Planning and Optimization

A6. Positive Normal Form, STRIPS and SAS$^+$

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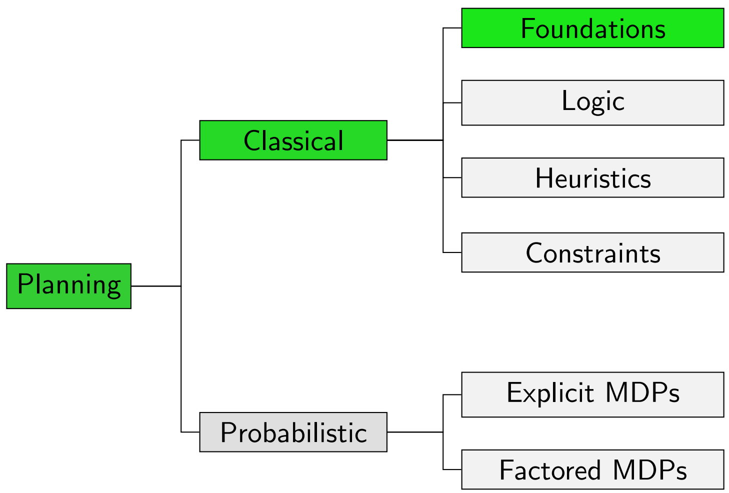
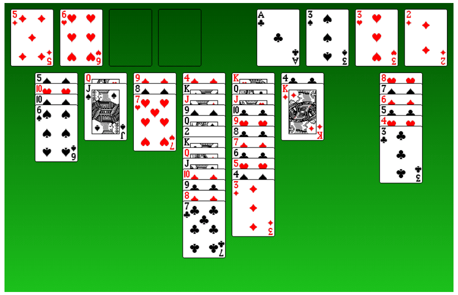
— A6. Positive Normal Form, STRIPS and SAS$^+$

Content of this Course



A6.1 Motivation

Example: Freecell



Example (Good and Bad Effects)

If we move K♢ to a free tableau position,
the good effect is that 4♣ is now accessible.
The bad effect is that we lose one free tableau position.

What is a Good or Bad Effect?

Question: Which operator effects are good, and which are bad?

Difficult to answer in general, because it depends on context:

- Locking our door is good if we want to keep burglars out.
- Locking our door is bad if we want to enter.

We now consider a reformulation of propositional planning tasks that makes the distinction between good and bad effects obvious.

A6.2 Positive Normal Form

Positive Formulas, Operators and Tasks

Definition (Positive Formula)

A logical formula φ is positive if no negation symbols appear in φ.

Note: This includes the negation symbols implied by $\rightarrow$ and $\leftrightarrow$.

Definition (Positive Operator)

An operator o is positive if $pre(o)$ and all effect conditions in $eff(o)$ are positive.

Definition (Positive Propositional Planning Task)

A propositional planning task $\langle V, I, O, \gamma \rangle$ is positive if all operators in O and the goal γ are positive.

Positive Normal Form

Definition (Positive Normal Form)

A propositional planning task is in positive normal form if it is positive and all operator effects are flat.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$\begin{aligned}
V &= \{\mathit{home}, \mathit{uni}, \mathit{lecture}, \mathit{bike}, \mathit{bike\text{-}locked}\}\\
I &= \{\mathit{home} \mapsto \mathsf{T}, \mathit{bike} \mapsto \mathsf{T}, \mathit{bike\text{-}locked} \mapsto \mathsf{T},\\
&\quad\ \mathit{uni} \mapsto \mathsf{F}, \mathit{lecture} \mapsto \mathsf{F}\}\\
O &= \{\langle \mathit{home} \land \mathit{bike} \land \lnot \mathit{bike\text{-}locked}, \lnot \mathit{home} \land \mathit{uni}\rangle,\\
&\quad\ \langle \mathit{bike} \land \mathit{bike\text{-}locked}, \lnot \mathit{bike\text{-}locked}\rangle,\\
&\quad\ \langle \mathit{bike} \land \lnot \mathit{bike\text{-}locked}, \mathit{bike\text{-}locked}\rangle,\\
&\quad\ \langle \mathit{uni}, \mathit{lecture} \land ((\mathit{bike} \land \lnot \mathit{bike\text{-}locked}) \rhd \lnot \mathit{bike})\rangle\}\\
\gamma &= \mathit{lecture} \land \mathit{bike}
\end{aligned}$$

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$\begin{aligned}
V &= \{\mathit{home}, \mathit{uni}, \mathit{lecture}, \mathit{bike}, \mathit{bike\text{-}locked}\}\\
I &= \{\mathit{home} \mapsto \mathsf{T}, \mathit{bike} \mapsto \mathsf{T}, \mathit{bike\text{-}locked} \mapsto \mathsf{T},\\
&\quad\ \mathit{uni} \mapsto \mathsf{F}, \mathit{lecture} \mapsto \mathsf{F}\}\\
O &= \{\langle \mathit{home} \land \mathit{bike} \land \lnot \mathit{bike\text{-}locked}, \lnot \mathit{home} \land \mathit{uni}\rangle,\\
&\quad\ \langle \mathit{bike} \land \mathit{bike\text{-}locked}, \lnot \mathit{bike\text{-}locked}\rangle,\\
&\quad\ \langle \mathit{bike} \land \lnot \mathit{bike\text{-}locked}, \mathit{bike\text{-}locked}\rangle,\\
&\quad\ \langle \mathit{uni}, \mathit{lecture} \land ((\mathit{bike} \land \lnot \mathit{bike\text{-}locked}) \rhd \lnot \mathit{bike})\rangle\}\\
\gamma &= \mathit{lecture} \land \mathit{bike}
\end{aligned}$$

Identify state variable v occurring negatively in conditions.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$\begin{aligned}
V &= \{\mathit{home}, \mathit{uni}, \mathit{lecture}, \mathit{bike}, \mathit{bike\text{-}locked}, \mathit{bike\text{-}unlocked}\}\\
I &= \{\mathit{home} \mapsto \mathsf{T}, \mathit{bike} \mapsto \mathsf{T}, \mathit{bike\text{-}locked} \mapsto \mathsf{T},\\
&\quad\ \mathit{uni} \mapsto \mathsf{F}, \mathit{lecture} \mapsto \mathsf{F}, \mathit{bike\text{-}unlocked} \mapsto \mathsf{F}\}\\
O &= \{\langle \mathit{home} \land \mathit{bike} \land \lnot \mathit{bike\text{-}locked}, \lnot \mathit{home} \land \mathit{uni}\rangle,\\
&\quad\ \langle \mathit{bike} \land \mathit{bike\text{-}locked}, \lnot \mathit{bike\text{-}locked}\rangle,\\
&\quad\ \langle \mathit{bike} \land \lnot \mathit{bike\text{-}locked}, \mathit{bike\text{-}locked}\rangle,\\
&\quad\ \langle \mathit{uni}, \mathit{lecture} \land ((\mathit{bike} \land \lnot \mathit{bike\text{-}locked}) \rhd \lnot \mathit{bike})\rangle\}\\
\gamma &= \mathit{lecture} \land \mathit{bike}
\end{aligned}$$

Introduce new variable $\hat{v}$ with complementary initial value.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$\begin{aligned}
V &= \{\mathit{home}, \mathit{uni}, \mathit{lecture}, \mathit{bike}, \mathit{bike\text{-}locked}, \mathit{bike\text{-}unlocked}\} \\
I &= \{\mathit{home} \mapsto \mathrm{T}, \mathit{bike} \mapsto \mathrm{T}, \mathit{bike\text{-}locked} \mapsto \mathrm{T}, \\
&\quad\ \mathit{uni} \mapsto \mathrm{F}, \mathit{lecture} \mapsto \mathrm{F}, \mathit{bike\text{-}unlocked} \mapsto \mathrm{F}\} \\
O &= \{\langle \mathit{home} \land \mathit{bike} \land \lnot \mathit{bike\text{-}locked}, \lnot \mathit{home} \land \mathit{uni}\rangle, \\
&\quad\ \langle \mathit{bike} \land \mathit{bike\text{-}locked}, \lnot \mathit{bike\text{-}locked}\rangle, \\
&\quad\ \langle \mathit{bike} \land \lnot \mathit{bike\text{-}locked}, \mathit{bike\text{-}locked}\rangle, \\
&\quad\ \langle \mathit{uni}, \mathit{lecture} \land ((\mathit{bike} \land \lnot \mathit{bike\text{-}locked}) \rhd \lnot \mathit{bike})\rangle\} \\
\gamma &= \mathit{lecture} \land \mathit{bike}
\end{aligned}$$

Identify effects on variable v.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$\begin{aligned}
V &= \{\mathit{home}, \mathit{uni}, \mathit{lecture}, \mathit{bike}, \mathit{bike\text{-}locked}, \mathit{bike\text{-}unlocked}\} \\
I &= \{\mathit{home} \mapsto \mathrm{T}, \mathit{bike} \mapsto \mathrm{T}, \mathit{bike\text{-}locked} \mapsto \mathrm{T}, \\
&\quad\ \mathit{uni} \mapsto \mathrm{F}, \mathit{lecture} \mapsto \mathrm{F}, \mathit{bike\text{-}unlocked} \mapsto \mathrm{F}\} \\
O &= \{\langle \mathit{home} \land \mathit{bike} \land \lnot \mathit{bike\text{-}locked}, \lnot \mathit{home} \land \mathit{uni}\rangle, \\
&\quad\ \langle \mathit{bike} \land \mathit{bike\text{-}locked}, \lnot \mathit{bike\text{-}locked} \land \mathit{bike\text{-}unlocked}\rangle, \\
&\quad\ \langle \mathit{bike} \land \lnot \mathit{bike\text{-}locked}, \mathit{bike\text{-}locked} \land \lnot \mathit{bike\text{-}unlocked}\rangle, \\
&\quad\ \langle \mathit{uni}, \mathit{lecture} \land ((\mathit{bike} \land \lnot \mathit{bike\text{-}locked}) \rhd \lnot \mathit{bike})\rangle\} \\
\gamma &= \mathit{lecture} \land \mathit{bike}
\end{aligned}$$

Introduce complementary effects for $\hat{v}$.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$\begin{aligned}
V &= \{\mathit{home}, \mathit{uni}, \mathit{lecture}, \mathit{bike}, \mathit{bike\text{-}locked}, \mathit{bike\text{-}unlocked}\} \\
I &= \{\mathit{home} \mapsto \mathrm{T}, \mathit{bike} \mapsto \mathrm{T}, \mathit{bike\text{-}locked} \mapsto \mathrm{T}, \\
&\quad\ \mathit{uni} \mapsto \mathrm{F}, \mathit{lecture} \mapsto \mathrm{F}, \mathit{bike\text{-}unlocked} \mapsto \mathrm{F}\} \\
O &= \{\langle \mathit{home} \land \mathit{bike} \land \lnot \mathit{bike\text{-}locked}, \lnot \mathit{home} \land \mathit{uni}\rangle, \\
&\quad\ \langle \mathit{bike} \land \mathit{bike\text{-}locked}, \lnot \mathit{bike\text{-}locked} \land \mathit{bike\text{-}unlocked}\rangle, \\
&\quad\ \langle \mathit{bike} \land \lnot \mathit{bike\text{-}locked}, \mathit{bike\text{-}locked} \land \lnot \mathit{bike\text{-}unlocked}\rangle, \\
&\quad\ \langle \mathit{uni}, \mathit{lecture} \land ((\mathit{bike} \land \lnot \mathit{bike\text{-}locked}) \rhd \lnot \mathit{bike})\rangle\} \\
\gamma &= \mathit{lecture} \land \mathit{bike}
\end{aligned}$$

Identify negative conditions for v.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$\begin{aligned}
V &= \{\mathit{home}, \mathit{uni}, \mathit{lecture}, \mathit{bike}, \mathit{bike\text{-}locked}, \mathit{bike\text{-}unlocked}\} \\
I &= \{\mathit{home} \mapsto \mathrm{T}, \mathit{bike} \mapsto \mathrm{T}, \mathit{bike\text{-}locked} \mapsto \mathrm{T}, \\
&\quad\ \mathit{uni} \mapsto \mathrm{F}, \mathit{lecture} \mapsto \mathrm{F}, \mathit{bike\text{-}unlocked} \mapsto \mathrm{F}\} \\
O &= \{\langle \mathit{home} \land \mathit{bike} \land \mathit{bike\text{-}unlocked}, \lnot \mathit{home} \land \mathit{uni}\rangle, \\
&\quad\ \langle \mathit{bike} \land \mathit{bike\text{-}locked}, \lnot \mathit{bike\text{-}locked} \land \mathit{bike\text{-}unlocked}\rangle, \\
&\quad\ \langle \mathit{bike} \land \mathit{bike\text{-}unlocked}, \mathit{bike\text{-}locked} \land \lnot \mathit{bike\text{-}unlocked}\rangle, \\
&\quad\ \langle \mathit{uni}, \mathit{lecture} \land ((\mathit{bike} \land \mathit{bike\text{-}unlocked}) \rhd \lnot \mathit{bike})\rangle\} \\
\gamma &= \mathit{lecture} \land \mathit{bike}
\end{aligned}$$

Replace by positive condition $\hat{v}$.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$V = \{home, uni, lecture, bike, bike\text{-}locked, bike\text{-}unlocked\}$
$I = \{home \mapsto \mathrm{T}, bike \mapsto \mathrm{T}, bike\text{-}locked \mapsto \mathrm{T},$
$uni \mapsto \mathrm{F}, lecture \mapsto \mathrm{F}, bike\text{-}unlocked \mapsto \mathrm{F}\}$
$O = \{\langle home \land bike \land bike\text{-}unlocked, \neg home \land uni\rangle,$
$\langle bike \land bike\text{-}locked, \neg bike\text{-}locked \land bike\text{-}unlocked\rangle,$
$\langle bike \land bike\text{-}unlocked, bike\text{-}locked \land \neg bike\text{-}unlocked\rangle,$
$\langle uni, lecture \land ((bike \land bike\text{-}unlocked) \rhd \neg bike)\rangle\}$
$\gamma = lecture \land bike$

Positive Normal Form: Existence

Theorem (Positive Normal Form)
For every propositional planning task Π, there is an equivalent propositional planning task Π' in positive normal form. Moreover, Π' can be computed from Π in polynomial time.

Note: Equivalence here means that the transition systems induced by Π and Π', restricted to the reachable states, are isomorphic.

We prove the theorem by describing a suitable algorithm.
(However, we do not prove its correctness or complexity.)

Positive Normal Form: Algorithm

Transformation of $\langle V, I, O, \gamma\rangle$ to Positive Normal Form

Replace all operators with equivalent conflict-free operators.
Convert all conditions to negation normal form (NNF).
while any condition contains a negative literal $\neg v$:
  Let v be a variable which occurs negatively in a condition.
  $V := V \cup \{\hat{v}\}$ for some new propositional state variable $\hat{v}$
  $$I(\hat{v}) := \begin{cases} \mathrm{F} & \text{if } I(v) = \mathrm{T} \\ \mathrm{T} & \text{if } I(v) = \mathrm{F} \end{cases}$$
  Replace the effect v by $(v \land \neg\hat{v})$ in all operators $o \in O$.
  Replace the effect $\neg v$ by $(\neg v \land \hat{v})$ in all operators $o \in O$.
  Replace $\neg v$ by $\hat{v}$ in all conditions.
Convert all operators $o \in O$ to flat operators.

Here, all conditions refers to all operator preconditions, operator effect conditions and the goal.

Why Positive Normal Form is Interesting

In the absence of nontrivial conditional effects, positive normal form allows us to distinguish good and bad effects easily:

- Effects that make state variables true (add effects) are good.
- Effects that make state variables false (delete effects) are bad.

This is particularly useful for planning algorithms based on delete relaxation, which we will study later in this course.

(Why restriction "in the absence of nontrivial conditional effects"?)

A6.3 STRIPS

STRIPS Operators and Planning Tasks

Definition (STRIPS Operator)

An operator o of a prop. planning task is a STRIPS operator if

- $pre(o)$ is a conjunction of state variables, and
- $\mathit{eff}(o)$ is a conflict-free conjunction of atomic effects.

Definition (STRIPS Planning Task)

A propositional planning task $\langle V, O, I, \gamma \rangle$ is a STRIPS planning task if all operators $o \in O$ are STRIPS operators and γ is a conjunction of state variables.

Note: STRIPS operators are conflict-free and flat.
(For "flat", we think of atomic effects ℓ as $\top \rhd \ell$ here.)
STRIPS is a special case of positive normal form.

STRIPS Operators: Remarks

- Every STRIPS operator is of the form

$$\langle v_1 \wedge \dots \wedge v_n, \quad \ell_1 \wedge \dots \wedge \ell_m \rangle$$

  where v_i are state variables and ℓ_j are atomic effects.
- Often, STRIPS operators o are described via three sets of state variables:
  - the preconditions (state variables occurring in $pre(o)$)
  - the add effects (state variables occurring positively in $\mathit{eff}(o)$)
  - the delete effects (state variables occurring negatively in $\mathit{eff}(o)$)
- Definitions of STRIPS in the literature often do not require conflict-freeness. But it is easy to achieve and makes many things simpler.
- There exists a variant called STRIPS with negation where negative literals are also allowed in conditions.

Why STRIPS is Interesting

- STRIPS is particularly simple, yet expressive enough to capture general planning tasks.
- In particular, STRIPS planning is no easier than planning in general (as we will see in Chapter A8).
- Many algorithms in the planning literature are only presented for STRIPS planning tasks (generalization is often, but not always, obvious).

STRIPS

STanford Research Institute Problem Solver
(Fikes & Nilsson, 1971)

Transformation to STRIPS

- Not every operator is equivalent to a STRIPS operator.
- However, each operator can be transformed into a set of STRIPS operators whose "combination" is equivalent to the original operator. (How?)
- However, this transformation may exponentially increase the number of operators. There are planning tasks for which such a blow-up is unavoidable.
- There are polynomial transformations of propositional planning tasks to STRIPS, but these do not lead to isomorphic transition systems (auxiliary states are needed). (They are, however, equivalent in a weaker sense.)

A6.4 SAS$^+$

SAS$^+$ Operators and Planning Tasks

Definition (SAS$^+$ Operator)

An operator o of an FDR planning task is a SAS$^+$ operator if

- $pre(o)$ is a satisfiable conjunction of atoms, and
- $eff(o)$ is a conflict-free conjunction of atomic effects.

Definition (SAS$^+$ Planning Task)

An FDR planning task $\langle V, O, I, \gamma \rangle$ is a SAS$^+$ planning task if all operators $o \in O$ are SAS$^+$ operators and γ is a satisfiable conjunction of atoms.

Note: SAS$^+$ operators are conflict-free and flat.
(Same comments as for STRIPS operators apply.)

SAS$^+$ Operators: Remarks

- Every SAS$^+$ operator is of the form

$$\langle v_1 = d_1 \wedge \dots \wedge v_n = d_n, \quad v'_1 := d'_1 \wedge \dots \wedge v'_m := d'_m \rangle$$

  where all v_i are distinct and all v'_j are distinct.
- Often, SAS$^+$ operators o are described via two sets of partial assignments:
  - the preconditions $\{v_1 \mapsto d_1, \dots, v_n \mapsto d_n\}$
  - the effects $\{v'_1 \mapsto d'_1, \dots, v'_m \mapsto d'_m\}$

SAS$^+$ vs. STRIPS

- SAS$^+$ is an analogue of STRIPS planning tasks for FDR, but there is no special role of "positive" conditions.
- Apart from this difference, all comments for STRIPS apply analogously.
- If all variable domains are binary, SAS$^+$ is essentially STRIPS with negation.

SAS$^+$
Derives from SAS = Simplified Action Structures (Bäckström & Klein, 1991)

A6.5 Summary

Summary

- A positive task helps distinguish good and bad effects. The notion of positive tasks only exists for propositional tasks.
- A positive task with flat operators is in positive normal form.
- STRIPS is even more restrictive than positive normal form, forbidding complex preconditions and conditional effects.
- Both forms are expressive enough to capture general propositional planning tasks.
- Transformation to positive normal form is possible with polynomial size increase.
- Isomorphic transformations of propositional planning tasks to STRIPS can increase the number of operators exponentially; non-isomorphic polynomial transformations exist.
- SAS$^+$ is the analogue of STRIPS for FDR planning tasks.