better heuristic estimates than the maximum.
- For example, the canonical heuristic for PDBs sums up where addition is admissible (by an additivity criterion) and takes the maximum otherwise.
- **Cost partitioning** provides a more general additivity criterion, based on an adaption of the operator costs.

Additivity

When is it impossible to sum up abstraction heuristics admissibly?

- Abstraction heuristics are consistent and goal-aware.
- Sum of goal-aware heuristics is goal aware.
- \Rightarrow Sum of consistent heuristics not necessarily consistent.

Combining Heuristics Admissibly: Example

Example

Consider an FDR planning task $\langle V, I, \{o_1, o_2, o_3, o_4\}, \gamma \rangle$ with $V = \{v_1, v_2, v_3\}$ with $\text{dom}(v_1) = \{A, B\}$ and $\text{dom}(v_2) = \text{dom}(v_3) = \{A, B, C\}$, $I = \{v_1 \mapsto A, v_2 \mapsto A, v_3 \mapsto A\}$,

$$o_1 = \langle v_1 = A, v_1 := B, 1 \rangle$$

$$o_2 = \langle v_2 = A \wedge v_3 = A, v_2 := B \wedge v_3 := B, 1 \rangle$$

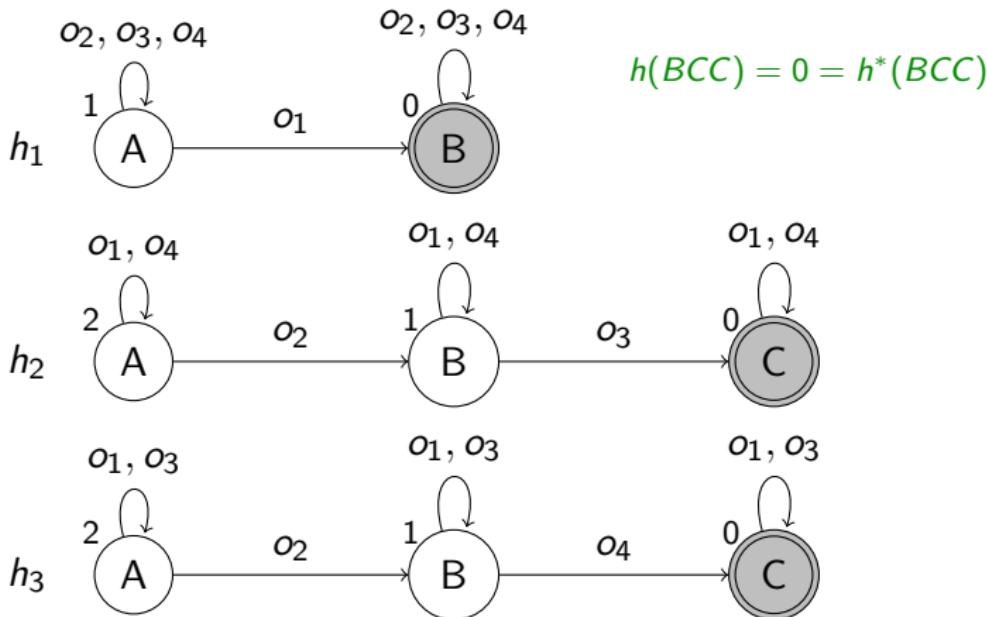
$$o_3 = \langle v_2 = B, v_2 := C, 1 \rangle$$

$$o_4 = \langle v_3 = B, v_3 := C, 1 \rangle$$

and $\gamma = (v_1 = B) \wedge (v_2 = C) \wedge (v_3 = C)$.

Combining Heuristics Admissibly: Example

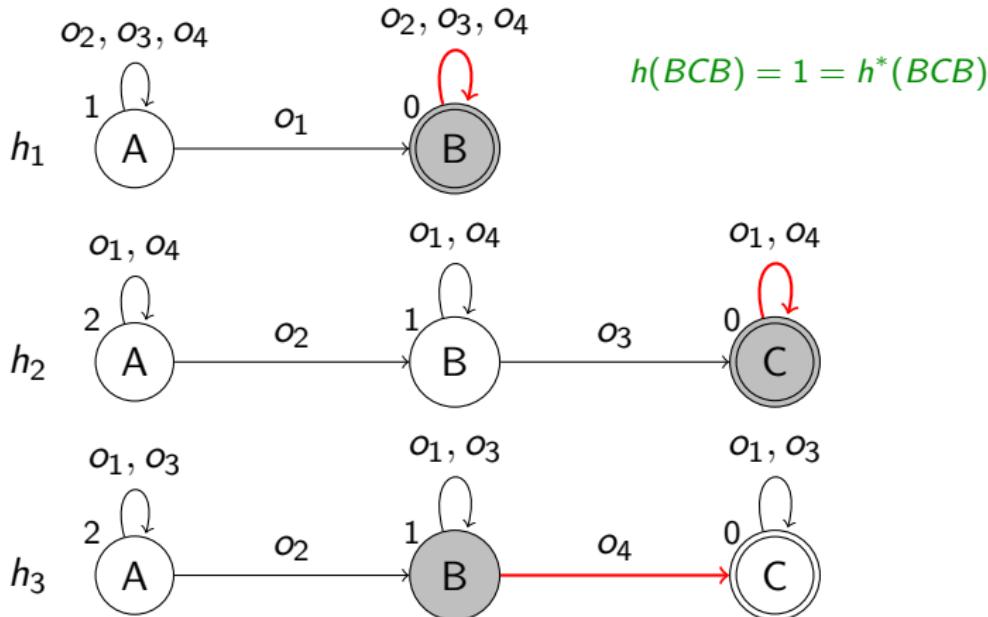
Let $h = h_1 + h_2 + h_3$. Where is consistency constraint violated?



Consider solution $\langle o_1, o_2, o_3, o_4 \rangle$

Combining Heuristics Admissibly: Example

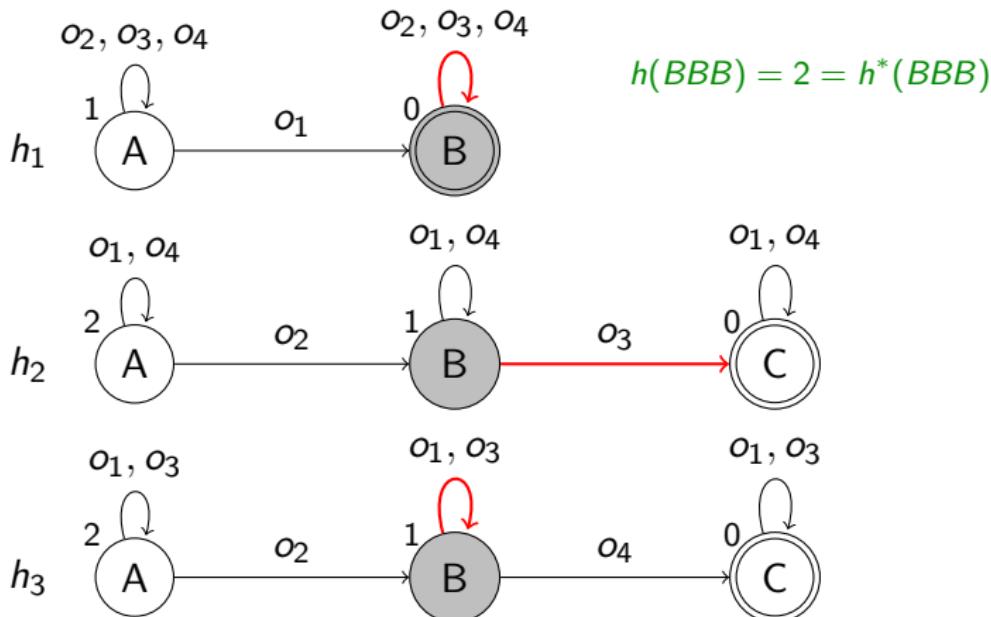
Let $h = h_1 + h_2 + h_3$. Where is consistency constraint violated?



Consider solution $\langle o_1, o_2, o_3, \textcolor{red}{o_4} \rangle$

Combining Heuristics Admissibly: Example

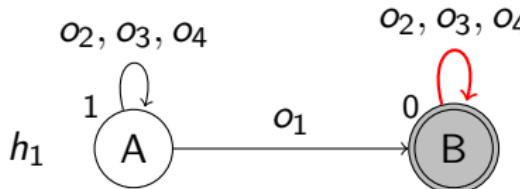
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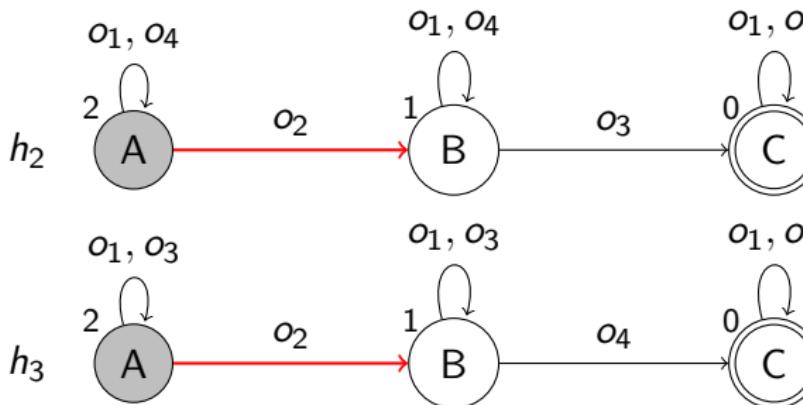
Consider solution $\langle o_1, o_2, \textcolor{red}{o}_3, o_4 \rangle$

Combining Heuristics Admissibly: Example

Let $h = h_1 + h_2 + h_3$. Where is consistency constraint violated?



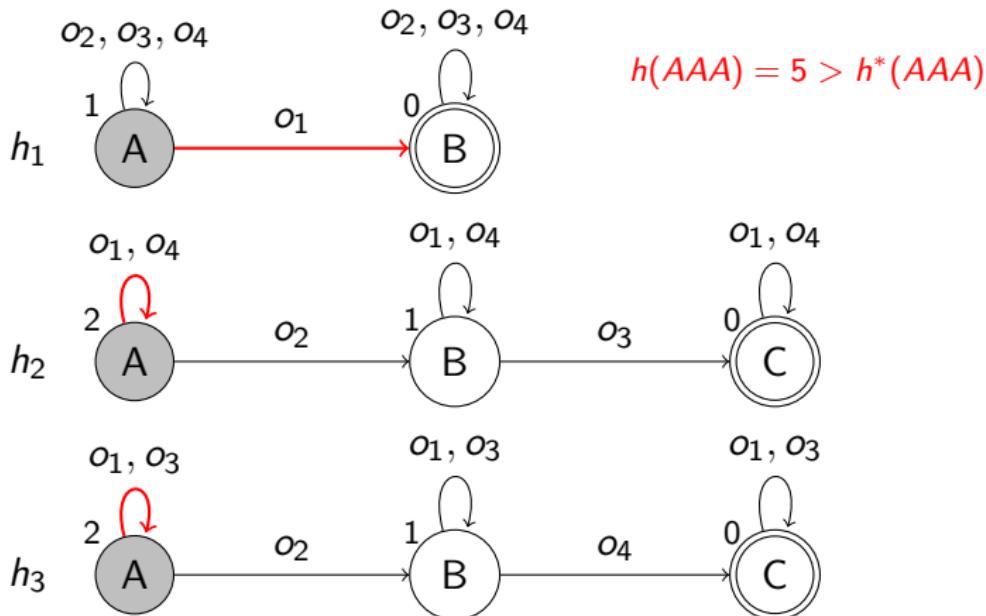
$h(BAA) = 4 > h^*(BAA)$ and
 $h(BAA) > h(BBB) + \text{cost}(o_2)$
 $\Rightarrow h$ inconsistent and inadmissible



Consider solution $\langle o_1, \textcolor{red}{o_2}, o_3, o_4 \rangle$

Combining Heuristics Admissibly: Example

Let $h = h_1 + h_2 + h_3$. Where is consistency constraint violated?



Consider solution $\langle o_1, o_2, o_3, o_4 \rangle$

Solution: Cost partitioning

h is not admissible because $cost(o_2)$ is considered in h_2 and h_3

Is there anything we can do about this?

Solution: Cost partitioning

h is not admissible because $cost(o_2)$ is considered in h_2 and h_3

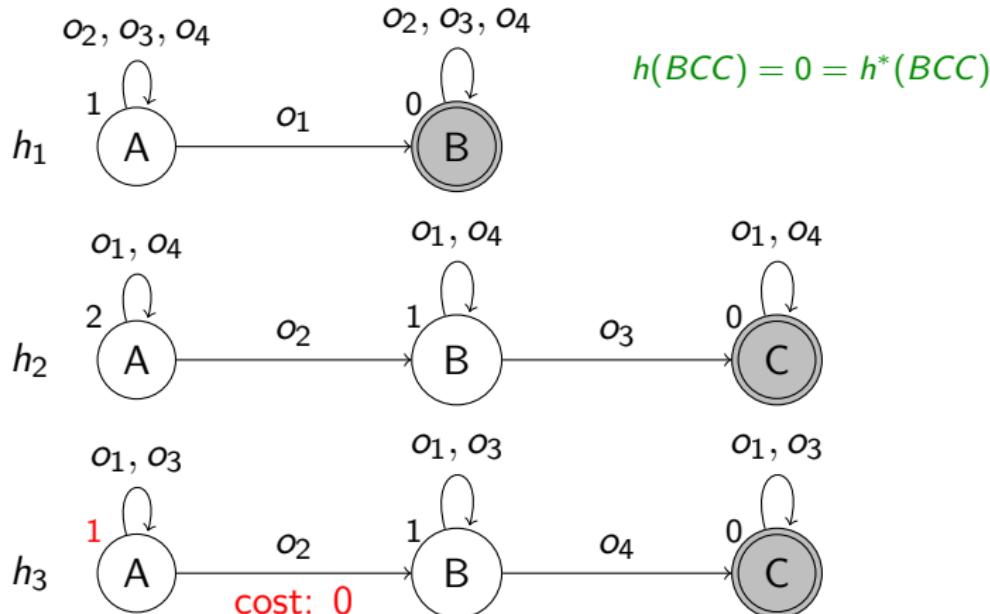
Is there anything we can do about this?

Solution 1:

We can ignore the cost of o_2 in h_2 or h_3 by setting its cost to 0.

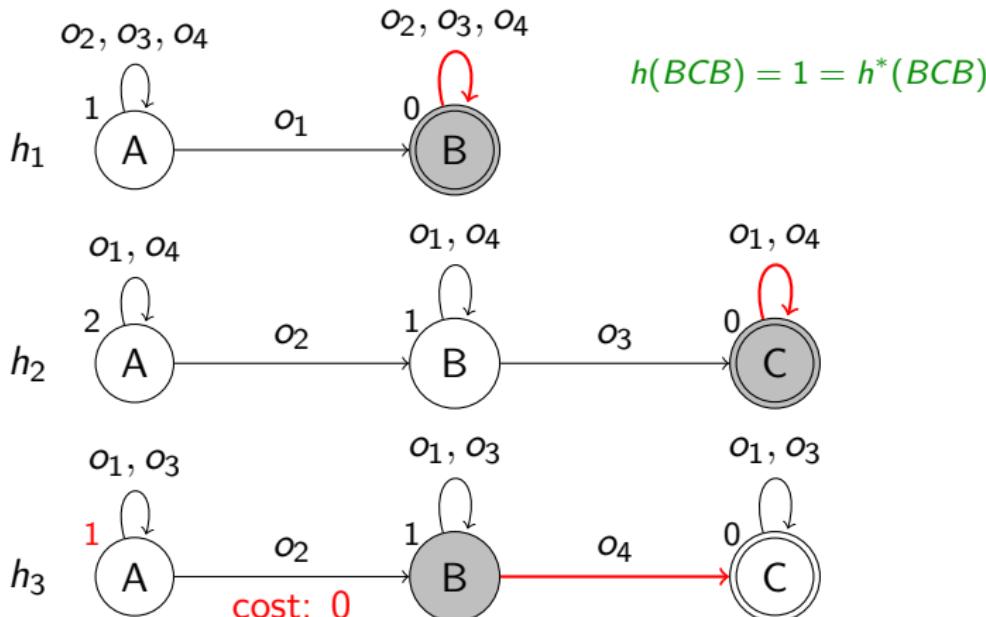
Combining Heuristics Admissibly: Example

Assume $cost_3(o_2) = 0$

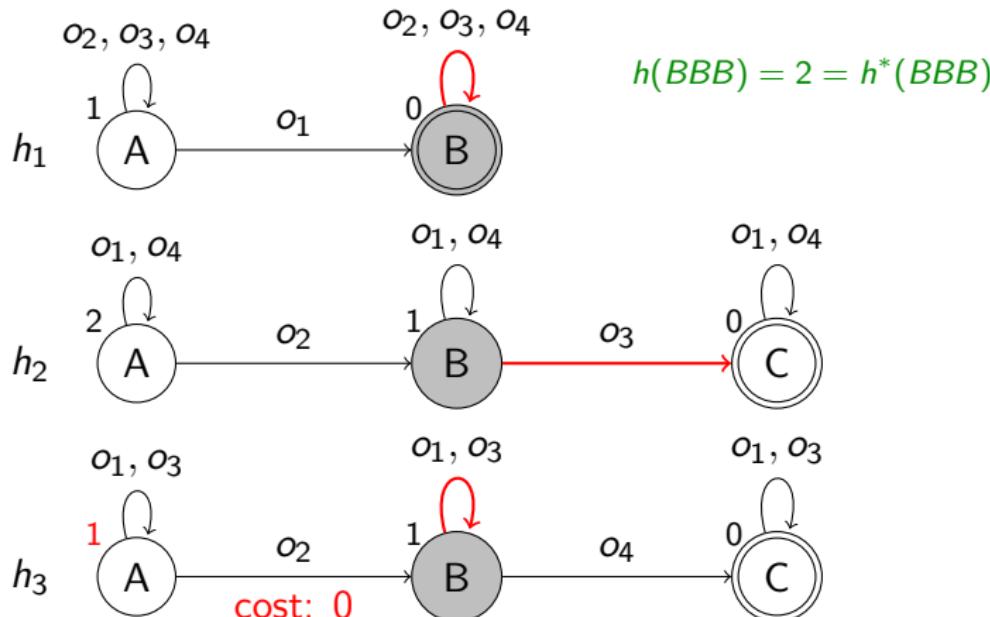


Consider solution $\langle o_1, o_2, o_3, o_4 \rangle$

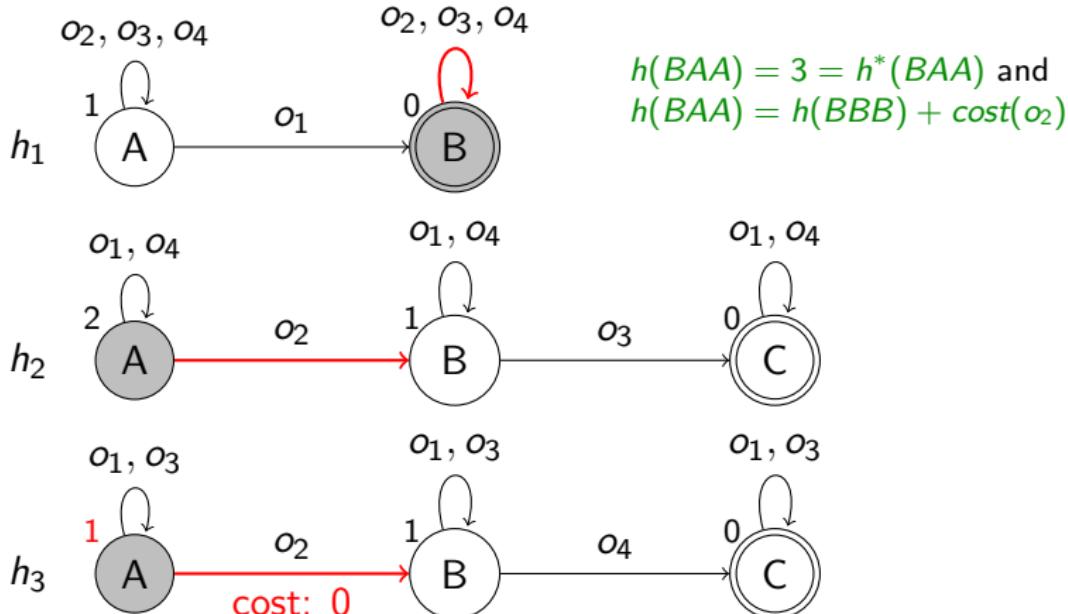
Combining Heuristics Admissibly: Example

Assume $cost_3(o_2) = 0$ Consider solution $\langle o_1, o_2, o_3, o_4 \rangle$

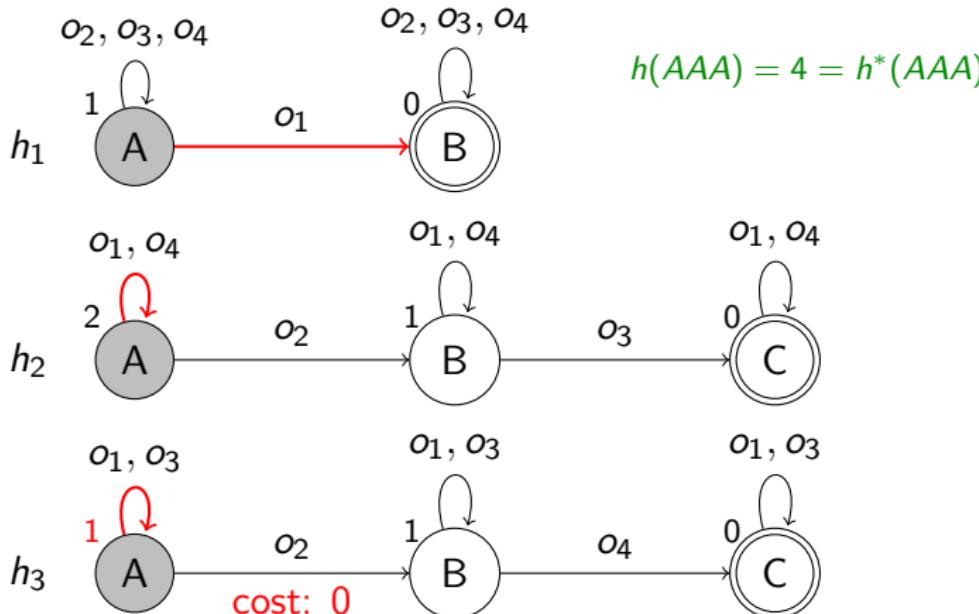
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Assume $cost_3(o_2) = 0$ Consider solution $\langle o_1, o_2, o_3, o_4 \rangle$

Solution: Cost partitioning

h is not admissible because $cost(o_2)$ is considered in h_2 and h_3

Is there anything we can do about this?

Solution 1:

We can ignore the cost of o_2 in h_2 or h_3 by setting its cost to 0.
This is called a **zero-one cost partitioning**.

Solution: Cost partitioning

h is not admissible because $\text{cost}(o_2)$ is considered in h_2 and h_3

Is there anything we can do about this?

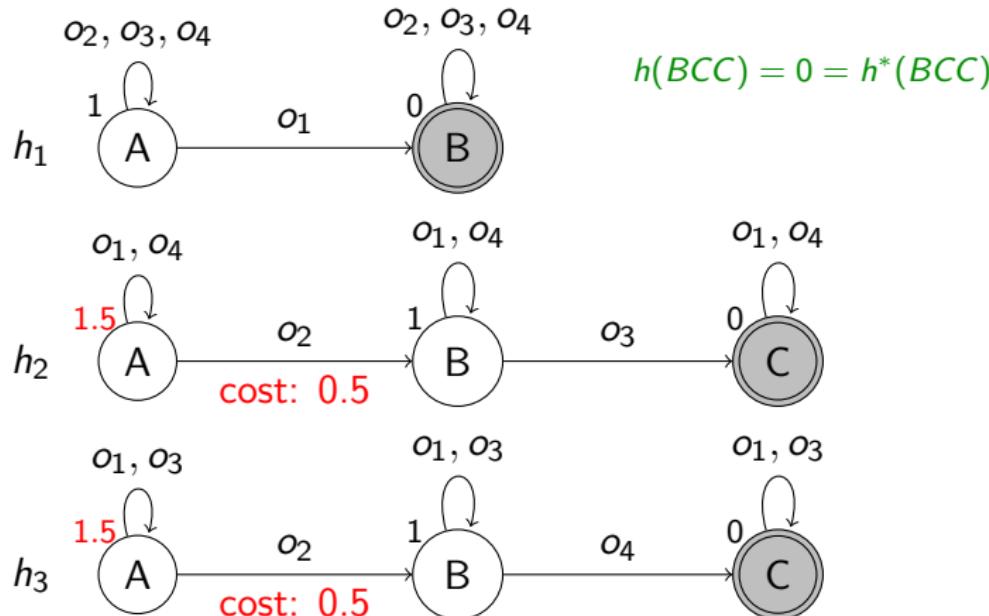
Solution 1:

We can ignore the cost of o_2 in h_2 or h_3 by setting its cost to 0.
This is called a **zero-one cost partitioning**.

Solution 2: Consider a cost of $\frac{1}{2}$ for o_2 both in h_2 and h_3 .

Combining Heuristics Admissibly: Example

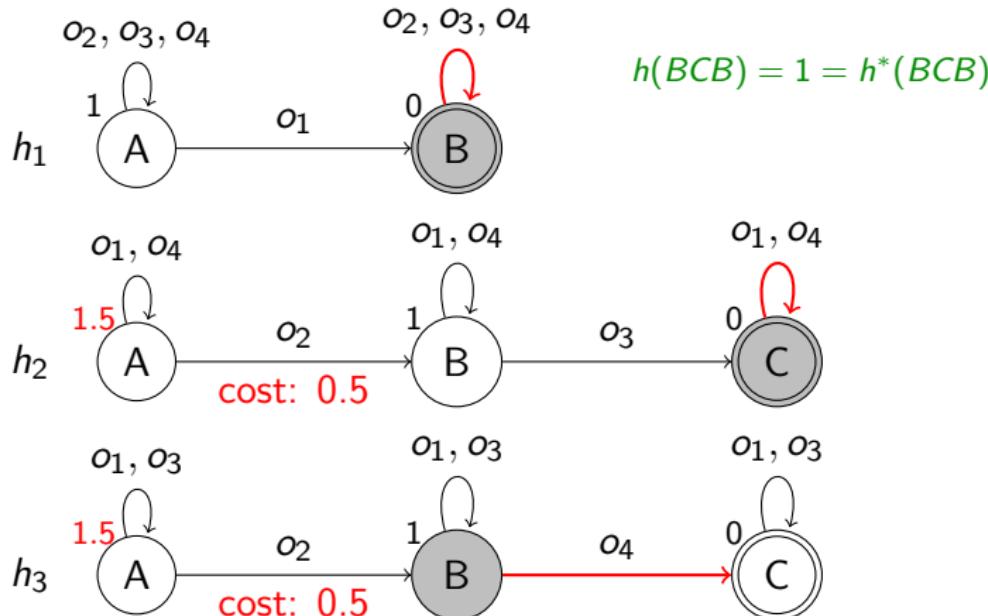
Assume $cost_2(o_2) = cost_3(o_2) = \frac{1}{2}$



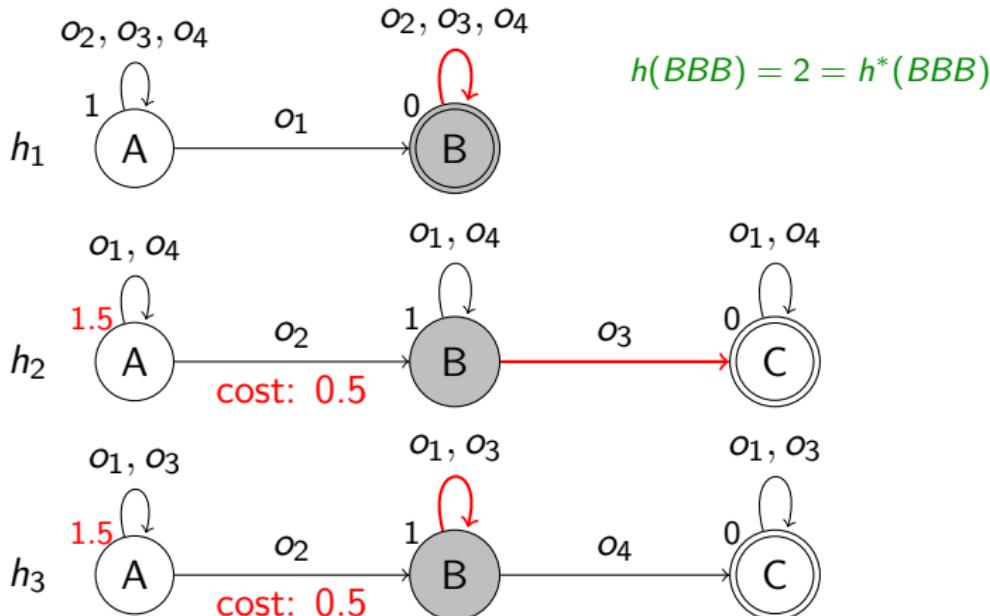
$$h(BCC) = 0 = h^*(BCC)$$

Consider solution $\langle o_1, o_2, o_3, o_4 \rangle$

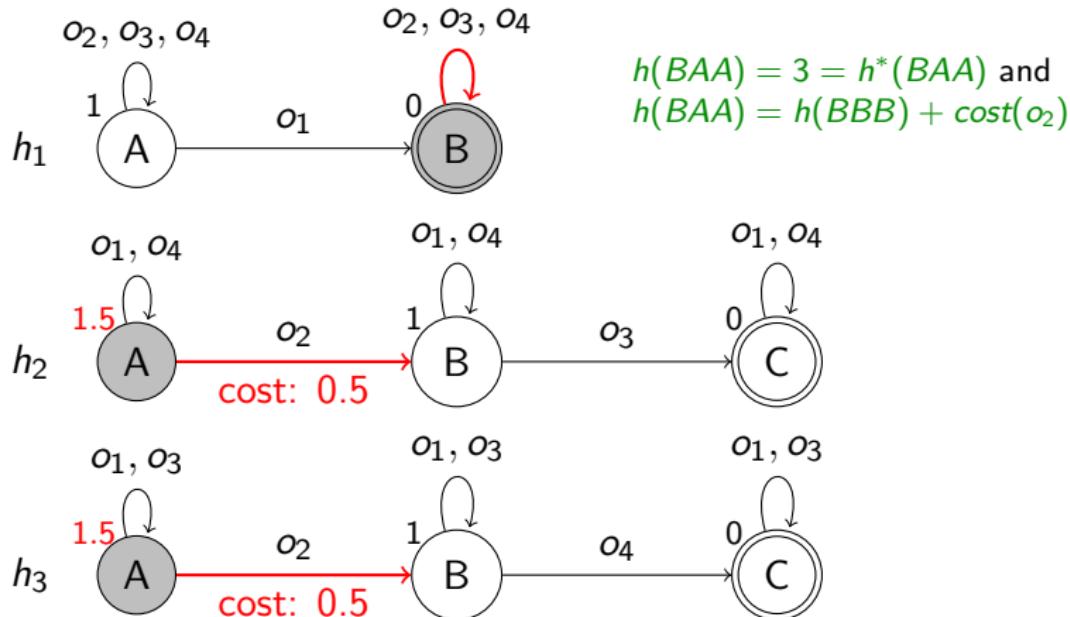
Combining Heuristics Admissibly: Example

Assume $cost_2(o_2) = cost_3(o_2) = \frac{1}{2}$ Consider solution $\langle o_1, o_2, o_3, \textcolor{red}{o_4} \rangle$

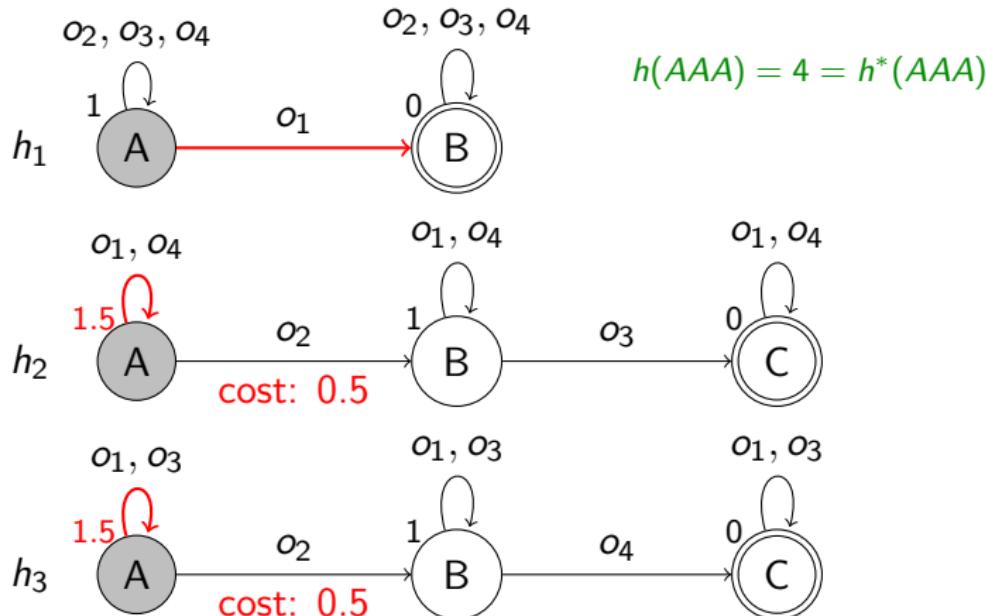
Combining Heuristics Admissibly: Example

Assume $cost_2(o_2) = cost_3(o_2) = \frac{1}{2}$ Consider solution $\langle o_1, o_2, \textcolor{red}{o_3}, o_4 \rangle$

Combining Heuristics Admissibly: Example

Assume $cost_2(o_2) = cost_3(o_2) = \frac{1}{2}$ Consider solution $\langle o_1, \textcolor{red}{o_2}, o_3, o_4 \rangle$

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Solution: Cost partitioning

h is not admissible because $cost(o_2)$ is considered in h_2 and h_3

Is there anything we can do about this?

Solution 1:

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This is called a **zero-one cost partitioning**.

Solution 2: Consider a cost of $\frac{1}{2}$ for o_2 both in h_2 and h_3 .
This is called a **uniform cost partitioning**.

Solution: Cost partitioning

h is not admissible because $cost(o_2)$ is considered in h_2 and h_3

Is there anything we can do about this?

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Solution 2: Consider a cost of $\frac{1}{2}$ for o_2 both in h_2 and h_3 .
This is called a **uniform cost partitioning**.

General solution: satisfy **cost partitioning constraint**

$$\sum_{i=1}^n cost_i(o) \leq cost(o) \text{ for all } o \in O$$

Solution: Cost partitioning

h is not admissible because $cost(o_2)$ is considered in h_2 and h_3

Is there anything we can do about this?

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General solution: satisfy **cost partitioning constraint**

$$\sum_{i=1}^n cost_i(o) \leq cost(o) \text{ for all } o \in O$$

What about o_1 , o_3 and o_4 ?

Cost Partitioning

Cost Partitioning

Definition (Cost Partitioning)

Let Π be a planning task with operators O .

A **cost partitioning** for Π is a tuple $\langle cost_1, \dots, cost_n \rangle$, where

- $cost_i : O \rightarrow \mathbb{R}_0^+$ for $1 \leq i \leq n$ and
- $\sum_{i=1}^n cost_i(o) \leq cost(o)$ for all $o \in O$.

The cost partitioning induces a tuple $\langle \Pi_1, \dots, \Pi_n \rangle$ of planning tasks, where each Π_i is identical to Π except that the cost of each operator o is $cost_i(o)$.

Cost Partitioning: Admissibility (1)

Theorem (Sum of Solution Costs is Admissible)

Let Π be a planning task, $\langle cost_1, \dots, cost_n \rangle$ be a cost partitioning and $\langle \Pi_1, \dots, \Pi_n \rangle$ be the tuple of induced tasks.

Then the sum of the solution costs of the induced tasks is an admissible heuristic for Π , i.e., $\sum_{i=1}^n h_{\Pi_i}^* \leq h_{\Pi}^*$.

Cost Partitioning: Admissibility (2)

Proof of Theorem.

If there is no plan for state s of Π , both sides are ∞ . Otherwise, let $\pi = \langle o_1, \dots, o_m \rangle$ be an optimal plan for state s of Π . Then

$$\begin{aligned} \sum_{i=1}^n h_{\Pi_i}^*(s) &\leq \sum_{i=1}^n \sum_{j=1}^m \text{cost}_i(o_j) && (\pi \text{ plan in each } \Pi_i) \\ &= \sum_{j=1}^m \sum_{i=1}^n \text{cost}_i(o_j) && (\text{comm./ass. of sum}) \\ &\leq \sum_{j=1}^m \text{cost}(o_j) && (\text{cost partitioning}) \\ &= h_{\Pi}^*(s) && (\pi \text{ optimal plan in } \Pi) \end{aligned}$$



Cost Partitioning Preserves Admissibility

In the rest of the chapter, we write h_{Π} to denote heuristic h evaluated on task Π .

Corollary (Sum of Admissible Estimates is Admissible)

Let Π be a planning task and let $\langle \Pi_1, \dots, \Pi_n \rangle$ be induced by a cost partitioning.

For admissible heuristics h_1, \dots, h_n , the sum $h(s) = \sum_{i=1}^n h_{i,\Pi_i}(s)$ is an admissible estimate for s in Π .

Cost Partitioning Preserves Consistency

Theorem (Cost Partitioning Preserves Consistency)

Let Π be a planning task and let $\langle \Pi_1, \dots, \Pi_n \rangle$ be induced by a cost partitioning $\langle cost_1, \dots, cost_n \rangle$.

If h_1, \dots, h_n are consistent heuristics then $h = \sum_{i=1}^n h_{i,\Pi_i}$ is a consistent heuristic for Π .

Proof.

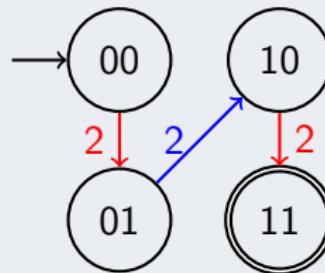
Let o be an operator that is applicable in state s .

$$\begin{aligned} h(s) &= \sum_{i=1}^n h_{i,\Pi_i}(s) \leq \sum_{i=1}^n (cost_i(o) + h_{i,\Pi_i}(s[o])) \\ &= \sum_{i=1}^n cost_i(o) + \sum_{i=1}^n h_{i,\Pi_i}(s[o]) \leq cost(o) + h(s[o]) \end{aligned}$$



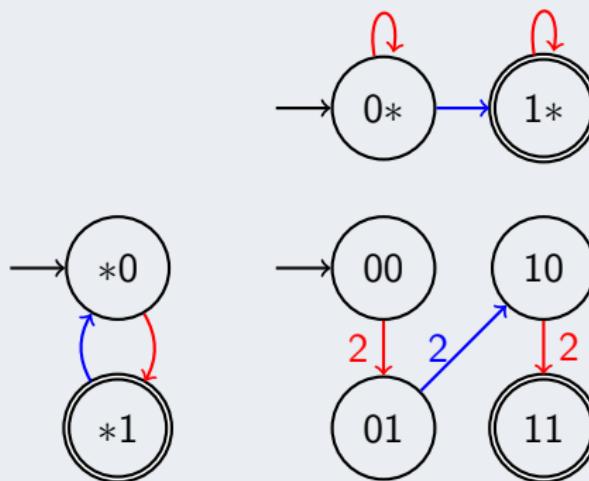
Cost Partitioning: Example

Example



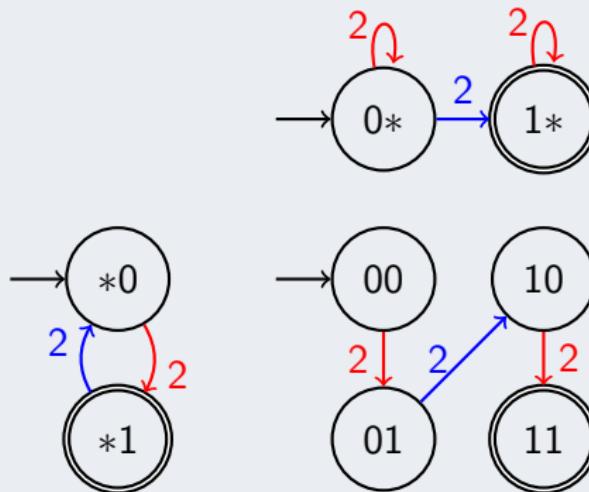
Cost Partitioning: Example

Example



Cost Partitioning: Example

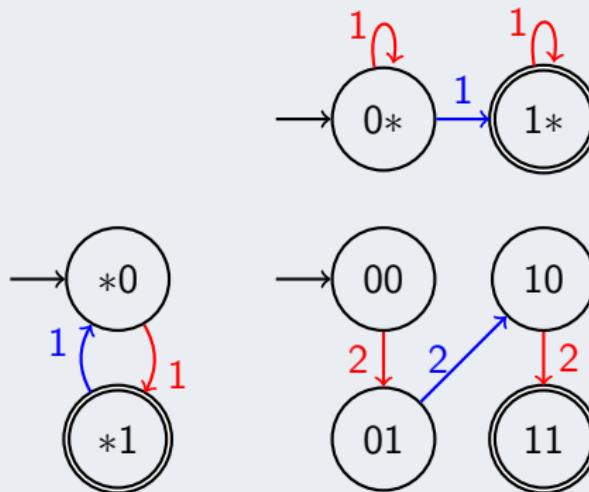
Example (No Cost Partitioning)



Heuristic value: $\max\{2, 2\} = 2$

Cost Partitioning: Example

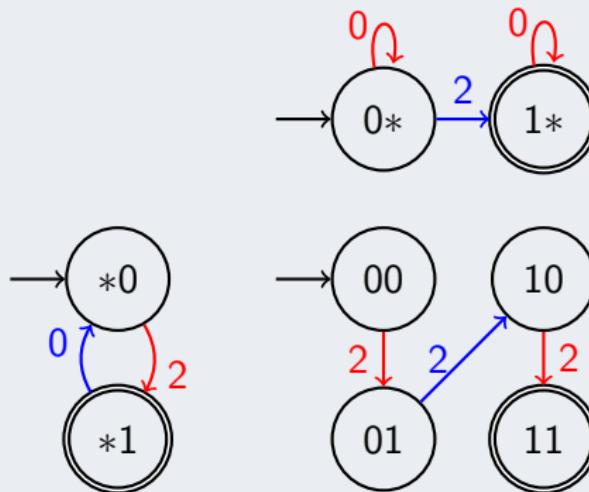
Example (Cost Partitioning 1)



Heuristic value: $1 + 1 = 2$

Cost Partitioning: Example

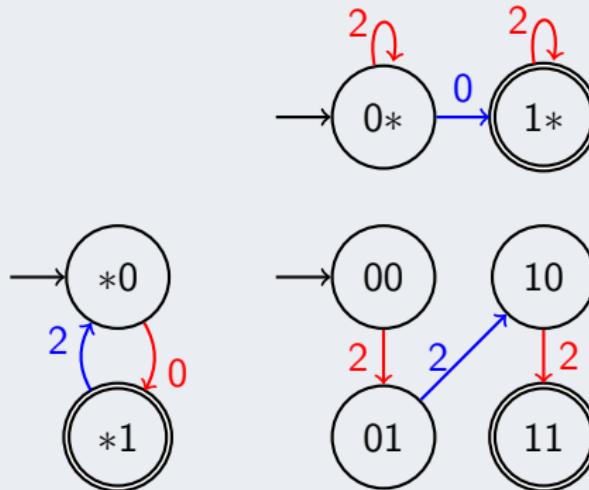
Example (Cost Partitioning 2)



Heuristic value: $2 + 2 = 4$

Cost Partitioning: Example

Example (Cost Partitioning 3)



Heuristic value: $0 + 0 = 0$

Cost Partitioning: Quality

- $h(s) = h_{1,\Pi_1}(s) + \cdots + h_{n,\Pi_n}(s)$
can be **better or worse** than any $h_{i,\Pi}(s)$
→ depending on cost partitioning
- strategies for defining cost-functions
 - uniform
 - zero-one
 - saturated (now)
 - optimal (next chapter)

Saturated Cost Partitioning

Idea

Heuristics do not always “need” all operator costs

- Pick a heuristic and use minimum costs **preserving all estimates**
- Continue with **remaining cost** until all heuristics were picked

Saturated cost partitioning (SCP) currently offers the best tradeoff between computation time and heuristic guidance in practice.

Saturated Cost Function

Definition (Saturated Cost Function)

Let Π be a planning task and h be a heuristic.

A cost function scf is **saturated** for h and $cost$ if

- ① $scf(o) \leq cost(o)$ for all operators o and
- ② $h_{\Pi_{scf}}(s) = h_{\Pi}(s)$ for all states s ,
where Π_{scf} is Π with cost function scf.

Minimal Saturated Cost Function

For **abstractions**, there exists a unique
minimal saturated cost function (MSCF).

Definition (MSCF for Abstractions)

Let Π be a planning task and α be an abstraction for Π .
The **minimal saturated cost function** for α is

$$\text{mscf}(o) = \max_{\alpha(s) \xrightarrow{o} \alpha(t)} \max\{h^\alpha(s) - h^\alpha(t), 0\}$$

Algorithm

Saturated Cost Partitioning: Seipp & Helmert (2014)

Iterate:

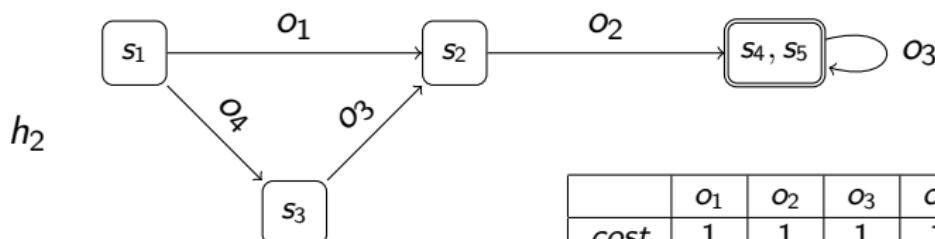
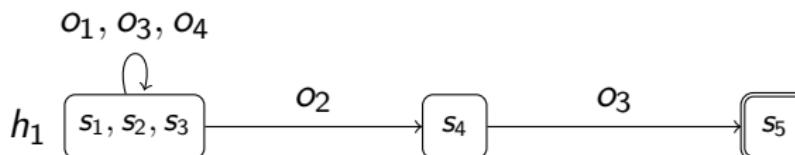
- ① Pick a heuristic h_i that hasn't been picked before.
Terminate if none is left.
- ② Compute h_i given current *cost*
- ③ Compute minimal saturated cost function mscf_i for h_i
- ④ Decrease $\text{cost}(o)$ by $\text{mscf}_i(o)$ for all operators o

$\langle \text{mscf}_1, \dots, \text{mscf}_n \rangle$ is **saturated cost partitioning (SCP)**

for $\langle h_1, \dots, h_n \rangle$ (in pick order)

Example

Consider the abstraction heuristics h_1 and h_2

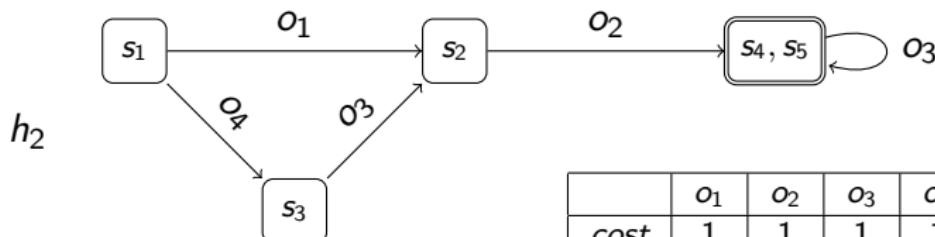
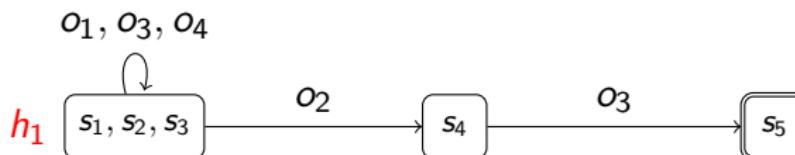


	o_1	o_2	o_3	o_4
<i>cost</i>	1	1	1	1

Example

Consider the abstraction heuristics h_1 and h_2

- 1 Pick a heuristic h_i

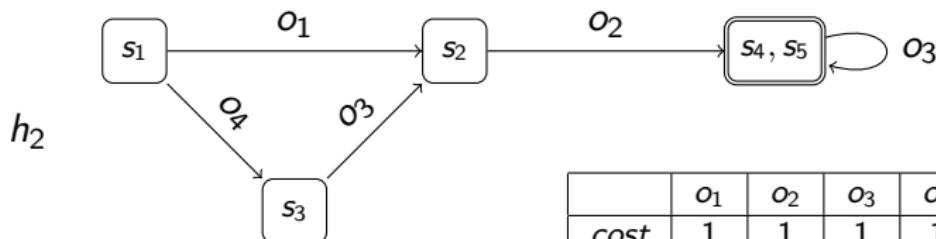
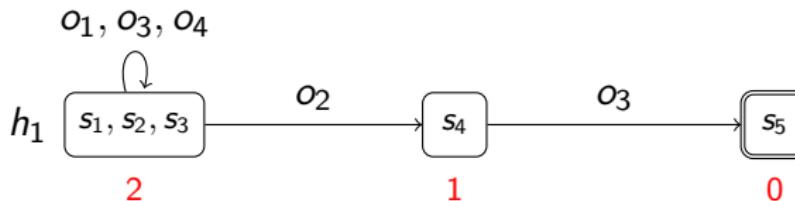


	o_1	o_2	o_3	o_4
$cost$	1	1	1	1

Example

Consider the abstraction heuristics h_1 and h_2

② Compute h_i

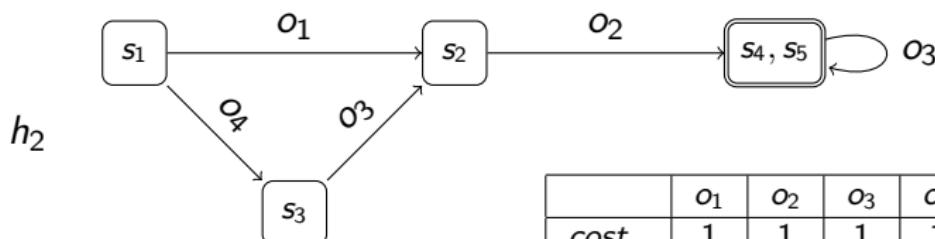
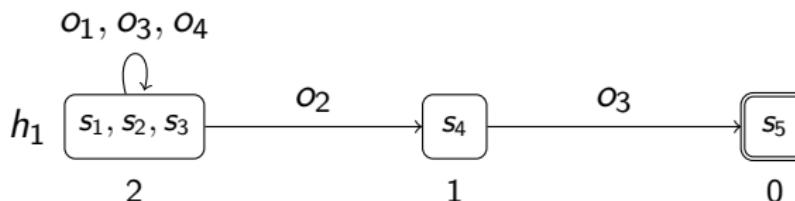


	o_1	o_2	o_3	o_4
<i>cost</i>	1	1	1	1

Example

Consider the abstraction heuristics h_1 and h_2

③ Compute minimal saturated cost function $mscf_i$ for h_i

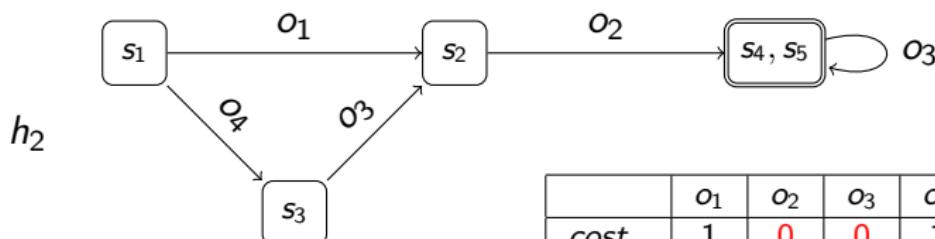
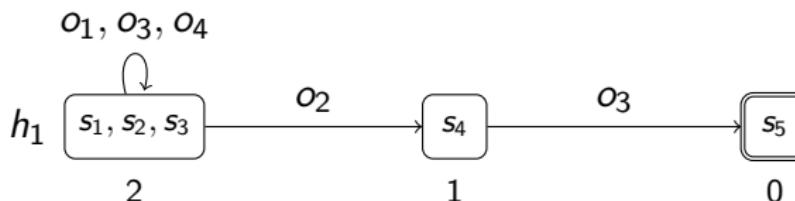


	o_1	o_2	o_3	o_4
$cost$	1	1	1	1
$mscf_1$	0	1	1	0

Example

Consider the abstraction heuristics h_1 and h_2

④ Decrease $cost(o)$ by $mscf_i(o)$ for all operators o

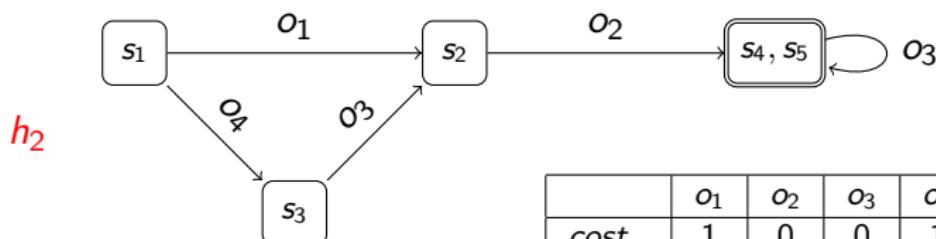
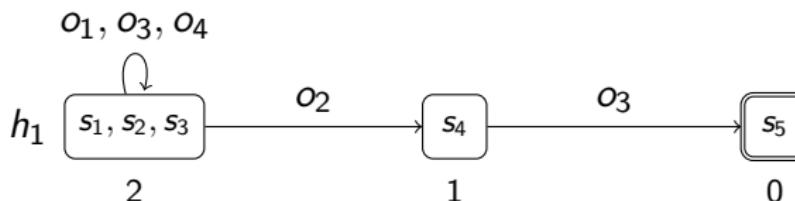


	o_1	o_2	o_3	o_4
$cost$	1	0	0	1
$mscf_1$	0	1	1	0

Example

Consider the abstraction heuristics h_1 and h_2

- 1 Pick a heuristic h_i

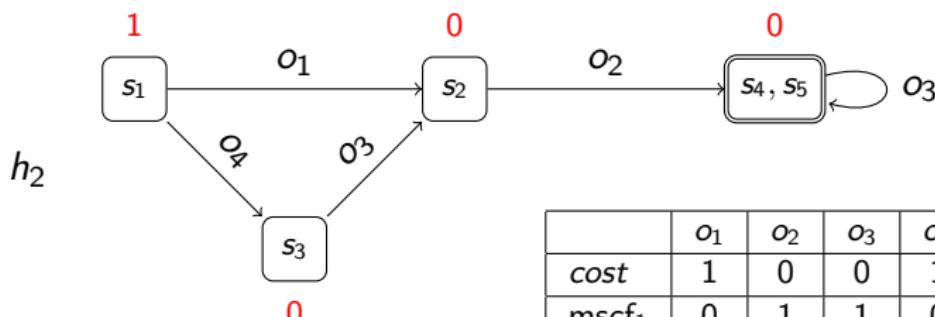
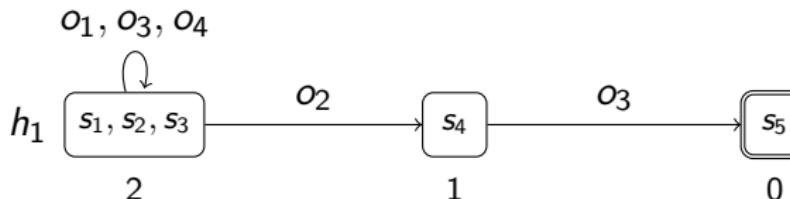


	o_1	o_2	o_3	o_4
<i>cost</i>	1	0	0	1
<i>mscf₁</i>	0	1	1	0

Example

Consider the abstraction heuristics h_1 and h_2

② Compute h_i

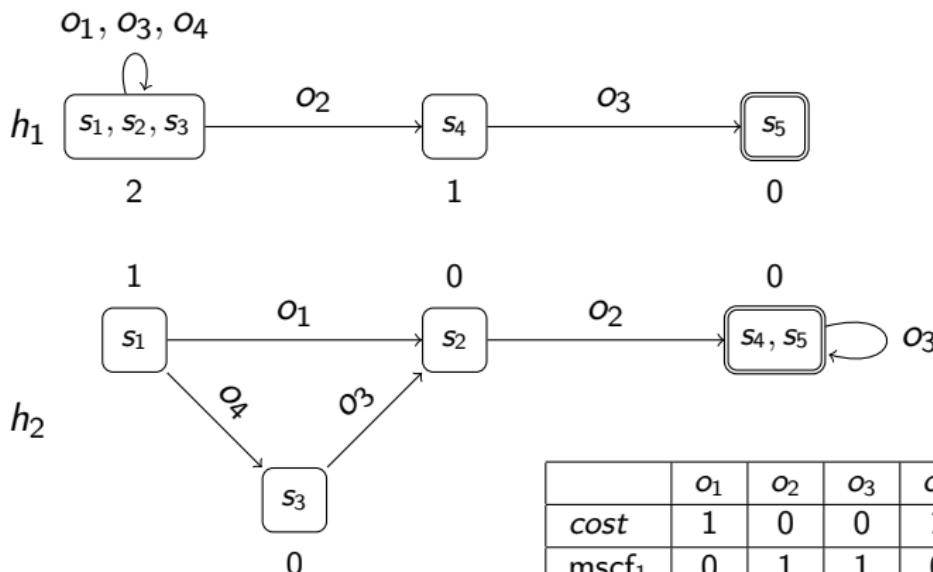


	o_1	o_2	o_3	o_4
$cost$	1	0	0	1
$mscf_1$	0	1	1	0

Example

Consider the abstraction heuristics h_1 and h_2

③ Compute minimal saturated cost function $mscf_i$ for h_i

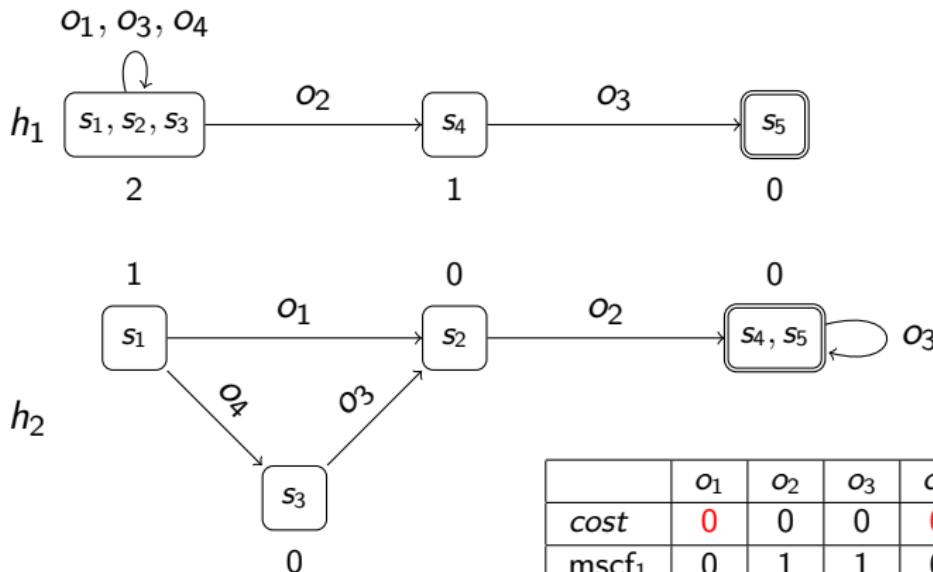


	o_1	o_2	o_3	o_4
<i>cost</i>	1	0	0	1
$mscf_1$	0	1	1	0
$mscf_2$	1	0	0	1

Example

Consider the abstraction heuristics h_1 and h_2

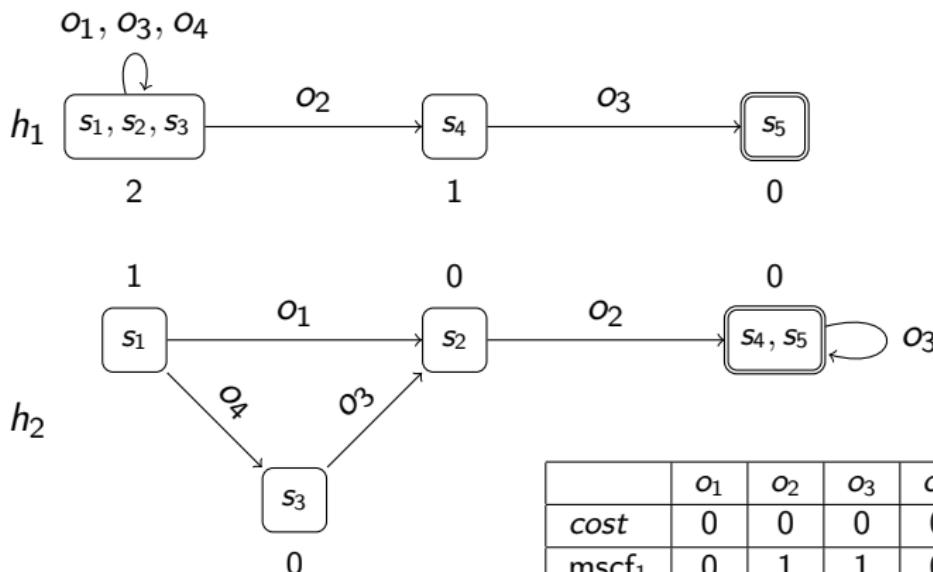
④ Decrease $cost(o)$ by $mscf_i(o)$ for all operators o



Example

Consider the abstraction heuristics h_1 and h_2

- 1 Pick a heuristic h_i . **Terminate if none is left.**



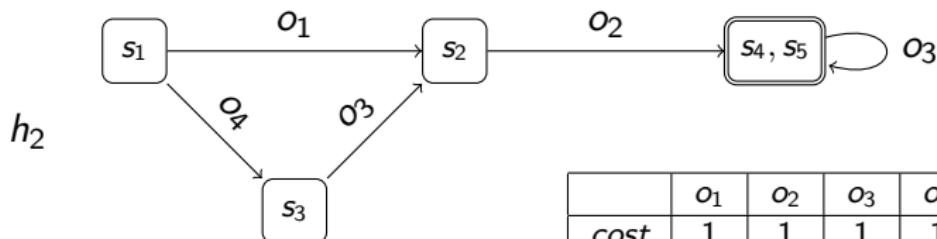
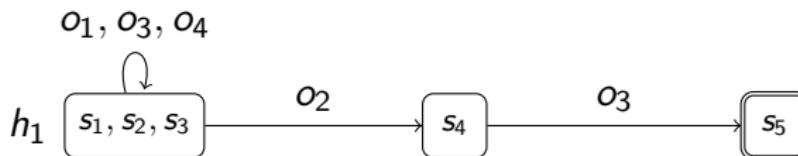
	o_1	o_2	o_3	o_4
<i>cost</i>	0	0	0	0
<i>mscf</i> ₁	0	1	1	0
<i>mscf</i> ₂	1	0	0	1

Influence of Selected Order

- quality **highly susceptible to selected order**
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- but there are also often orders where SCP performs worse

Saturated Cost Partitioning: Order

Consider the abstraction heuristics h_1 and h_2

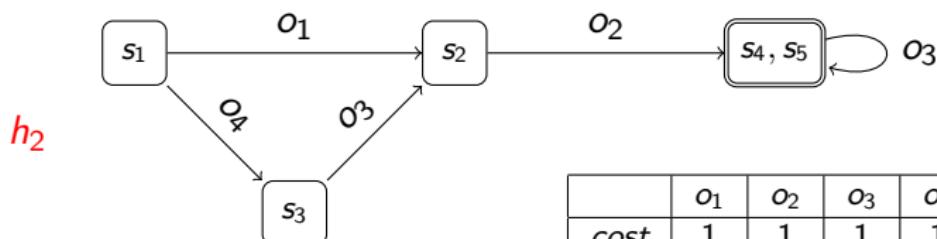
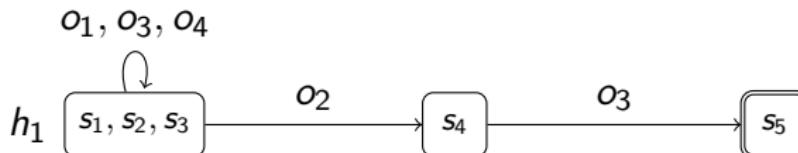


	o_1	o_2	o_3	o_4
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Saturated Cost Partitioning: Order

Consider the abstraction heuristics h_1 and h_2

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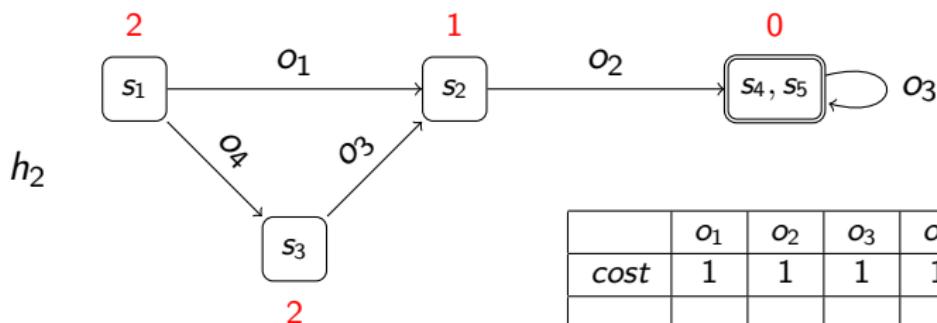


	o_1	o_2	o_3	o_4
$cost$	1	1	1	1

Saturated Cost Partitioning: Order

Consider the abstraction heuristics h_1 and h_2

② Compute h_i

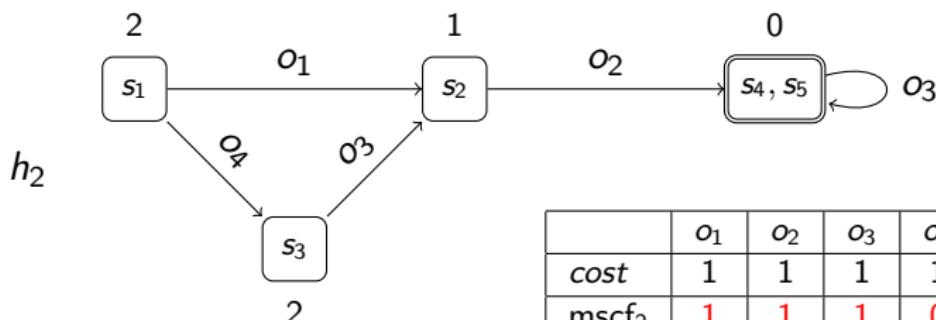
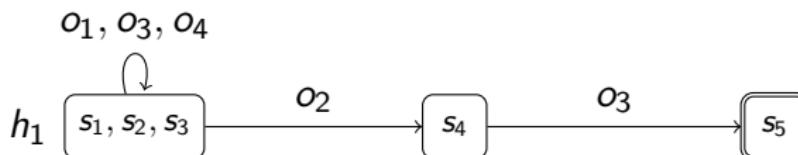


	o_1	o_2	o_3	o_4
$cost$	1	1	1	1

Saturated Cost Partitioning: Order

Consider the abstraction heuristics h_1 and h_2

③ Compute minimal saturated cost function $mscf_i$ for h_i

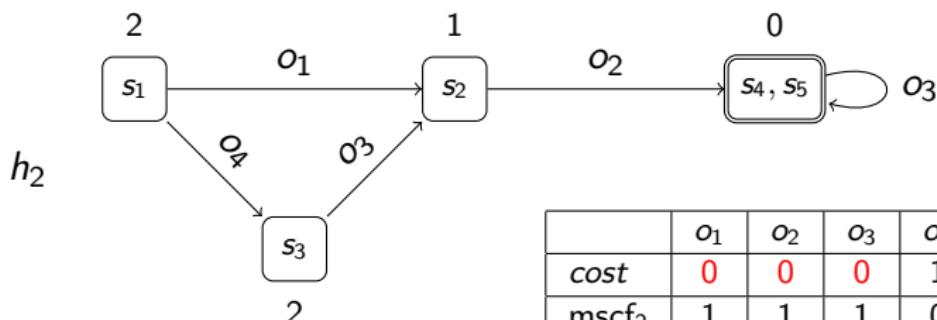
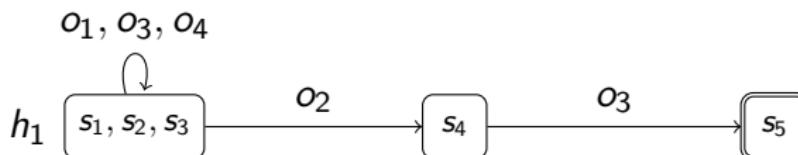


	o_1	o_2	o_3	o_4
$cost$	1	1	1	1
$mscf_2$	1	1	1	0

Saturated Cost Partitioning: Order

Consider the abstraction heuristics h_1 and h_2

- ④ Decrease $cost(o)$ by $mscf_i(o)$ for all operators o

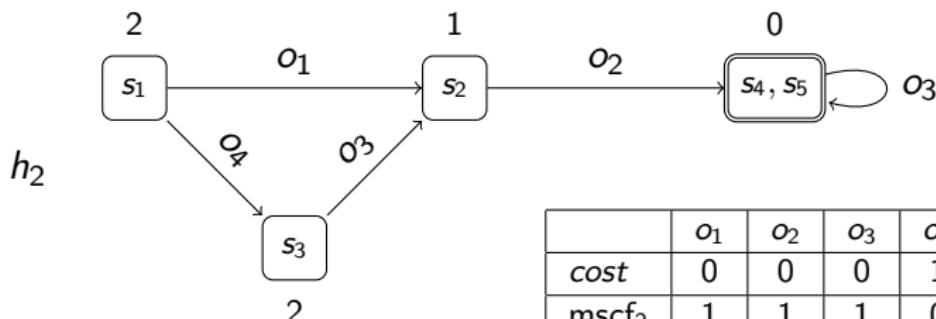
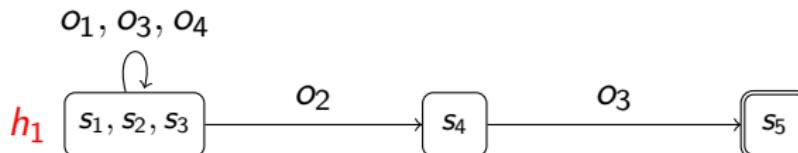


	o_1	o_2	o_3	o_4
$cost$	0	0	0	1
$mscf_2$	1	1	1	0

Saturated Cost Partitioning: Order

Consider the abstraction heuristics h_1 and h_2

- 1 Pick a heuristic h_i

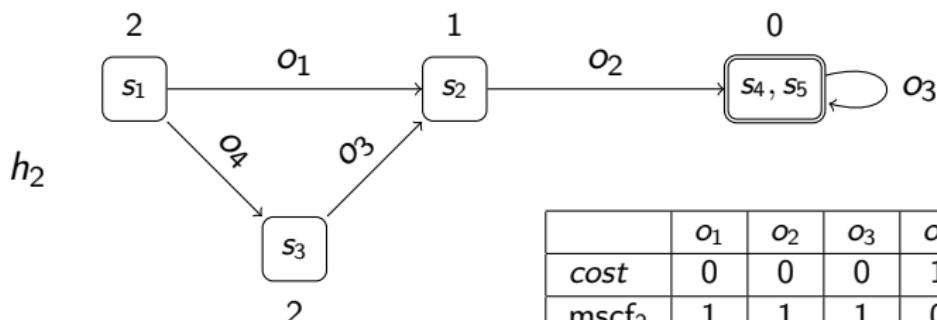
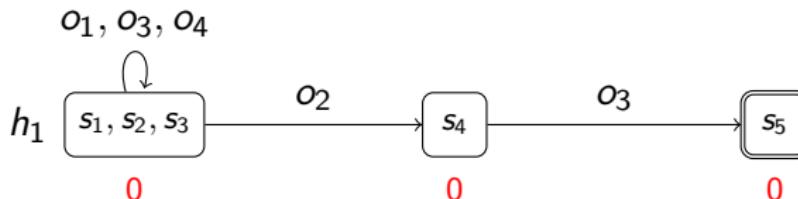


	o_1	o_2	o_3	o_4
$cost$	0	0	0	1
$mscf_2$	1	1	1	0

Saturated Cost Partitioning: Order

Consider the abstraction heuristics h_1 and h_2

② Compute h_i

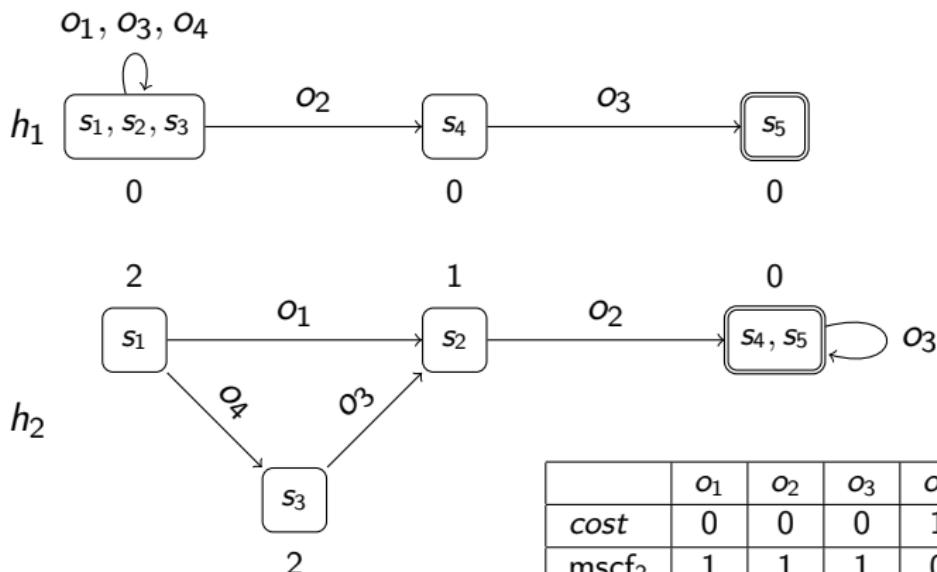


	o_1	o_2	o_3	o_4
<i>cost</i>	0	0	0	1
<i>mscf₂</i>	1	1	1	0

Saturated Cost Partitioning: Order

Consider the abstraction heuristics h_1 and h_2

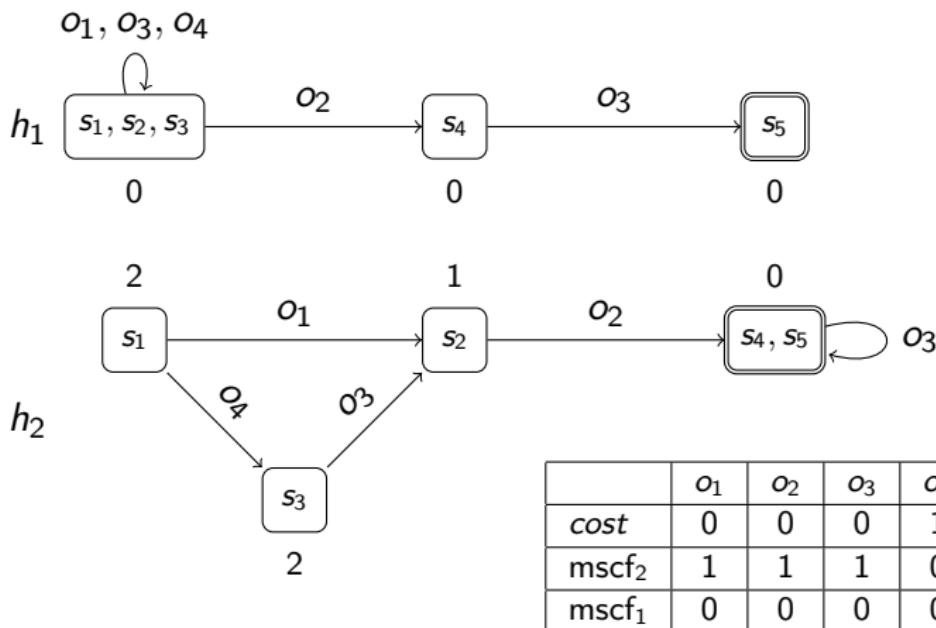
③ Compute minimal saturated cost function mscf_i for h_i



Saturated Cost Partitioning: Order

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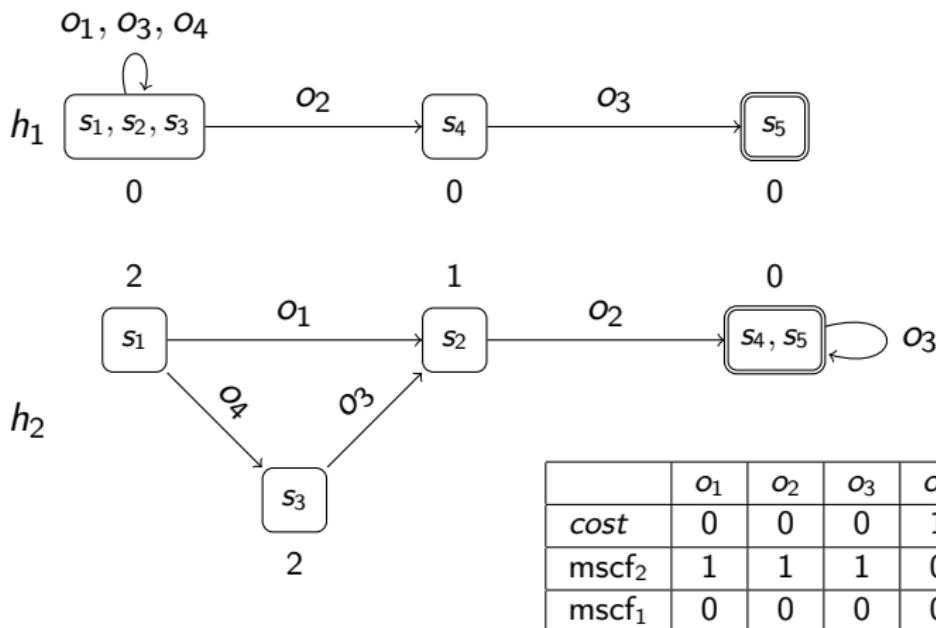
- ④ Decrease $cost(o)$ by $mscf_i(o)$ for all operators o



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Maximizing over multiple orders good solution in practice

SCP for Disjunctive Action Landmarks

Same algorithm can be used for disjunctive action landmarks, where we also have a **minimal saturated cost function**.

Definition (MSCF for Disjunctive Action Landmark)

Let Π be a planning task and \mathcal{L} be a disjunctive action landmark. The **minimal saturated cost function** for \mathcal{L} is

$$\text{mscf}(o) = \begin{cases} \min_{o \in \mathcal{L}} \text{cost}(o) & \text{if } o \in \mathcal{L} \\ 0 & \text{otherwise} \end{cases}$$

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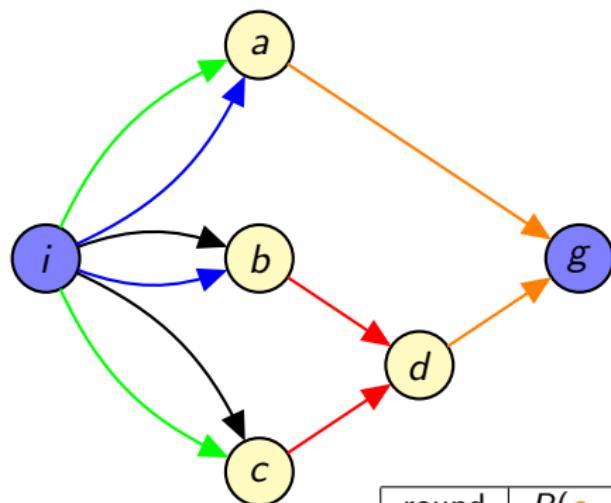
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Does this look familiar?

Reminder: LM-Cut



$$o_{blue} = \langle \{i\}, \{a, b\}, \{\}, 4 \rangle$$

$$o_{green} = \langle \{i\}, \{a, c\}, \{\}, 5 \rangle$$

$$o_{black} = \langle \{i\}, \{b, c\}, \{\}, 3 \rangle$$

$$o_{red} = \langle \{b, c\}, \{d\}, \{\}, 2 \rangle$$

$$o_{orange} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$$

round	$P(o_{orange})$	$P(o_{red})$	landmark	cost
1	d	b	{ o_{red} }	2
2	a	b	{ o_{green} , o_{blue} }	4
3	d	c	{ o_{green} , o_{black} }	1
$h^{\text{LM-cut}}(I)$				7

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LM-Cut computes SCP over disjunctive action landmarks

Introduction
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Cost Partitioning
oooooooo

Saturated Cost Partitioning
oooooooooooo

Summary
●○

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Summary

- **Cost partitioning** allows to admissibly add up estimates of several heuristics.
- This can be better or worse than the best individual heuristic on the original problem, depending on the cost partitioning.
- **Saturated cost partitioning** offers good tradeoff between computation time and heuristic guidance
- LM-Cut computes SCP over disjunctive action landmarks