Planning and Optimization

E3. Landmarks: LM-Cut Heuristic

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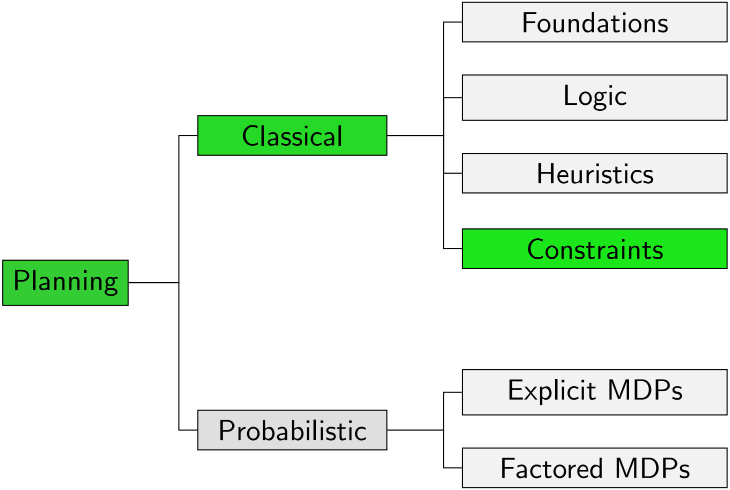
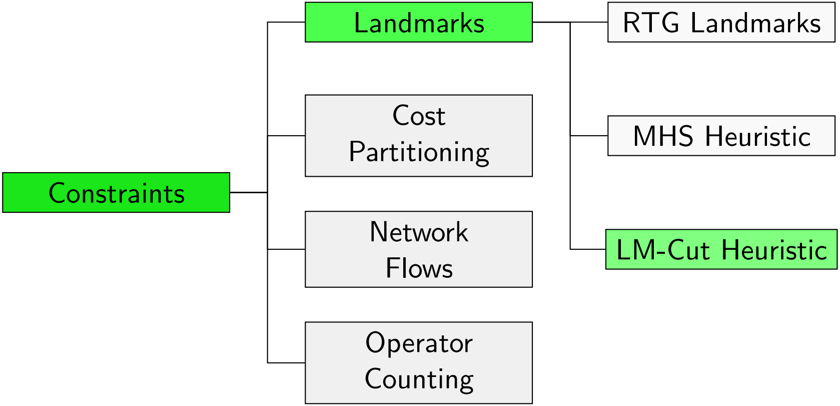
Universität Basel

Content of this Course



Content of this Course: Constraints



Roadmap for this Chapter

- We first introduce a new normal form for delete-free STRIPS tasks that simplifies later definitions.
- We then present a method that computes disjunctive action landmarks for such tasks.
- We conclude with the LM-cut heuristic that builds on this method.

i-g Form

Delete-Free STRIPS Planning Task in i-g Form (1)

In this chapter, we only consider delete-free STRIPS tasks in a special form:

Definition (i-g Form for Delete-free STRIPS)

A delete-free STRIPS planning task $\langle V, I, O, \gamma \rangle$ is in i-g form if

- V contains atoms i and g
- Initially exactly i is true: $I(v) = \mathrm{T}$ iff $v = i$
- g is the only goal atom: $\gamma = \{g\}$
- Every action has at least one precondition.

Transformation to i-g Form

Every delete-free STRIPS task $\Pi = \langle V, I, O, \gamma \rangle$ can easily be transformed into an analogous task in i-g form.

- If i or g are in V already, rename them everywhere.
- Add i and g to V.
- Add an operator $\langle \{i\}, \{v \in V \mid I(v) = \mathrm{T}\}, \{\}, 0 \rangle$.
- Add an operator $\langle \gamma, \{g\}, \{\}, 0 \rangle$.
- Replace all operator preconditions $\top$ with i.
- Replace initial state and goal.

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- Add an operator $\langle \gamma, \{g\}, \{\}, 0 \rangle$.
- Replace all operator preconditions $\top$ with i.
- Replace initial state and goal.

For the remainder of this chapter, we assume tasks in i-g form.

Example: Delete-Free Planning Task in i-g Form

Example

Consider a delete-free STRIPS planning task $\langle V, I, O, \gamma \rangle$ with $V = \{i, a, b, c, d, g\}$, $I = \{i \mapsto \mathrm{T}\} \cup \{v \mapsto \mathrm{F} \mid v \in V \setminus \{i\}\}$, $\gamma = \{g\}$ and operators

- $o_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 4 \rangle$,
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optimal solution?

Example: Delete-Free Planning Task in i-g Form

Example

Consider a delete-free STRIPS planning task $\langle V, I, O, \gamma \rangle$ with $V = \{i, a, b, c, d, g\}$, $I = \{i \mapsto \mathrm{T}\} \cup \{v \mapsto \mathrm{F} \mid v \in V \setminus \{i\}\}$, $\gamma = \{g\}$ and operators

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optimal solution to reach g from i:

- plan: $\langle o_{\mathrm{blue}}, o_{\mathrm{black}}, o_{\mathrm{red}}, o_{\mathrm{orange}} \rangle$
- cost: $4 + 3 + 2 + 0 = 9$ $\quad (= h^{+}(I)$ because plan is optimal)

Cut Landmarks

Justification Graphs

Definition (Precondition Choice Function)

A precondition choice function (pcf) $P : O \to V$ for a delete-free STRIPS task $\Pi = \langle V, I, O, \gamma \rangle$ in i-g form maps each operator to one of its preconditions (i.e. $P(o) \in pre(o)$ for all $o \in O$).

Definition (Justification Graphs)

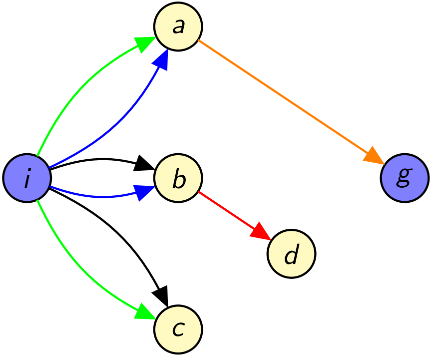
Let P be a pcf for $\langle V, I, O, \gamma \rangle$ in i-g form. The justification graph for P is the directed, edge-labeled graph $J = \langle V, E \rangle$, where

- the vertices are the variables from V, and
- E contains an edge $P(o) \xrightarrow{o} a$ for each $o \in O$, $a \in add(o)$.

Example: Justification Graph

Example (Precondition Choice Function)

$P(o_{\text{blue}}) = P(o_{\text{green}}) = P(o_{\text{black}}) = i, \ P(o_{\text{red}}) = b, \ P(o_{\text{orange}}) = a$



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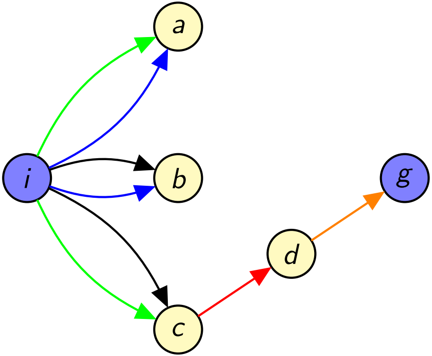
Example: Justification Graph

Example (Precondition Choice Function)

$P(o_{\text{blue}}) = P(o_{\text{green}}) = P(o_{\text{black}}) = i,\ P(o_{\text{red}}) = b,\ P(o_{\text{orange}}) = a$

$P'(o_{\text{blue}}) = P'(o_{\text{green}}) = P'(o_{\text{black}}) = i,\ P'(o_{\text{red}}) = c,\ P'(o_{\text{orange}}) = d$



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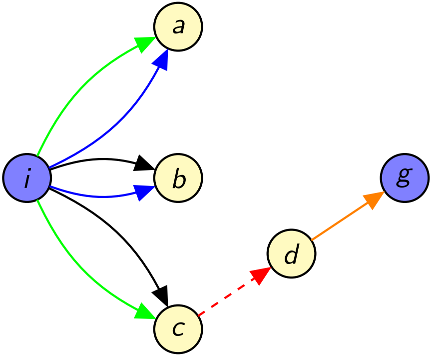
Cuts

Definition (Cut)

A cut in a justification graph is a subset C of its edges such that all paths from i to g contain an edge from C.



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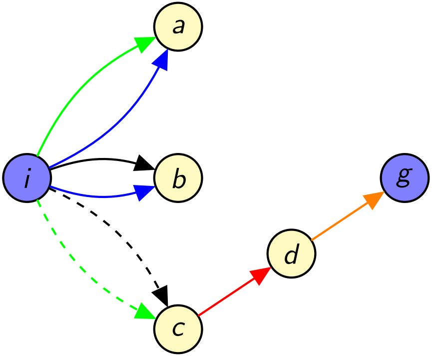

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Cuts are Disjunctive Action Landmarks

Theorem (Cuts are Disjunctive Action Landmarks)

Let P be a pcf for $\langle V, I, O, \gamma\rangle$ (in i-g form) and C be a cut in the justification graph for P.

The set of edge labels from C (formally $\{o \mid \langle v, o, v'\rangle \in C\}$) is a disjunctive action landmark for I.

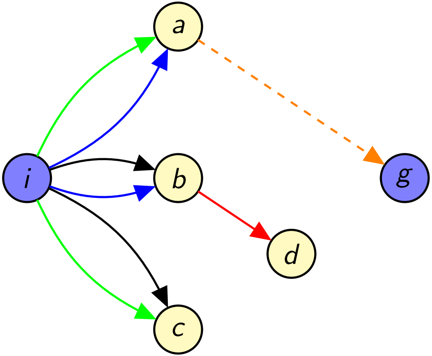
Proof idea:

- The justification graph corresponds to a simpler problem where some preconditions (those not picked by the pcf) are ignored.
- Cuts are landmarks for this simplified problem.
- Hence they are also landmarks for the original problem.

Example: Cuts in Justification Graphs

Example (Landmarks)

- $L_1 = \{o_{\text{orange}}\}$ (cost $= 0$)



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\end{aligned}
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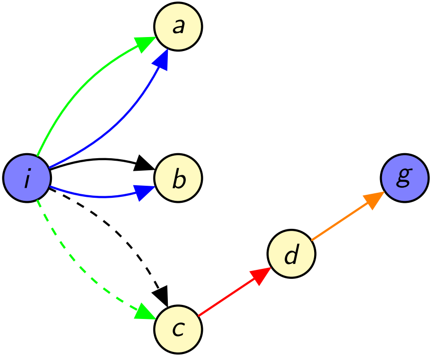
Example: Cuts in Justification Graphs

Example (Landmarks)

- $L_1 = \{o_{\text{orange}}\}$ (cost = 0)
- $L_2 = \{o_{\text{green}}, o_{\text{black}}\}$ (cost = 3)



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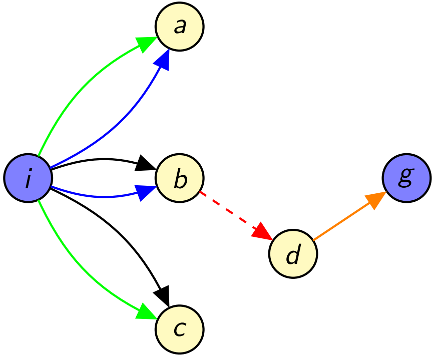
Example: Cuts in Justification Graphs

Example (Landmarks)

- $L_1 = \{o_{\text{orange}}\}$ (cost $= 0$)
- $L_2 = \{o_{\text{green}}, o_{\text{black}}\}$ (cost $= 3$)
- $L_3 = \{o_{\text{red}}\}$ (cost $= 2$)



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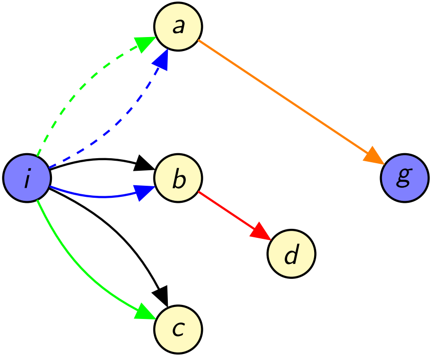
Example: Cuts in Justification Graphs

Example (Landmarks)

- $L_1 = \{o_{\text{orange}}\}$ (cost $= 0$)
- $L_2 = \{o_{\text{green}}, o_{\text{black}}\}$ (cost $= 3$)
- $L_3 = \{o_{\text{red}}\}$ (cost $= 2$)
- $L_4 = \{o_{\text{green}}, o_{\text{blue}}\}$ (cost $= 4$)



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Power of Cuts in Justification Graphs

- Which landmarks can be computed with the cut method?

Power of Cuts in Justification Graphs

- Which landmarks can be computed with the cut method?
- all interesting ones!

Proposition (perfect hitting set heuristics)

Let $\mathcal{L}$ be the set of all "cut landmarks" of a given planning task with initial state I. Then $h^{MHS}(\mathcal{L}) = h^{+}(I)$.

$\rightsquigarrow$ Hitting set heuristic for $\mathcal{L}$ is perfect.

Power of Cuts in Justification Graphs

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$\rightsquigarrow$ Hitting set heuristic for $\mathcal{L}$ is perfect.

Proof idea:

- Show 1:1 correspondence of hitting sets H for $\mathcal{L}$ and plans, i.e., each hitting set for $\mathcal{L}$ corresponds to a plan, and vice versa.

The LM-Cut Heuristic

LM-Cut Heuristic: Motivation

- In general, there are exponentially many pcfs, hence computing all relevant landmarks is not tractable.
- The LM-cut heuristic is a method that chooses pcfs and computes cuts in a goal-oriented way.
- As a side effect, it computes
  - a cost partitioning over multiple instances of $h^{\max}$ that is also
  - a saturated cost partitioning over disjunctive action landmarks.

⇝ currently one of the best admissible planning heuristic

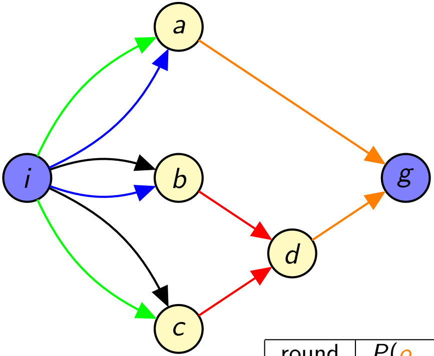
LM-Cut Heuristic

$h^{\text{LM-cut}}$: Helmert & Domshlak (2009)

Initialize $h^{\text{LM-cut}}(I) := 0$. Then iterate:

1. Compute $h^{\max}$ values of the variables. Stop if $h^{\max}(g) = 0$.
2. Compute justification graph G for the P that chooses preconditions with maximal $h^{\max}$ value
3. Determine the goal zone V_g of G that consists of all nodes that have a zero-cost path to g.
4. Compute the cut L that contains the labels of all edges $\langle v, o, v' \rangle$ such that $v \notin V_g$, $v' \in V_g$ and v can be reached from i without traversing a node in V_g.
   It is guaranteed that $cost(L) > 0$.
5. Increase $h^{\text{LM-cut}}(I)$ by $cost(L)$.
6. Decrease $cost(o)$ by $cost(L)$ for all $o \in L$.

Example: Computation of LM-Cut



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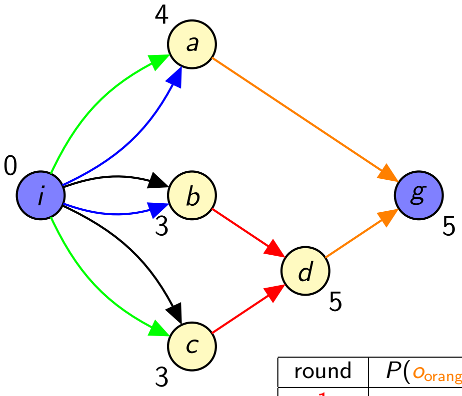
round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
			$h^{\text{LM-cut}}(I)$	0

Example: Computation of LM-Cut

1. Compute $h^{\max}$ values of the variables



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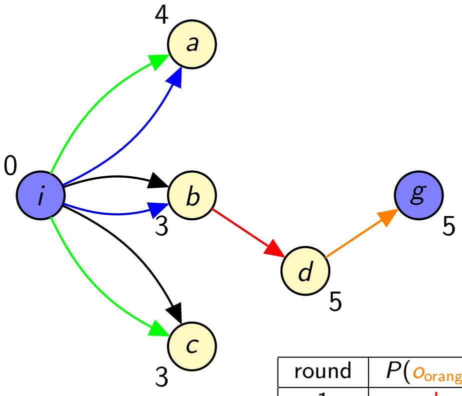
round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1				
			$h^{\text{LM-cut}}(I)$	0

Example: Computation of LM-Cut

2. Compute justification graph



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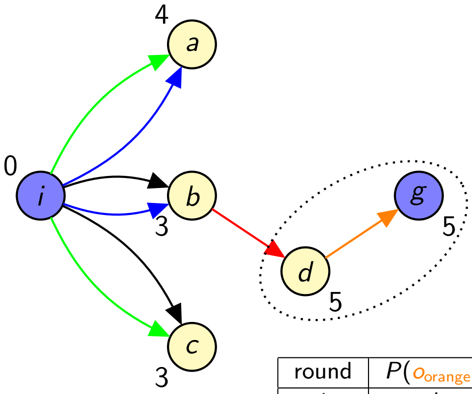
round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b		
			$h^{\text{LM-cut}}(I)$	0

Example: Computation of LM-Cut

3. Determine goal zone



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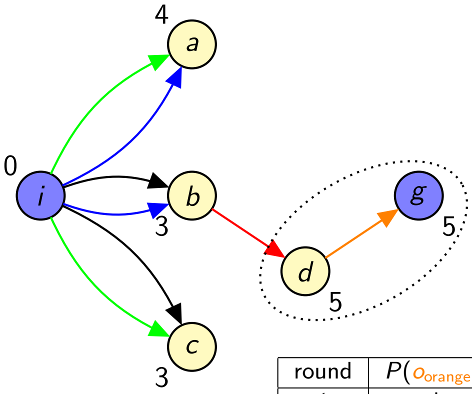
round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b		
			$h^{\text{LM-cut}}(I)$	0

Example: Computation of LM-Cut

4. Compute cut



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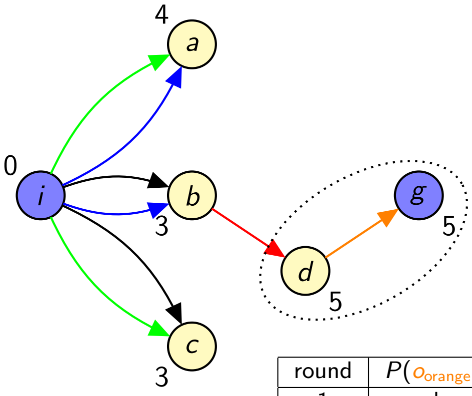
$o_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
			$h^{\text{LM-cut}}(I)$	0

Example: Computation of LM-Cut

5. Increase $h^{\text{LM-cut}}(I)$ by $cost(L)$



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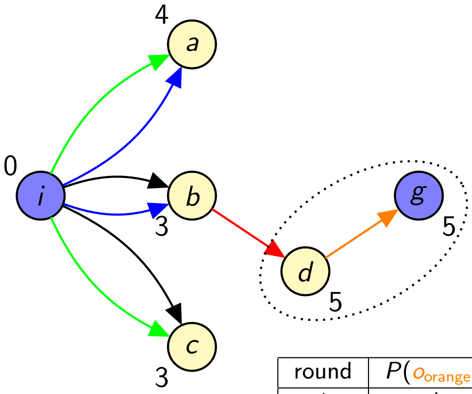
round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
			$h^{\text{LM-cut}}(I)$	2

Example: Computation of LM-Cut

6. Decrease $cost(o)$ by $cost(L)$ for all $o \in L$



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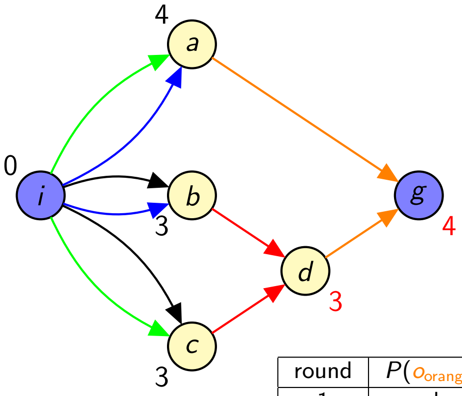
round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
			$h^{\text{LM-cut}}(I)$	2

Example: Computation of LM-Cut

1. Compute $h^{\max}$ values of the variables



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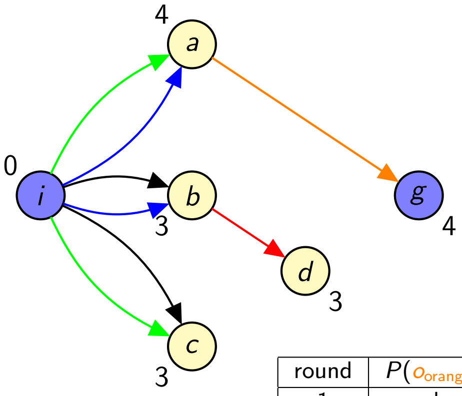
round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
2				
			$h^{\text{LM-cut}}(I)$	2

Example: Computation of LM-Cut

2. Compute justification graph



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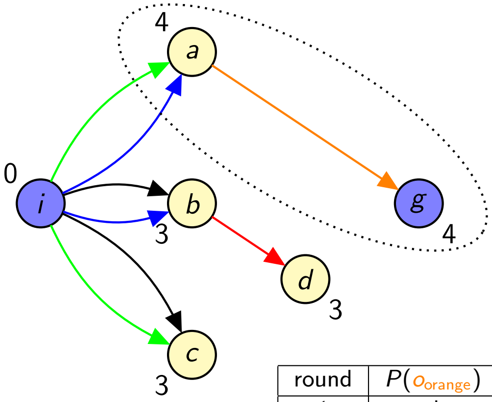
$o_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
2	a	b		
			$h^{\text{LM-cut}}(I)$	2

Example: Computation of LM-Cut

3. Determine goal zone



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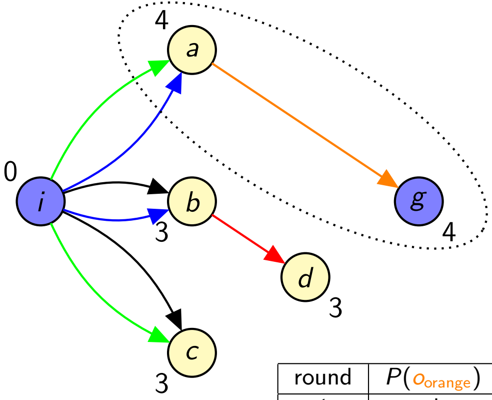
round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
2	a	b		
			$h^{\text{LM-cut}}(I)$	2

Example: Computation of LM-Cut

4. Compute cut

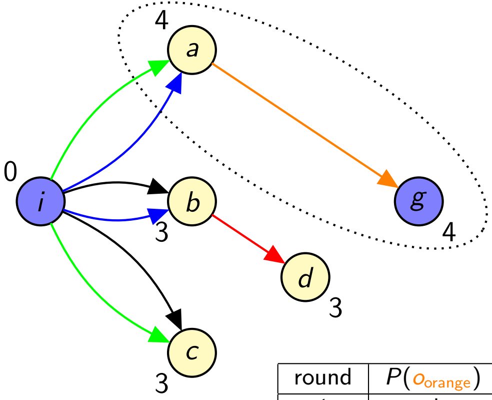


$o_{\text{blue}} = \langle\{i\}, \{a, b\}, \{\}, 4\rangle$
$o_{\text{green}} = \langle\{i\}, \{a, c\}, \{\}, 5\rangle$
$o_{\text{black}} = \langle\{i\}, \{b, c\}, \{\}, 3\rangle$
$o_{\text{red}} = \langle\{b, c\}, \{d\}, \{\}, 0\rangle$
$o_{\text{orange}} = \langle\{a, d\}, \{g\}, \{\}, 0\rangle$

round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
2	a	b	$\{o_{\text{green}}, o_{\text{blue}}\}$	4
			$h^{\text{LM-cut}}(I)$	2

Example: Computation of LM-Cut

5. Increase $h^{\text{LM-cut}}(I)$ by $cost(L)$

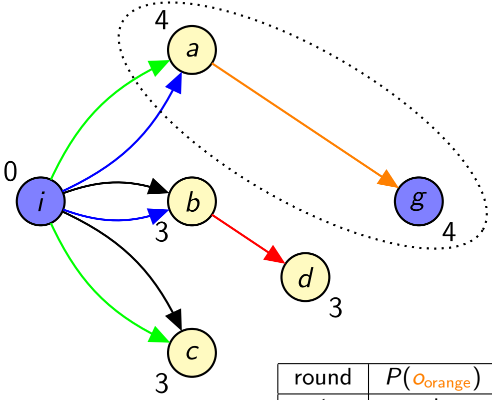


$o_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 4 \rangle$
$o_{\text{green}} = \langle \{i\}, \{a, c\}, \{\}, 5 \rangle$
$o_{\text{black}} = \langle \{i\}, \{b, c\}, \{\}, 3 \rangle$
$o_{\text{red}} = \langle \{b, c\}, \{d\}, \{\}, 0 \rangle$
$o_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
2	a	b	$\{o_{\text{green}}, o_{\text{blue}}\}$	4
			$h^{\text{LM-cut}}(I)$	6

Example: Computation of LM-Cut

6. Decrease $cost(o)$ by $cost(L)$ for all $o \in L$

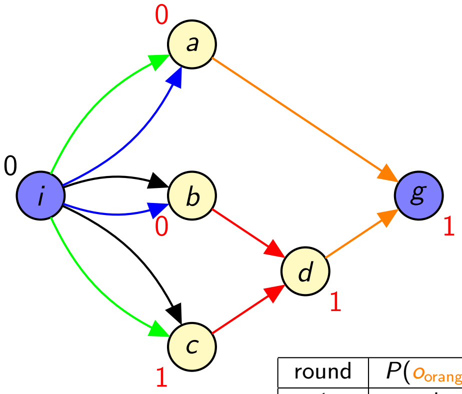


$o_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 0 \rangle$
$o_{\text{green}} = \langle \{i\}, \{a, c\}, \{\}, 1 \rangle$
$o_{\text{black}} = \langle \{i\}, \{b, c\}, \{\}, 3 \rangle$
$o_{\text{red}} = \langle \{b, c\}, \{d\}, \{\}, 0 \rangle$
$o_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
2	a	b	$\{o_{\text{green}}, o_{\text{blue}}\}$	4
			$h^{\text{LM-cut}}(I)$	6

Example: Computation of LM-Cut

1. Compute $h^{\max}$ values of the variables

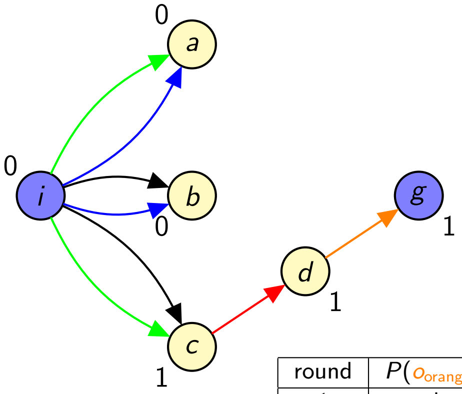


$o_{\text{blue}} = \langle\{i\}, \{a, b\}, \{\}, 0\rangle$
$o_{\text{green}} = \langle\{i\}, \{a, c\}, \{\}, 1\rangle$
$o_{\text{black}} = \langle\{i\}, \{b, c\}, \{\}, 3\rangle$
$o_{\text{red}} = \langle\{b, c\}, \{d\}, \{\}, 0\rangle$
$o_{\text{orange}} = \langle\{a, d\}, \{g\}, \{\}, 0\rangle$

round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
2	a	b	$\{o_{\text{green}}, o_{\text{blue}}\}$	4
3				
			$h^{\text{LM-cut}}(I)$	6

Example: Computation of LM-Cut

2. Compute justification graph

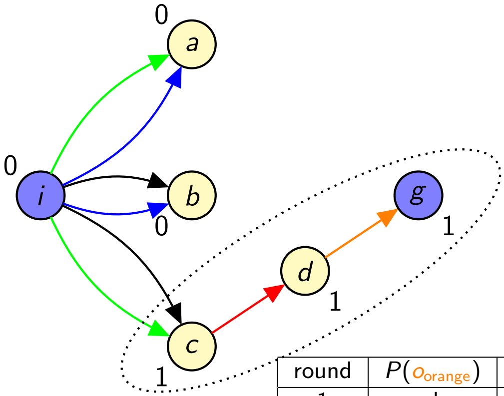


$$
\begin{aligned}
o_{\text{blue}} &= \langle \{i\}, \{a, b\}, \{\}, 0 \rangle \\
o_{\text{green}} &= \langle \{i\}, \{a, c\}, \{\}, 1 \rangle \\
o_{\text{black}} &= \langle \{i\}, \{b, c\}, \{\}, 3 \rangle \\
o_{\text{red}} &= \langle \{b, c\}, \{d\}, \{\}, 0 \rangle \\
o_{\text{orange}} &= \langle \{a, d\}, \{g\}, \{\}, 0 \rangle
\end{aligned}
$$

round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
2	a	b	$\{o_{\text{green}}, o_{\text{blue}}\}$	4
3	d	c		
			$h^{\text{LM-cut}}(I)$	6

Example: Computation of LM-Cut

3. Determine goal zone

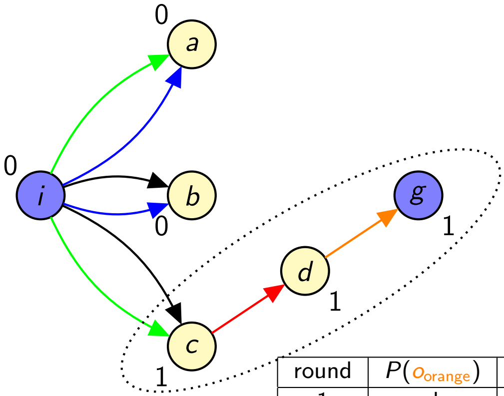


$o_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 0 \rangle$
$o_{\text{green}} = \langle \{i\}, \{a, c\}, \{\}, 1 \rangle$
$o_{\text{black}} = \langle \{i\}, \{b, c\}, \{\}, 3 \rangle$
$o_{\text{red}} = \langle \{b, c\}, \{d\}, \{\}, 0 \rangle$
$o_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
2	a	b	$\{o_{\text{green}}, o_{\text{blue}}\}$	4
3	d	c		
			$h^{\text{LM-cut}}(I)$	6

Example: Computation of LM-Cut

4. Compute cut

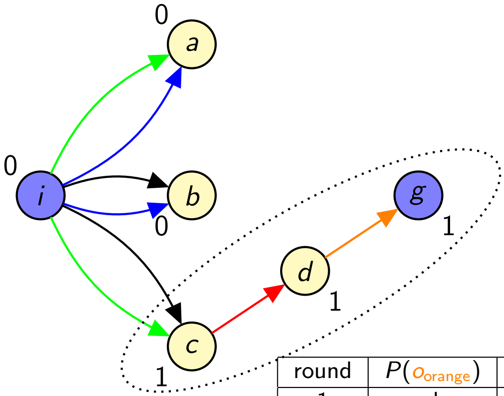


$o_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 0 \rangle$
$o_{\text{green}} = \langle \{i\}, \{a, c\}, \{\}, 1 \rangle$
$o_{\text{black}} = \langle \{i\}, \{b, c\}, \{\}, 3 \rangle$
$o_{\text{red}} = \langle \{b, c\}, \{d\}, \{\}, 0 \rangle$
$o_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
2	a	b	$\{o_{\text{green}}, o_{\text{blue}}\}$	4
3	d	c	$\{o_{\text{green}}, o_{\text{black}}\}$	1
			$h^{\text{LM-cut}}(I)$	6

Example: Computation of LM-Cut

5. Increase $h^{\text{LM-cut}}(I)$ by $cost(L)$

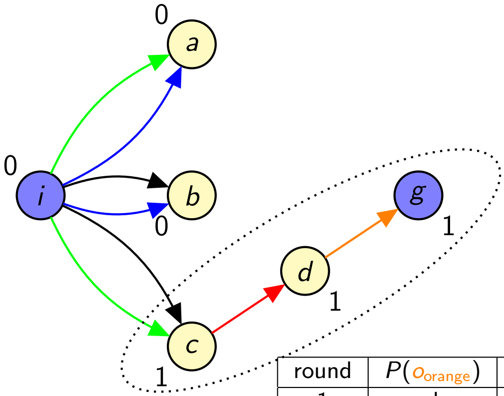


$o_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 0 \rangle$
$o_{\text{green}} = \langle \{i\}, \{a, c\}, \{\}, 1 \rangle$
$o_{\text{black}} = \langle \{i\}, \{b, c\}, \{\}, 3 \rangle$
$o_{\text{red}} = \langle \{b, c\}, \{d\}, \{\}, 0 \rangle$
$o_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
2	a	b	$\{o_{\text{green}}, o_{\text{blue}}\}$	4
3	d	c	$\{o_{\text{green}}, o_{\text{black}}\}$	1
			$h^{\text{LM-cut}}(I)$	7

Example: Computation of LM-Cut

6. Decrease $cost(o)$ by $cost(L)$ for all $o \in L$

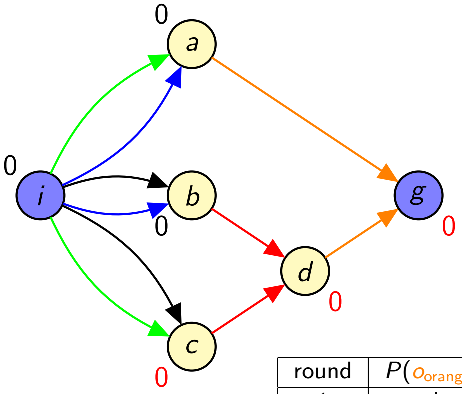


$o_{\text{blue}} = \langle\{i\}, \{a, b\}, \{\}, 0\rangle$
$o_{\text{green}} = \langle\{i\}, \{a, c\}, \{\}, 0\rangle$
$o_{\text{black}} = \langle\{i\}, \{b, c\}, \{\}, 2\rangle$
$o_{\text{red}} = \langle\{b, c\}, \{d\}, \{\}, 0\rangle$
$o_{\text{orange}} = \langle\{a, d\}, \{g\}, \{\}, 0\rangle$

round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
2	a	b	$\{o_{\text{green}}, o_{\text{blue}}\}$	4
3	d	c	$\{o_{\text{green}}, o_{\text{black}}\}$	1
			$h^{\text{LM-cut}}(I)$	7

Example: Computation of LM-Cut

1. Compute $h^{\max}$ values of the variables. Stop if $h^{\max}(g) = 0$.



$$o_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 0 \rangle$$
$$o_{\text{green}} = \langle \{i\}, \{a, c\}, \{\}, 0 \rangle$$
$$o_{\text{black}} = \langle \{i\}, \{b, c\}, \{\}, 2 \rangle$$
$$o_{\text{red}} = \langle \{b, c\}, \{d\}, \{\}, 0 \rangle$$
$$o_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$$

round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
2	a	b	$\{o_{\text{green}}, o_{\text{blue}}\}$	4
3	d	c	$\{o_{\text{green}}, o_{\text{black}}\}$	1
			$h^{\text{LM-cut}}(I)$	7

Properties of LM-Cut Heuristic

Theorem

Let $\langle V, I, O, \gamma \rangle$ be a delete-free STRIPS task in i-g normal form. The LM-cut heuristic is admissible: $h^{\text{LM-cut}}(I) \leq h^*(I)$.

Proof omitted.

If Π is not delete-free, we can compute $h^{\text{LM-cut}}$ on Π^+.
Then $h^{\text{LM-cut}}$ is bounded by h^+.

Summary & Outlook

Summary

- Cuts in justification graphs are a general method to find disjunctive action landmarks.
- The minimum hitting set over all cut landmarks is a perfect heuristic for delete-free planning tasks.
- The LM-cut heuristic is an admissible heuristic based on these ideas.

Literature (1)

References on landmark heuristics:

Julie Porteous, Laura Sebastia and Joerg Hoffmann.
On the Extraction, Ordering, and Usage of Landmarks in Planning.
Proc. ECP 2001, pp. 174–182, 2013.
Introduces landmarks.

Malte Helmert and Carmel Domshlak.
Landmarks, Critical Paths and Abstractions: What's the Difference Anyway?
Proc. ICAPS 2009, pp. 162–169, 2009.
Introduces cut landmarks and LM-cut heuristic.

Literature (2)

- Lin Zhu and Robert Givan.
  Landmark Extraction via Planning Graph Propagation.
  Doctoral Consortium ICAPS 2003, 2003.
  Core idea for complete landmark generation.

- Emil Keyder, Silvia Richter and Malte Helmert.
  Sound and Complete Landmarks for And/Or Graphs
  Proc. ECAI 2010 , pp. 335–340, 2010.
  Introduces landmarks from AND/OR graphs
  and usage of Π^m compilation.

Literature (3)

Silvia Richter and Matthias Westphal.
The LAMA Planner: Guiding Cost-Based Anytime Planning with Landmarks.
JAIR 39 (2010) , pp. 127–177, 2010.
Introduces landmark-count heuristic and contains another landmark generation method.

Erez Karpas and Carmel Domshlak.
Cost-Optimal Planning with Landmarks.
Proc. IJCAI 2009, pp. 1728–1733, 2009.
Introduces admissible variant of landmark heuristic.