# Planning and Optimization

## A3. Transition Systems and Propositional Logic

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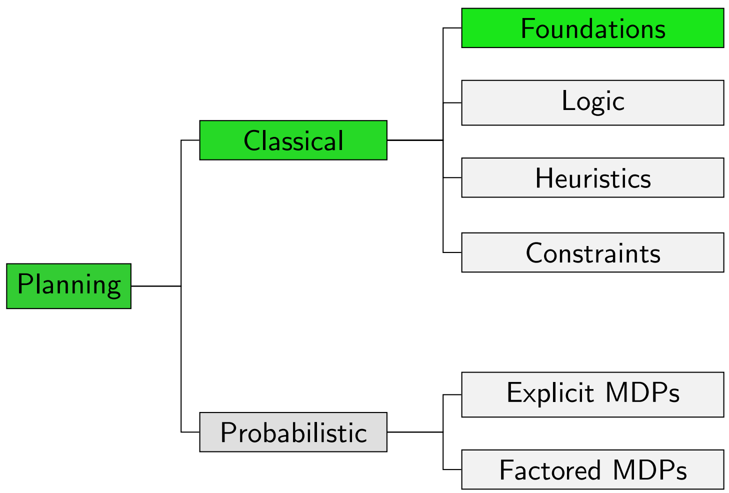
## Content of this Course






## Goals for Today

Today:

- introduce a mathematical model for planning tasks: transition systems
  ⇝ Chapter A3
- introduce compact representations for planning tasks suitable as input for planning algorithms
  ⇝ Chapter A4

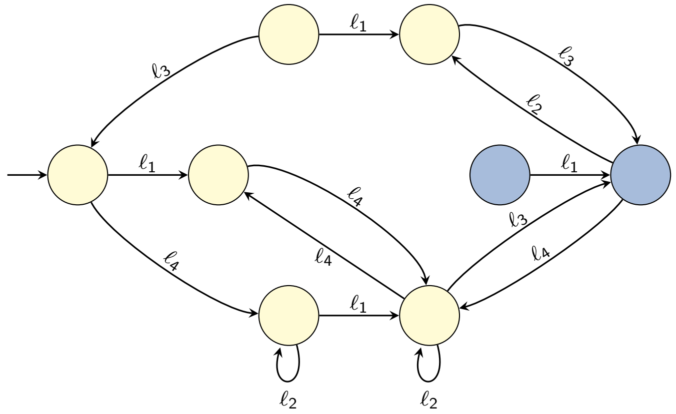
# A3.1 Transition Systems





## Transition System Example

Transition systems are often depicted as directed arc-labeled graphs with decorations to indicate the initial state and goal states.




$c(\ell_1) = 1,\ c(\ell_2) = 1,\ c(\ell_3) = 5,\ c(\ell_4) = 0$





## Transition Systems

**Definition (Transition System)**

A transition system is a 6-tuple $\mathcal{T} = \langle S, L, c, T, s_0, S_\star \rangle$ where

- $S$ is a finite set of states,
- $L$ is a finite set of (transition) labels,
- $c : L \to \mathbb{R}_0^+$ is a label cost function,
- $T \subseteq S \times L \times S$ is the transition relation,
- $s_0 \in S$ is the initial state, and
- $S_\star \subseteq S$ is the set of goal states.

We say that $\mathcal{T}$ has the transition $\langle s, \ell, s' \rangle$ if $\langle s, \ell, s' \rangle \in T$.

We also write this as $s \xrightarrow{\ell} s'$, or $s \to s'$ when not interested in $\ell$.

Note: Transition systems are also called state spaces.





## Deterministic Transition Systems

**Definition (Deterministic Transition System)**

A transition system is called deterministic if for all states $s$ and all labels $\ell$, there is at most one state $s'$ with $s \xrightarrow{\ell} s'$.

Example: previously shown transition system

# Transition System Terminology (1)

We use common terminology from graph theory:

- $s'$ successor of $s$ if $s \to s'$
- $s$ predecessor of $s'$ if $s \to s'$





# Transition System Terminology (2)

We use common terminology from graph theory:

- $s'$ reachable from $s$ if there exists a sequence of transitions $s^0 \xrightarrow{\ell_1} s^1, \ldots, s^{n-1} \xrightarrow{\ell_n} s^n$ s.t. $s^0 = s$ and $s^n = s'$
  - Note: $n = 0$ possible; then $s = s'$
  - $s^0, \ldots, s^n$ is called (state) path from $s$ to $s'$
  - $\ell_1, \ldots, \ell_n$ is called (label) path from $s$ to $s'$
  - $s^0 \xrightarrow{\ell_1} s^1, \ldots, s^{n-1} \xrightarrow{\ell_n} s^n$ is called trace from $s$ to $s'$
  - length of path/trace is $n$
  - cost of label path/trace is $\sum_{i=1}^{n} c(\ell_i)$





# Transition System Terminology (3)

We use common terminology from graph theory:

- $s'$ reachable (without reference state) means reachable from initial state $s_0$
- solution or goal path from $s$: path from $s$ to some $s' \in S_\star$
  - if $s$ is omitted, $s = s_0$ is implied
- transition system solvable if a goal path from $s_0$ exists





# A3.2 Example: Blocks World

## Running Example: Blocks World

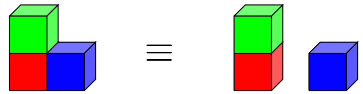
- Throughout the course, we occasionally use the blocks world domain as an example.
- In the blocks world, a number of different blocks are arranged on a table.
- Our job is to rearrange them according to a given goal.



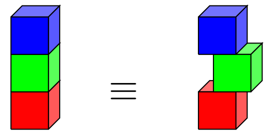


## Blocks World Rules (1)

Location on the table does not matter.



Location on a block does not matter.

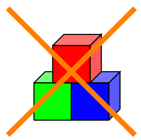




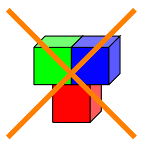


## Blocks World Rules (2)

At most one block may be below a block.



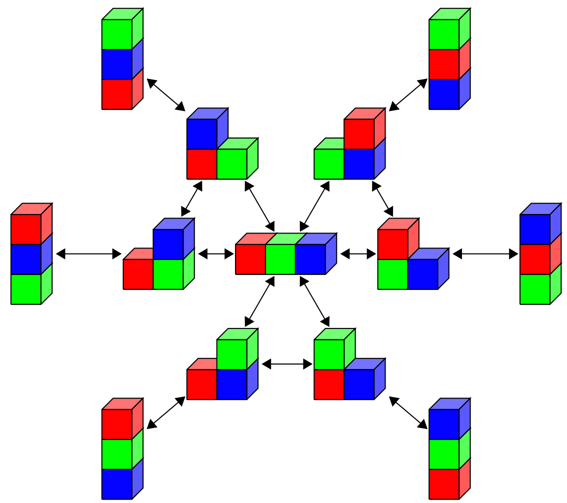
At most one block may be on top of a block.







## Blocks World Transition System for Three Blocks



Labels omitted for clarity. All label costs are 1. Initial/goal states not marked.

## Blocks World Computational Properties

| blocks | states | blocks | states |
|---|---|---|---|
| 1 | 1 | 10 | 58941091 |
| 2 | 3 | 11 | 824073141 |
| 3 | 13 | 12 | 12470162233 |
| 4 | 73 | 13 | 202976401213 |
| 5 | 501 | 14 | 3535017524403 |
| 6 | 4051 | 15 | 65573803186921 |
| 7 | 37633 | 16 | 1290434218669921 |
| 8 | 394353 | 17 | 26846616451246353 |
| 9 | 4596553 | 18 | 588633468315403843 |

- Finding solutions is possible in linear time in the number of blocks: move everything onto the table, then construct the goal configuration.
- Finding a shortest solution is NP-complete given a compact description of the problem.





## The Need for Compact Descriptions

- We see from the blocks world example that transition systems are often far too large to be directly used as inputs to planning algorithms.
- We therefore need compact descriptions of transition systems.
- For this purpose, we will use propositional logic, which allows expressing information about $2^n$ states as logical formulas over $n$ state variables.





# A3.3 Reminder: Propositional Logic





## More on Propositional Logic

**Need to Catch Up?**

- This section is a reminder. We assume you are already well familiar with propositional logic.
- If this is not the case, we recommend Chapters B1–B3 of the Theory of Computer Science course at `https://dmi.unibas.ch/en/academics/computer-science/courses-in-spring-semester-2020/lecture-theory-of-computer-science/`

## Syntax of Propositional Logic

### Definition (Logical Formula)

Let $A$ be a set of atomic propositions.

The logical formulas over $A$ are constructed by finite application of the following rules:

- $\top$ and $\bot$ are logical formulas (truth and falsity).
- For all $a \in A$, $a$ is a logical formula (atom).
- If $\varphi$ is a logical formula, then so is $\neg\varphi$ (negation).
- If $\varphi$ and $\psi$ are logical formulas, then so are $(\varphi \vee \psi)$ (disjunction) and $(\varphi \wedge \psi)$ (conjunction).





## Syntactical Conventions for Propositional Logic

Abbreviations:

- $(\varphi \rightarrow \psi)$ is short for $(\neg\varphi \vee \psi)$ (implication)
- $(\varphi \leftrightarrow \psi)$ is short for $((\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi))$ (equijunction)
- parentheses omitted when not necessary:
  - $(\neg)$ binds more tightly than binary connectives
  - $(\wedge)$ binds more tightly than $(\vee)$, which binds more tightly than $(\rightarrow)$, which binds more tightly than $(\leftrightarrow)$





## Semantics of Propositional Logic

### Definition (Valuation, Model)

A valuation of propositions $A$ is a function $v : A \rightarrow \{\mathrm{T}, \mathrm{F}\}$.

Define the notation $v \models \varphi$ ($v$ satisfies $\varphi$; $v$ is a model of $\varphi$; $\varphi$ is true under $v$) for valuations $v$ and formulas $\varphi$ by

- $v \models \top$
- $v \not\models \bot$
- $v \models a$ iff $v(a) = \mathrm{T}$ (for all $a \in A$)
- $v \models \neg\varphi$ iff $v \not\models \varphi$
- $v \models (\varphi \vee \psi)$ iff $(v \models \varphi$ or $v \models \psi)$
- $v \models (\varphi \wedge \psi)$ iff $(v \models \varphi$ and $v \models \psi)$

Note: Valuations are also called interpretations or truth assignments.





## Propositional Logic Terminology (1)

- A logical formula $\varphi$ is satisfiable if there is at least one valuation $v$ such that $v \models \varphi$.
- Otherwise it is unsatisfiable.
- A logical formula $\varphi$ is valid or a tautology if $v \models \varphi$ for all valuations $v$.
- A logical formula $\psi$ is a logical consequence of a logical formula $\varphi$, written $\varphi \models \psi$, if $v \models \psi$ for all valuations $v$ with $v \models \varphi$.
- Two logical formulas $\varphi$ and $\psi$ are logically equivalent, written $\varphi \equiv \psi$, if $\varphi \models \psi$ and $\psi \models \varphi$.

Question: How to phrase these in terms of models?

## Propositional Logic Terminology (2)

- A logical formula that is a proposition $a$ or a negated proposition $\neg a$ for some atomic proposition $a \in A$ is a literal.
- A formula that is a disjunction of literals is a clause. This includes unit clauses $\ell$ consisting of a single literal and the empty clause $\bot$ consisting of zero literals.
- A formula that is a conjunction of literals is a monomial. This includes unit monomials $\ell$ consisting of a single literal and the empty monomial $\top$ consisting of zero literals.

Normal forms:

- negation normal form (NNF)
- conjunctive normal form (CNF)
- disjunctive normal form (DNF)





# A3.4 Summary





## Summary

- Transition systems are (typically huge) directed graphs that encode how the state of the world can change.
- Propositional logic allows us to compactly describe complex information about large sets of valuations as logical formulas.