

# Planning and Optimization

## A5. Equivalent Operators and Normal Forms

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### — A5. Equivalent Operators and Normal Forms

#### A5.1 Reminder & Motivation

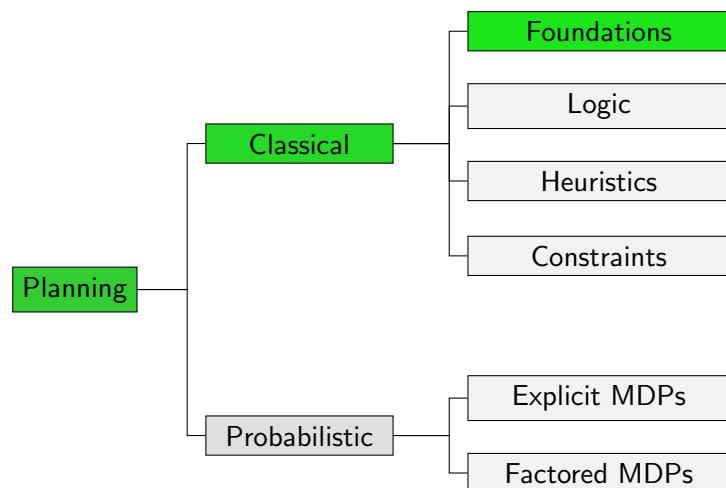
#### A5.2 Equivalence Transformations

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## Content of this Course



## A5.1 Reminder & Motivation

## Reminder: Syntax of Effects

### Definition (Effect)

Effects over state variables  $V$  are inductively defined as follows:

- ▶ If  $v \in V$  is a propositional state variable, then  $v$  and  $\neg v$  are effects (atomic effect).
- ▶ If  $v \in V$  is a finite-domain state variable and  $d \in \text{dom}(v)$ , then  $v := d$  is an effect (atomic effect).
- ▶ If  $e_1, \dots, e_n$  are effects, then  $(e_1 \wedge \dots \wedge e_n)$  is an effect (conjunctive effect).
- ▶ The special case with  $n = 0$  is the empty effect  $\top$ .
- ▶ If  $\chi$  is a formula over  $V$  and  $e$  is an effect, then  $(\chi \triangleright e)$  is an effect (conditional effect).

Arbitrary nesting of conjunctive and conditional effects!  
 ↵ Can we make our life easier?

## Motivation

Similarly to normal forms in propositional logic (DNF, CNF, NNF), we can define normal forms for effects, operators and planning tasks.

Among other things, we consider normal forms that avoid complicated nesting and subtleties of conflicts.

This is useful because algorithms (and proofs) then only need to deal with effects, operators and tasks in normal form.

## Reminder: Semantics of Effects

- ▶  $\text{effcond}(e, e')$ : condition that must be true in the current state for the effect  $e'$  to trigger the atomic effect  $e$
- ▶ add-after-delete semantics (propositional tasks): if an operator with effect  $e$  is applied in state  $s$  and we have both  $s \models \text{effcond}(v, e)$  and  $s \models \text{effcond}(\neg v, e)$ , then  $s'(v) = \top$  in the resulting state  $s'$ .
- ▶ consistency semantics (finite-domain tasks): applying an operator with effect  $e$  in a state  $s$  where both  $s \models \text{effcond}(v := d, e)$  and  $s \models \text{effcond}(v := d', e)$  for values  $d \neq d'$  is forbidden  
 ↵ tested via the consistency condition  $\text{consist}(e)$

These are very subtle details!  
 ↵ Can we make our life easier?

## Motivation

Similarly to normal forms in propositional logic (DNF, CNF, NNF), we can define normal forms for effects, operators and planning tasks.

Among other things, we consider normal forms that avoid complicated nesting and subtleties of conflicts.

This is useful because algorithms (and proofs) then only need to deal with effects, operators and tasks in normal form.

## Notation: Applying Effects and Operator Sequences

### Existing notation:

- ▶ We already write  $s[o]$  for the resulting state after applying operator  $o$  in state  $s$ .

### New extended notation:

- ▶ If we want to consider an effect  $e$  without a precondition, we write  $s[e]$  for  $s[\langle \top, e \rangle]$ .
- ▶ For a sequence  $\pi = \langle o_1, \dots, o_n \rangle$  of operators that are consecutively applicable in  $s$ , we write  $s[\pi]$  for  $s[o_1][o_2] \dots [o_n]$ .

## A5.2 Equivalence Transformations

### Equivalence of Operators and Effects: Definition

#### Definition (Equivalent Effects)

Two effects  $e$  and  $e'$  over state variables  $V$  are **equivalent**, written  $e \equiv e'$ , if  $s[e] = s[e']$  for all states  $s$ .

For consistency semantics, this includes the requirement that  $s[e]$  is defined iff  $s[e']$  is.

#### Definition (Equivalent Operators)

Two operators  $o$  and  $o'$  over state variables  $V$  are **equivalent**, written  $o \equiv o'$ , if  $\text{cost}(o) = \text{cost}(o')$  and for all states  $s, s'$  over  $V$ ,  $o$  induces the transition  $s \xrightarrow{o} s'$  iff  $o'$  induces the transition  $s \xrightarrow{o'} s'$ .

### Equivalence of Operators and Effects: Theorem

#### Theorem

Let  $o$  and  $o'$  be operators with  $\text{pre}(o) \equiv \text{pre}(o')$ ,  $\text{eff}(o) \equiv \text{eff}(o')$  and  $\text{cost}(o) = \text{cost}(o')$ . Then  $o \equiv o'$ .

Note: The converse is not true. (Why not?)

### Equivalence Transformations for Effects

$$e_1 \wedge e_2 \equiv e_2 \wedge e_1 \quad (1)$$

$$(e_1 \wedge \dots \wedge e_n) \wedge (e'_1 \wedge \dots \wedge e'_m) \equiv e_1 \wedge \dots \wedge e_n \wedge e'_1 \wedge \dots \wedge e'_m \quad (2)$$

$$\top \wedge e \equiv e \quad (3)$$

$$\chi \triangleright e \equiv \chi' \triangleright e \quad \text{if } \chi \equiv \chi' \quad (4)$$

$$\top \triangleright e \equiv e \quad (5)$$

$$\perp \triangleright e \equiv \top \quad (6)$$

$$\chi_1 \triangleright (\chi_2 \triangleright e) \equiv (\chi_1 \wedge \chi_2) \triangleright e \quad (7)$$

$$\chi \triangleright (e_1 \wedge \dots \wedge e_n) \equiv (\chi \triangleright e_1) \wedge \dots \wedge (\chi \triangleright e_n) \quad (8)$$

$$(\chi_1 \triangleright e) \wedge (\chi_2 \triangleright e) \equiv (\chi_1 \vee \chi_2) \triangleright e \quad (9)$$

## A5.3 Conflict-Free Operators

### Conflict-Free Effects and Operators

#### Definition (Conflict-Free)

An **effect**  $e$  over **propositional** state variables  $V$  is called **conflict-free** if  $\text{effcond}(v, e) \wedge \text{effcond}(\neg v, e)$  is unsatisfiable for all  $v \in V$ .

An **effect**  $e$  over **finite-domain** state variables  $V$  is called **conflict-free** if  $\text{effcond}(v := d, e) \wedge \text{effcond}(v := d', e)$  is unsatisfiable for all  $v \in V$  and  $d, d' \in \text{dom}(v)$  with  $d \neq d'$ .

An **operator**  $o$  is called **conflict-free** if  $\text{eff}(o)$  is conflict-free.

**Note:** This fixes both of our issues.

In particular, observe that  $\text{consist}(o) \equiv \top$  for conflict-free  $o$ .

### Conflict-Freeness: Motivation

- ▶ The add-after-delete semantics makes effects like  $(a \triangleright c) \wedge (b \triangleright \neg c)$  somewhat unintuitive to interpret.
  - ↝ What happens in states where  $a \wedge b$  is true?
- ▶ Similarly, it may be unintuitive that an effect like  $(u = a \triangleright w := a) \wedge (v = b \triangleright w := b)$  introduces an applicability condition “through the back door”
- ▶ It would be nicer if
  - ▶  $\text{effcond}(e, e')$  always were the condition under which the atomic effect  $e$  actually materializes (because of add-after-delete, it is not)
  - ▶  $\text{pre}(o)$  always fully described the applicability of  $o$  (because of the consistency condition, is does not)
- ↝ introduce normal form where “complicated cases” never arise

### Making Operators Conflict-Free

- ▶ In general, testing whether an operator is conflict-free is a coNP-complete problem. ([Why?](#))
- ▶ However, we do not necessarily need such a test. Instead, we can **produce** an equivalent conflict-free operator in polynomial time.
- ▶ **Algorithm:** given operator  $o$ ,
  - ▶ replace all atomic effects  $\neg v$  by  $(\neg \text{effcond}(v, \text{eff}(o)) \triangleright \neg v)$
  - ▶ replace all atomic effects  $v := d$  by  $(\text{consist}(o) \triangleright v := d)$
  - ▶ replace  $\text{pre}(o)$  with  $\text{pre}(o) \wedge \text{consist}(o)$  in the FDR case
- The resulting operator  $o'$  is conflict-free and  $o \equiv o'$ . ([Why?](#))

## A5.4 Flat Effects

### Flat Effect

#### Definition (Flat Effect)

An effect  $e$  is **flat** if it is:

- ▶ a conjunctive effect
- ▶ whose conjuncts are conditional effects
- ▶ whose subeffects are atomic effects, and
- ▶ no atomic effect occurs in  $e$  multiple times.

An operator  $o$  is **flat** if  $\text{eff}(o)$  is flat.

**Note:** non-conjunctive effects can be considered as conjunctive effects with 1 conjunct

### Flat Effects: Motivation

- ▶ CNF and DNF limit the **nesting** of connectives in propositional logic.
- ▶ For example, a CNF formula is
  - ▶ a conjunction of 0 or more subformulas,
  - ▶ each of which is a disjunction of 0 or more subformulas,
  - ▶ each of which is a literal.
- ▶ Similarly, we can define a normal form that limits the nesting of effects.
- ▶ This is useful because we then do not have to consider arbitrarily structured effects, e.g., when representing them in a planning algorithm.

### Flat Effect

#### Definition (Flat Effect)

An effect  $e$  is **flat** if it is:

- ▶ a conjunctive effect
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- ▶ no atomic effect occurs in  $e$  multiple times.

An operator  $o$  is **flat** if  $\text{eff}(o)$  is flat.

**Note:** non-conjunctive effects can be considered as conjunctive effects with 1 conjunct

### Flat Effect: Example

#### Example

Consider the effect

$$c \wedge (a \triangleright (\neg b \wedge (c \triangleright (b \wedge \neg d \wedge \neg a)))) \wedge (\neg b \triangleright \neg a)$$

An equivalent flat (and conflict-free) effect is

$$\begin{aligned} & (\top \triangleright c) \wedge \\ & ((a \wedge \neg c) \triangleright \neg b) \wedge \\ & ((a \wedge c) \triangleright b) \wedge \\ & ((a \wedge c) \triangleright \neg d) \wedge \\ & ((\neg b \vee (a \wedge c)) \triangleright \neg a) \end{aligned}$$

**Note:** for simplicity, we often write  $(\top \triangleright e)$  as  $e$ , i.e., omit **trivial** effect conditions. We still consider such effects to be flat.

## Producing Flat Operators

### Theorem

For every operator, an equivalent flat operator and an equivalent flat, conflict-free operator can be computed in polynomial time.

## Producing Flat Operators: Proof

### Proof Sketch.

Let  $E$  be the set of atomic effects over variables  $V$ .

Every effect  $e'$  over variables  $V$  is equivalent to

$\bigwedge_{e \in E} (\text{effcond}(e, e') \triangleright e)$ , which is a flat effect.

(Conjuncts of the form  $(\chi \triangleright e)$  where  $\chi \equiv \perp$  can be omitted to simplify the effect.)

To compute a flat operator equivalent to operator  $o$ , replace  $\text{eff}(o)$  by an equivalent flat effect.

To compute an equivalent conflict-free and flat operator, first compute a conflict-free operator  $o'$  equivalent to  $o$ , then replace  $\text{eff}(o')$  by an equivalent flat effect.

(Why not do these in the opposite order?)

## A5.5 Summary

## Summary

- ▶ **Equivalences** can be used to simplify operators and effects.
- ▶ In **conflict-free** operators, the “complicated case” of operator semantics does not arise.
- ▶ For **flat** operators, the only permitted nesting is atomic effects within conditional effects within conjunctive effects, and all atomic effects must be distinct.
- ▶ For flat, conflict-free operators, it is easy to determine the **condition** under which a given **literal** is **made true** by applying the operator in a given state.
- ▶ Every operator can be **transformed** into an equivalent **flat and conflict-free** one in **polynomial time**.