Discrete Mathematics in Computer Science

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Exercise Sheet 11

Due: Thursday, December 9, 2021

Exercise 11.1 (1 mark)

Add all missing parentheses to formula $\varphi = A \vee B \wedge C \leftrightarrow \neg D \wedge E$. You do not need to expand the abbreviating $\rightarrow$ and $\leftrightarrow$.

Exercise 11.2 (2 marks)

Transform $\chi = (\neg C \leftrightarrow (A \vee \neg B))$ into CNF by applying the algorithm on Slide 16 (handout version) of Chapter E3. At the least you must show the formula after each step of the algorithm; we would however advise an even more fine-grained solution since according to experience smaller steps help to reduce the amount of errors.
You are allowed to abbreviate parentheses for conjunctions of conjunctions and disjunctions of disjunctions; for example you can write $(A_1 \wedge A_2 \wedge A_3)$.

Exercise 11.3 (2 marks)

Prove the contraposition theorem, that is, show for any set of formulas KB and formulas φ and ψ that

$$\mathrm{KB} \cup \{\varphi\} \models \neg\psi \text{ iff } \mathrm{KB} \cup \{\psi\} \models \neg\varphi$$

holds.

Exercise 11.4 (2 marks)

You'll find a Java program in the archive "proofchecker.zip" that checks proofs formulated in propositional logic. Use this program to prove the following statements. For a statement of the form WB $\models \varphi$ write a text file containing a derivation that only uses formulas from WB as assumptions and that has φ in its last line. An example for this is contained in the file `proof.txt`. The file `src/logic/proofs/rules/Calculus.java` contains an overview of all rules known to the program.
The program checks WB $\vdash \varphi$. Since the proof system used by the program is correct, this implies WB $\models \varphi$.
Note on the submission process: please create one text file for each exercise part which contains the derivation. The program must be able to parse the file and accept the derivation as correct.

(a) $\{(A \leftrightarrow (B \vee \neg C)), \neg(C \vee D)\} \models A$

(b) $\{(\neg C \vee A), (\neg A \wedge B)\} \models (B \wedge \neg C)$

Exercise 11.5 (3 marks)

Consider the following knowledge base

$$\mathrm{KB} = \{(D \rightarrow (A \leftrightarrow C)), ((\neg A \wedge B) \vee (D \wedge \neg A)), ((\neg A \wedge \neg C) \rightarrow \neg D)\}$$

Use the resolution calculus to show that KB $\models B$ holds.
Note: A proof using resolution consists of three steps (see lecture slides for an example). Use the notation from the lecture slides in particular in the last step, that is, use one line for each derived clause together with the derivation's justification. A complete proof has to include the argument why KB $\models B$ follows. Do not just list formulas or draw a tree of formulas. When using equivalences, you may implicitly use commutativity, double negation, and associativity.

Submission rules:
Upload a single archive containing a single PDF file (ending .pdf) generated using LaTeX as well as two text files for exercise 11.4 (a) and (b). If you did not do exercise 11.4, you may upload the single PDF directly.
Put the names of all group members on top of the first page of the PDF. Use page numbers or put your names on each page. Make sure your PDF has size A4 (fits the page size if printed on A4).