

# Planning and Optimization

## B4. Practical Issues of Regression Search

Malte Helmert and Gabriele Röger

Universität Basel

# Planning and Optimization

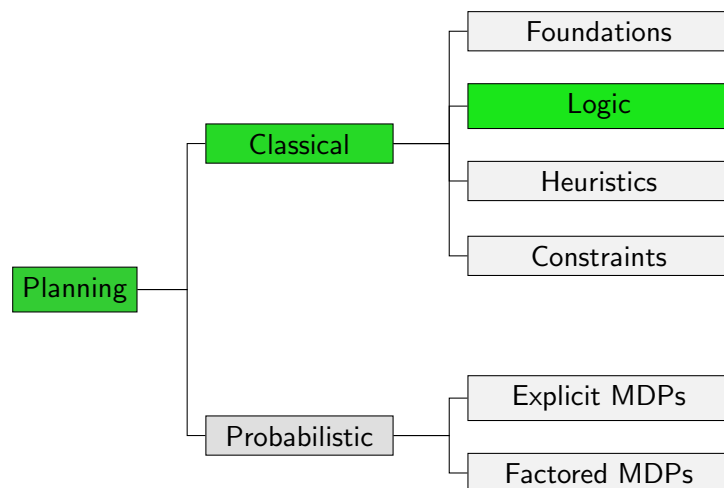
## — B4. Practical Issues of Regression Search

B4.1 Unpromising Branches

B4.2 Formula Growth

B4.3 Summary

## Content of this Course



## Regression Search

regression search

- ▶ backward search from goal to initial state
- ▶ formulas represent sets of states
- ▶ regression computes possible predecessor states for a set of states and an operator

## B4.1 Unpromising Branches

## Emptiness and Subsumption Testing

The following two tests are useful when performing regression searches to avoid exploring unpromising branches:

- ▶ Test that  $\text{regr}(\varphi, o)$  does not represent the empty set (which would mean that search is in a dead end).  
For example,  $\text{regr}(p, \langle a, \neg p \rangle) \equiv a \wedge (\perp \vee (p \wedge \neg \top)) \equiv \perp$ .
- ▶ Test that  $\text{regr}(\varphi, o)$  does not represent a subset of  $\varphi$  (which would mean that the resulting search state is worse than  $\varphi$  and can be pruned).  
For example,  $\text{regr}(a, \langle b, c \rangle) \equiv a \wedge b$ .

Both of these problems are **NP-complete**.

## B4.2 Formula Growth

## Formula Growth

The formula  $\text{regr}(\text{regr}(\dots \text{regr}(\varphi, o_n) \dots, o_2), o_1)$  may have size  $O(|\varphi| |o_1| |o_2| \dots |o_{n-1}| |o_n|)$ , i.e., the product of the sizes of  $\varphi$  and the operators.

$\rightsquigarrow$  worst-case **exponential** size  $\Omega(|\varphi|^n)$

### Logical Simplifications

- ▶  $\perp \wedge \varphi \equiv \perp$ ,  $\top \wedge \varphi \equiv \varphi$ ,  $\perp \vee \varphi \equiv \varphi$ ,  $\top \vee \varphi \equiv \top$
- ▶  $a \vee \varphi \equiv a \vee \varphi[\perp/a]$ ,  $\neg a \vee \varphi \equiv \neg a \vee \varphi[\top/a]$ ,  
 $a \wedge \varphi \equiv a \wedge \varphi[\top/a]$ ,  $\neg a \wedge \varphi \equiv \neg a \wedge \varphi[\perp/a]$
- ▶ idempotence, absorption, commutativity, associativity, ...

## Restricting Formula Growth in Search Trees

**Problem** very big formulas obtained by regression

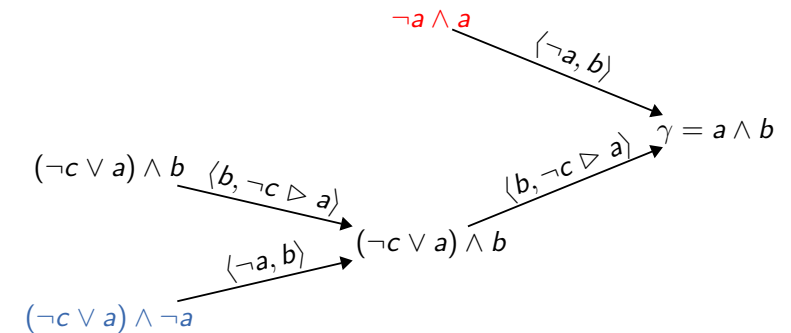
**Cause** **disjunctivity** in the (NNF) formulas  
(formulas **without disjunctions** easily convertible to conjunctions  $l_1 \wedge \dots \wedge l_n$  where  $l_i$  are literals and  $n$  is at most the number of state variables)

**Idea** split disjunctive formulas when generating search trees

## Unrestricted Regression: Search Tree Example

**Unrestricted regression:** do not treat disjunctions specially

Goal  $\gamma = a \wedge b$ , initial state  $I = \{a \mapsto \mathbf{F}, b \mapsto \mathbf{F}, c \mapsto \mathbf{F}\}$ .



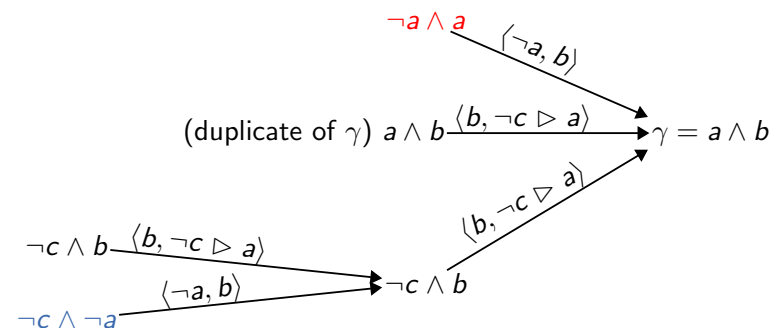
## Full Splitting: Search Tree Example

**Full splitting:** always split all disjunctive formulas

Goal  $\gamma = a \wedge b$ , initial state  $I = \{a \mapsto \mathbf{F}, b \mapsto \mathbf{F}, c \mapsto \mathbf{F}\}$ .

$(\neg c \vee a) \wedge b$  in DNF:  $(\neg c \wedge b) \vee (a \wedge b)$

$\rightsquigarrow$  split into  $\neg c \wedge b$  and  $a \wedge b$



## General Splitting Strategies

Alternatives:

- ① Do nothing (**unrestricted regression**).
- ② Always eliminate all disjunctivity (**full splitting**).
- ③ Reduce disjunctivity if formula becomes too big.

Discussion:

- ▶ **With unrestricted regression** formulas may have **sizes that are exponential** in the number of state variables.
- ▶ **With full splitting** search tree can be **exponentially bigger** than without splitting.
- ▶ The third option lies between these two extremes.

## B4.3 Summary

## Summary

When applying regression in practice, we need to consider

- ▶ **emptiness testing** to prune dead-end search states
- ▶ **subsumption testing** to prune dominated search states
- ▶ **logical simplifications** and **splitting** to restrict formula growth