Planning and Optimization

B4. Practical Issues of Regression Search

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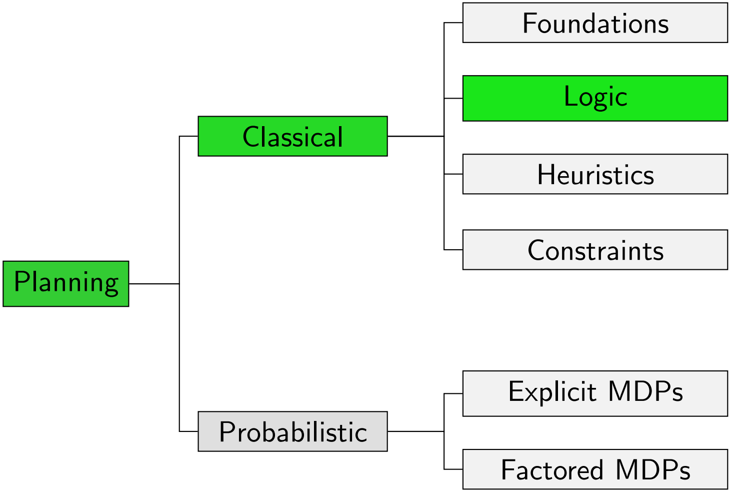
Content of this Course



Regression Search

regression search

- backward search from goal to initial state
- formulas represent sets of states
- regression computes possible predecessor states for a set of states and an operator

Unpromising Branches

Emptiness and Subsumption Testing

The following two tests are useful when performing regression searches to avoid exploring unpromising branches:

- Test that $\mathit{regr}(\varphi, o)$ does not represent the empty set (which would mean that search is in a dead end).
  For example, $\mathit{regr}(p, \langle a, \neg p\rangle) \equiv a \wedge (\bot \vee (p \wedge \neg\top)) \equiv \bot$.
- Test that $\mathit{regr}(\varphi, o)$ does not represent a subset of φ (which would mean that the resulting search state is worse than φ and can be pruned).
  For example, $\mathit{regr}(a, \langle b, c\rangle) \equiv a \wedge b$.

Both of these problems are NP-complete.

Formula Growth

Formula Growth

The formula $regr(regr(\ldots regr(\varphi, o_n)\ldots, o_2), o_1)$ may have size $O(|\varphi||o_1||o_2|\ldots|o_{n-1}||o_n|)$, i.e., the product of the sizes of φ and the operators.

$\rightsquigarrow$ worst-case exponential size $\Omega(|\varphi|^n)$

Logical Simplifications

- $\bot \land \varphi \equiv \bot$, $\top \land \varphi \equiv \varphi$, $\bot \lor \varphi \equiv \varphi$, $\top \lor \varphi \equiv \top$
- $a \lor \varphi \equiv a \lor \varphi[\bot/a]$, $\neg a \lor \varphi \equiv \neg a \lor \varphi[\top/a]$,
  $a \land \varphi \equiv a \land \varphi[\top/a]$, $\neg a \land \varphi \equiv \neg a \land \varphi[\bot/a]$
- idempotence, absorption, commutativity, associativity, . . .

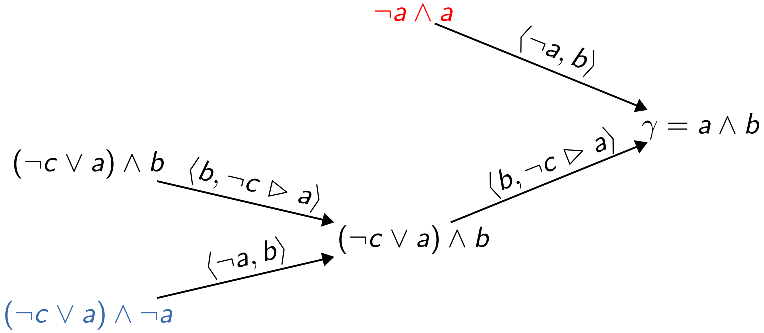
Restricting Formula Growth in Search Trees

Problem	very big formulas obtained by regression
Cause	disjunctivity in the (NNF) formulas (formulas without disjunctions easily convertible to conjunctions $\ell_1 \wedge \cdots \wedge \ell_n$ where ℓ_i are literals and n is at most the number of state variables)
Idea	split disjunctive formulas when generating search trees

Unrestricted Regression: Search Tree Example

Unrestricted regression: do not treat disjunctions specially

Goal $\gamma = a \wedge b$, initial state $I = \{a \mapsto \mathbf{F}, b \mapsto \mathbf{F}, c \mapsto \mathbf{F}\}$.


$\neg a \wedge a$
$\langle \neg a, b \rangle$
$\gamma = a \wedge b$
$(\neg c \vee a) \wedge b$
$\langle b, \neg c \rhd a \rangle$
$\langle b, \neg c \rhd a \rangle$
$(\neg c \vee a) \wedge b$
$\langle \neg a, b \rangle$
$(\neg c \vee a) \wedge \neg a$

Full Splitting: Search Tree Example

Full splitting: always split all disjunctive formulas

Goal $\gamma = a \land b$, initial state $I = \{a \mapsto \mathbf{F}, b \mapsto \mathbf{F}, c \mapsto \mathbf{F}\}$.
$(\neg c \lor a) \land b$ in DNF: $(\neg c \land b) \lor (a \land b)$
$\rightsquigarrow$ split into $\neg c \land b$ and $a \land b$

$$\begin{tikzcd}
 & & \neg a \land a \arrow[dr, "{\langle \neg a, b \rangle}"] & \\
 & & (\text{duplicate of } \gamma)\ a \land b \arrow[r, "{\langle b, \neg c \rhd a \rangle}"] & \gamma = a \land b \\
\neg c \land b \arrow[dr, "{\langle b, \neg c \rhd a \rangle}"] & & & \\
 & \neg c \land b \arrow[uurr, "{\langle b, \neg c \rhd a \rangle}"] & & \\
\neg c \land \neg a \arrow[ur, "{\langle \neg a, b \rangle}"] & & &
\end{tikzcd}$$

General Splitting Strategies

Alternatives:

1. Do nothing (unrestricted regression).
2. Always eliminate all disjunctivity (full splitting).
3. Reduce disjunctivity if formula becomes too big.

Discussion:

- With unrestricted regression formulas may have sizes that are exponential in the number of state variables.
- With full splitting search tree can be exponentially bigger than without splitting.
- The third option lies between these two extremes.

Summary

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When applying regression in practice, we need to consider

- emptiness testing to prune dead-end search states
- subsumption testing to prune dominated search states
- logical simplifications and splitting to restrict formula growth