Planning and Optimization
E2. Landmarks: RTG Landmarks & MHS Heuristic

Malte Helmert and Gabriele Röger

Universität Basel

Content of this Course
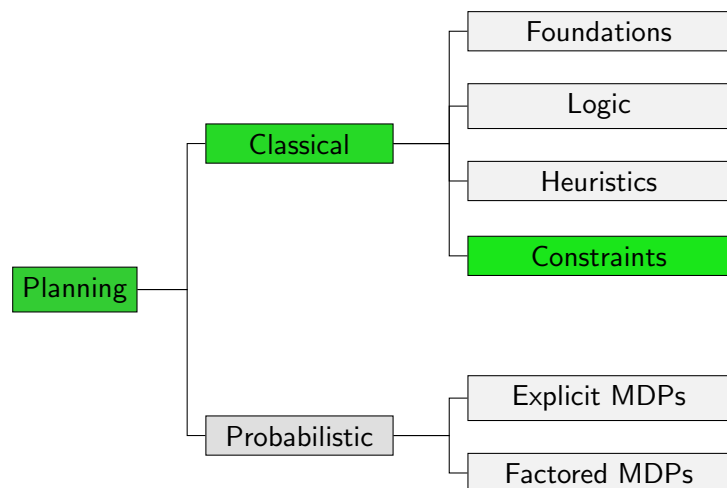
Content of this Course: Constraints
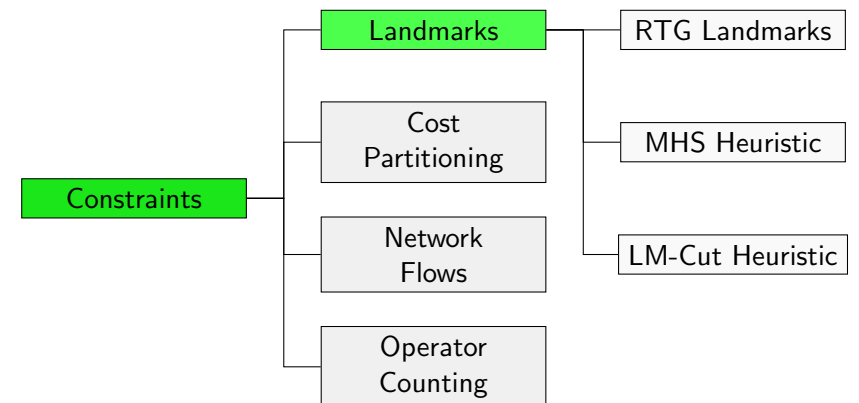
E2.1 Landmarks

Landmarks

Basic Idea: Something that must happen in every solution

For example

▶ some operator must be applied (action landmark)
▶ some atomic proposition must hold (fact landmark)
▶ some formula must be true (formula landmark)

→ Derive heuristic estimate from this kind of information.

We only consider fact and disjunctive action landmarks.

Definition

> **Definition (Disjunctive Action Landmark)**
>
> Let s be a state of planning task $\Pi = \langle V, I, O, \gamma \rangle$.
>
> A disjunctive action landmark for s is a set of operators $L \subseteq O$ such that every label path from s to a goal state contains an operator from L.
> The cost of landmark L is $cost(L) = \min_{o \in L} cost(o)$.

> **Definition (Fact Landmark)**
>
> Let s be a state of planning task $\Pi = \langle V, I, O, \gamma \rangle$.
>
> An atomic proposition $v = d$ for $v \in V$ and $d \in \text{dom}(v)$ is a fact landmark for s if every state path from s to a goal state contains a state s' with $s'(v) = d$.

If we talk about landmarks for the initial state, we omit "for I".

Landmarks: Example

> **Example**
>
> Consider a FDR planning task $\langle V, I, O, \gamma \rangle$ with
> ▶ $V = \{robot\text{-}at, dishes\text{-}at\}$ with
>   ▶ $\text{dom}(robot\text{-}at) = \{A1, \ldots, C3, B4, A5, \ldots, B6\}$
>   ▶ $\text{dom}(dishes\text{-}at) = \{Table, Robot, Dishwasher\}$
> ▶ $I = \{robot\text{-}at \mapsto C1, dishes\text{-}at \mapsto Table\}$
> ▶ operators
>   ▶ move-x-y to move from cell x to adjacent cell y
>   ▶ pickup dishes, and
>   ▶ load dishes into the dishwasher.
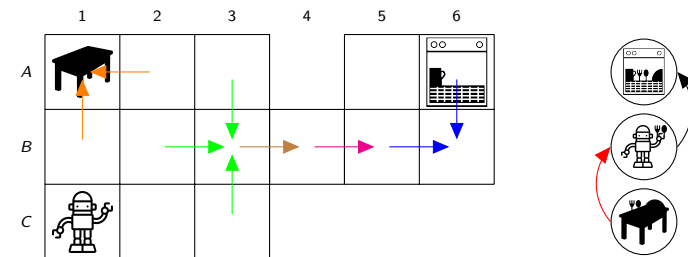> ▶ $\gamma = (robot\text{-}at = B6) \wedge (dishes\text{-}at = Dishwasher)$

Fact Landmarks: Example
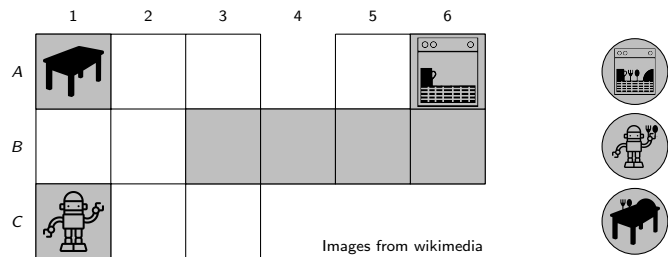
Each fact in gray is a fact landmark:
- robot-at $= x$ for $x \in \{A1, A6, B3, B4, B5, B6, C1\}$
- dishes-at $= x$ for $x \in \{\text{Dishwasher}, \text{Robot}, \text{Table}\}$

Disjunctive Action Landmarks: Example

Actions of same color form disjunctive action landmark:
- {pickup}
- {load}
- {move-B3-B4}
- {move-B4-B5}
- {move-A6-B6, move-B5-B6}
- {move-A3-B3, move-B2-B3, move-C3-B3}
- {move-B1-A1, move-A2-A1}
- . . .

Remarks

- Not every landmark is informative. Some examples:
  - The set of all operators is a disjunctive action landmark unless the initial state is already a goal state.
  - Every variable that is initially true is a fact landmark.
- Deciding whether a given variable is a fact landmark is as hard as the plan existence problem.
- Deciding whether a given operator set is a disjunctive action landmark is as hard as the plan existence problem.
- Every fact landmark v that is initially false induces a disjunctive action landmark consisting of all operators that possibly make v true.

E2.2 Landmarks from RTGs

Content of this Course: Constraints
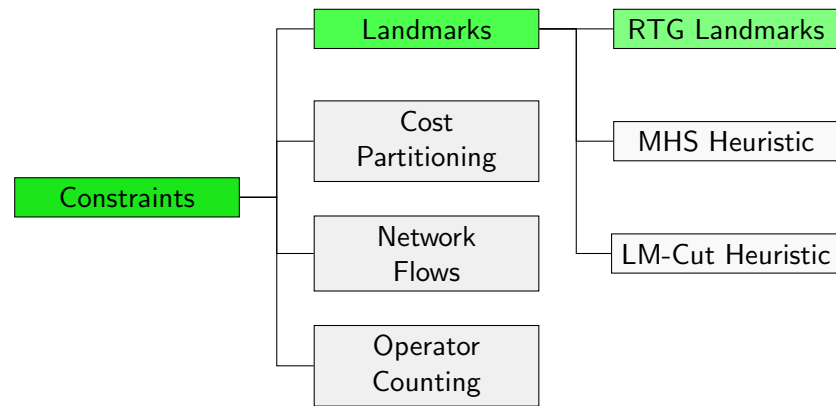
Computing Landmarks

How can we come up with landmarks?

Most landmarks are derived from the relaxed task graph:

▶ RHW landmarks: Richter, Helmert & Westphal. Landmarks Revisited. (AAAI 2008)

▶ LM-Cut: Helmert & Domshlak. Landmarks, Critical Paths and Abstractions: What's the Difference Anyway? (ICAPS 2009)

▶ h^m landmarks: Keyder, Richter & Helmert: Sound and Complete Landmarks for And/Or Graphs (ECAI 2010)

We discuss h^m landmarks restricted to $m = 1$
and to STRIPS planning tasks.

Incidental Landmarks: Example

Example (Incidental Landmarks)

Consider a STRIPS planning task $\langle V, I, \{o_1, o_2\}, \gamma \rangle$ with

$$V = \{a, b, c, d, e, f\},$$
$$I = \{a \mapsto T, b \mapsto T, c \mapsto F, d \mapsto F, e \mapsto T, f \mapsto F\},$$
$$o_1 = \langle \{a\}, \{c, d, e\}, \{a, b\} \rangle,$$
$$o_2 = \langle \{d, e\}, \{f\}, \{a, d\} \rangle, \text{ and}$$
$$\gamma = \{e, f\}.$$

Single solution: $\langle o_1, o_2 \rangle$

▶ All variables are fact landmarks.

▶ Variable b is initially true but irrelevant for the plan.

▶ Variable c gets true as "side effect" of o_1 but it is not necessary for the goal or to make an operator applicable.

Causal Landmarks

Definition (Causal Fact Landmark)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a STRIPS planning task.

An atomic proposition $v = T$ for $v \in V$ is a causal fact landmark

▶ if $v \in \gamma$

▶ or if for all goal paths $\pi = \langle o_1, \ldots, o_n \rangle$ there is an o_i with $v \in pre(o_i)$.

Causal Landmarks: Example

Example (Causal Landmarks)

Consider a STRIPS planning task $\langle V, I, \{o_1, o_2\}, \gamma \rangle$ with

$$V = \{a, b, c, d, e, f\},$$
$$I = \{a \mapsto \text{T}, b \mapsto \text{T}, c \mapsto \text{F}, d \mapsto \text{F}, e \mapsto \text{T}, f \mapsto \text{F}\},$$
$$o_1 = \langle \{a\}, \{c, d, e\}, \{a, b\} \rangle,$$
$$o_2 = \langle \{d, e\}, \{f\}, \{a, d\} \rangle, \text{ and}$$
$$\gamma = \{e, f\}.$$

Single solution: $\langle o_1, o_2 \rangle$

- All variables are fact landmarks for the initial state.
- Only a, d, e and f are causal landmarks.

What We Are Doing Next

- Causal landmarks are the desirable landmarks.
- We can use a simplified version of RTGs to compute causal landmarks for STRIPS planning tasks.
- We will define landmarks of AND/OR graphs, ...
- and show how they can be computed.
- Afterwards we establish that these are landmarks of the planning task.
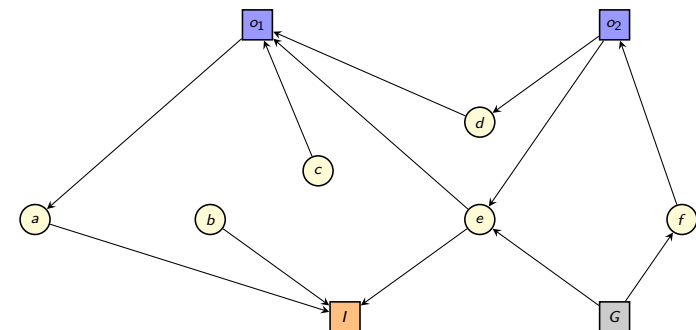
Simplified Relaxed Task Graph

Definition

For a STRIPS planning task $\Pi = \langle V, I, O, \gamma \rangle$, the simplified relaxed task graph $sRTG(\Pi^+)$ is the AND/OR graph $\langle N_{\text{and}} \cup N_{\text{or}}, A, type \rangle$ with

- $N_{\text{and}} = \{n_o \mid o \in O\} \cup \{v_I, v_G\}$
  with $type(n) = \wedge$ for all $n \in N_{\text{and}}$,
- $N_{\text{or}} = \{n_v \mid v \in V\}$
  with $type(n) = \vee$ for all $n \in N_{\text{or}}$, and
- $A = \{\langle n_a, n_o \rangle \mid o \in O, a \in add(o)\} \cup$
  $\{\langle n_o, n_p \rangle \mid o \in O, p \in pre(o)\} \cup$
  $\{\langle n_v, n_I \rangle \mid v \in I\} \cup$
  $\{\langle n_G, n_v \rangle \mid v \in \gamma\}$

Simplified RTG: Example

The simplified RTG for our example task is:

Characterizing Equation System

> **Theorem**
> Let $G = \langle N, A, type \rangle$ be an AND/OR graph. Consider the following system of equations:
>
> $$LM(n) = \{n\} \cup \bigcap_{\langle n,n' \rangle \in A} LM(n') \quad type(n) = \vee$$
>
> $$LM(n) = \{n\} \cup \bigcup_{\langle n,n' \rangle \in A} LM(n') \quad type(n) = \wedge$$
>
> The equation system has a unique maximal solution (maximal with regard to set inclusion), and for this solution it holds that
>
> $n' \in LM(n)$ iff n' is a landmark for reaching n in G.

Computation of Maximal Solution

> **Theorem**
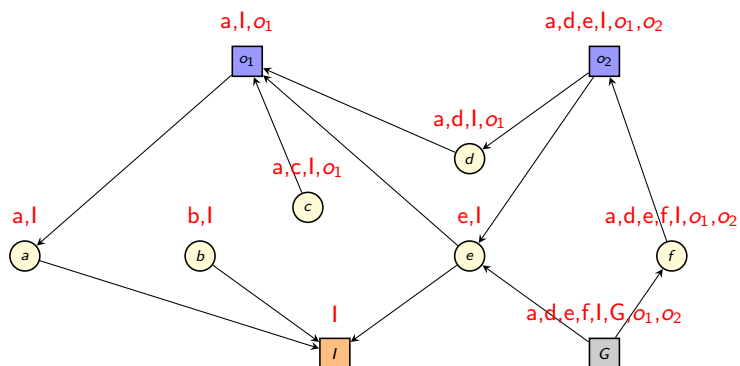> Let $G = \langle N, A, type \rangle$ be an AND/OR graph. Consider the following system of equations:
>
> $$LM(n) = \{n\} \cup \bigcap_{\langle n,n' \rangle \in A} LM(n') \quad type(n) = \vee$$
>
> $$LM(n) = \{n\} \cup \bigcup_{\langle n,n' \rangle \in A} LM(n') \quad type(n) = \wedge$$
>
> The equation system has a unique maximal solution (maximal with regard to set inclusion).

Computation: Initialize landmark sets as $LM(n) = N_{\text{and}} \cup N_{\text{or}}$ and apply equations as update rules until fixpoint.

Computation: Example

(cf. screen version of slides for step-wise computation)

Relation to Planning Task Landmarks

> **Theorem**
> Let $\Pi = \langle V, I, O, \gamma \rangle$ be a STRIPS planning task and let $\mathcal{L}$ be the set of landmarks for reaching n_G in $sRTG(\Pi^+)$.
>
> The set $\{v = \mathsf{T} \mid v \in V \text{ and } n_v \in \mathcal{L}\}$ is exactly the set of causal fact landmarks in Π^+.
>
> For operators $o \in O$, if $n_o \in \mathcal{L}$ then $\{o\}$ is a disjunctive action landmark in Π^+.
> There are no other disjunctive action landmarks of size 1.

(Proofs omitted.)

Computed RTG Landmarks: Example

Example (Computed RTG Landmarks)

Consider a STRIPS planning task $\langle V, I, \{o_1, o_2\}, \gamma \rangle$ with

$$V = \{a, b, c, d, e, f\},$$
$$I = \{a \mapsto T, b \mapsto T, c \mapsto F, d \mapsto F, e \mapsto T, f \mapsto F\},$$
$$o_1 = \langle \{a\}, \{c, d, e\}, \{a, b\}\rangle,$$
$$o_2 = \langle \{d, e\}, \{f\}, \{a, d\}\rangle, \text{ and}$$
$$\gamma = \{e, f\}.$$

- $LM(n_G) = \{a, d, e, f, I, G, o_1, o_2\}$
- a, d, e, and f are causal fact landmarks of Π^+.
- $\{o_1\}$ and $\{o_2\}$ are disjunctive action landmarks of Π^+.

Landmarks of Π^+ Are Landmarks of Π

Theorem

Let Π be a STRIPS planning task.

All fact landmarks of Π^+ are fact landmarks of Π and all disjunctive action landmarks of Π^+ are disjunctive action landmarks of Π.

Proof.

Let L be a disjunctive action landmark of Π^+ and π be a plan for Π. Then π is also a plan for Π^+ and, thus, π contains an operator from L.

Let f be a fact landmark of Π^+. If f is already true in the initial state, then it is also a landmark of Π. Otherwise, every plan for Π^+ contains an operator that adds f and the set of all these operators is a disjunctive action landmark of Π^+. Therefore, also each plan of Π contains such an operator, making f a fact landmark of Π. $\square$

E2.3 Minimum Hitting Set Heuristic

Content of this Course: Constraints
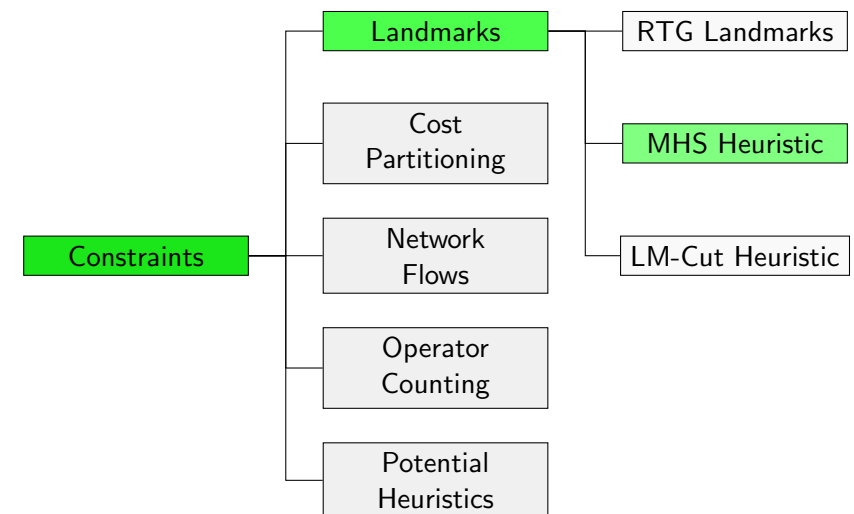
Exploiting Disjunctive Action Landmarks

▶ The cost $cost(L)$ of a disjunctive action landmark L is an admissible heuristic, but it is usually not very informative.

▶ Landmark heuristics typically aim to combine multiple disjunctive action landmarks.

How can we exploit a given set $\mathcal{L}$ of disjunctive action landmarks?

   ▶ Sum of costs $\sum_{L \in \mathcal{L}} cost(L)$?
   ⤳ not admissible!

   ▶ Maximize costs $\max_{L \in \mathcal{L}} cost(L)$?
   ⤳ usually very weak heuristic

   ▶ better: Hitting sets

Hitting Sets

Definition (Hitting Set)

Let X be a set, $\mathcal{F} = \{F_1, \ldots, F_n\} \subseteq 2^X$ be a family of subsets of X and $c : X \to \mathbb{R}_0^+$ be a cost function for X.

A hitting set is a subset $H \subseteq X$ that "hits" all subsets in $\mathcal{F}$, i.e., $H \cap F \neq \emptyset$ for all $F \in \mathcal{F}$. The cost of H is $\sum_{x \in H} c(x)$.

A minimum hitting set (MHS) is a hitting set with minimal cost.

MHS is a "classical" NP-complete problem (Karp, 1972)

Example: Hitting Sets

Example

$X = \{o_1, o_2, o_3, o_4\}$

$\mathcal{F} = \{\{o_4\}, \{o_1, o_2\}, \{o_1, o_3\}, \{o_2, o_3\}\}$

$c(o_1) = 3, \quad c(o_2) = 4, \quad c(o_3) = 5, \quad c(o_4) = 0$

What is a minimum hitting set?

Solution: $\{o_1, o_2, o_4\}$ with cost $3 + 4 + 0 = 7$

Hitting Sets for Disjunctive Action Landmarks

Idea: disjunctive action landmarks are interpreted as instance of minimum hitting set

Definition (Hitting Set Heuristic)

Let $\mathcal{L}$ be a set of disjunctive action landmarks. The hitting set heuristic $h^{\mathrm{MHS}}(\mathcal{L})$ is defined as the cost of a minimum hitting set for $\mathcal{L}$ with $c(o) = cost(o)$.

Proposition (Hitting Set Heuristic is Admissible)

Let $\mathcal{L}$ be a set of disjunctive action landmarks for state s. Then $h^{\mathrm{MHS}}(\mathcal{L})$ is an admissible estimate for s.

Hitting Set Heuristic: Discussion

- The hitting set heuristic is the best possible heuristic that only uses the given information. . .
- . . . but is NP-hard to compute.
- ⤳ Use approximations that can be efficiently computed.
  ⇒ LP-relaxation, cost partitioning (both discussed later)

E2.4 Summary

Summary

- Fact landmark: atomic proposition that is true in each state path to a goal
- Disjunctive action landmark: set L of operators such that every plan uses some operator from L
- Relaxed task graphs allows efficient computation of landmarks
- Hitting sets yield the most accurate heuristic for a given set of disjunctive action landmarks
- Computation of minimal hitting set is NP-hard