

Discrete Mathematics in Computer Science

M. Helmert, G. Röger
S. Eriksson
Fall Term 2021

University of Basel
Computer Science

Exercise Sheet 8

Due: Thursday, November 18, 2021

Exercise 8.1 (3 marks)

Prove or refute the following statements:

- (a) The induced graph of a DAG is a forest.
- (b) Every digraph representing a strict order relation is acyclic.
- (c) Every acyclic digraph (DAG) represents a strict order.

Exercise 8.2 (2 marks)

The archive `rooted-trees` contains a Java code skeleton for building rooted trees of a given (graph) tree, as well as a file `graph` describing such a tree. The skeleton already implements reading in the file and looping over all vertices, calling the (not yet implemented) method `buildTree` and printing the height of the resulting rooted tree for each vertex. You can compile the code with

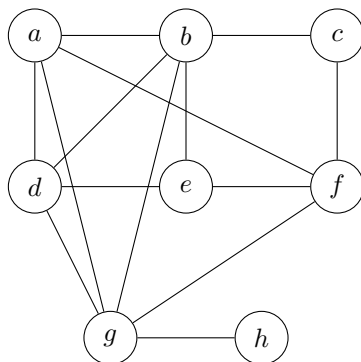
```
javac Main.java
```

and then run it with the given graph file with

```
java Main graph.
```

Implement the method `buildTree` in the file `Main.java`. For the tree given in `graph`, what are the minimal and maximal heights of the rooted trees, and which root nodes result in those heights?

For the remaining exercises we consider the following graph G :



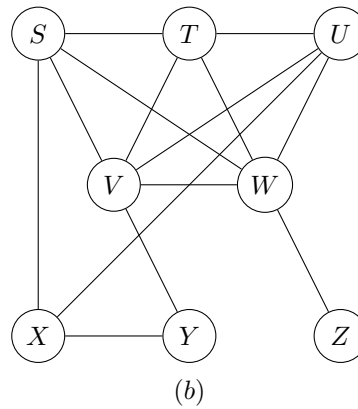
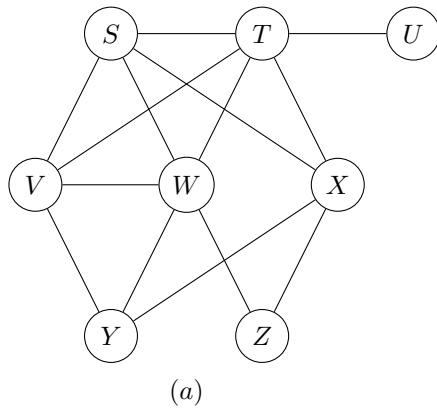
Exercise 8.3 (1 mark)

Draw the subgraph of G induced by $\{a, b, d, e, f, h\}$.

Exercise 8.4 (2 marks)

Are the following two graphs isomorphic to G ? Justify your answer by either providing the bijective mapping of the vertices or by explaining why the graph cannot be isomorphic to G .

Hint: Consider how many neighbours each vertex in G has. If for example G has only one vertex v with n neighbours, then an isomorphic graph must also have exactly one vertex v' with n neighbours and v must be mapped to v' .

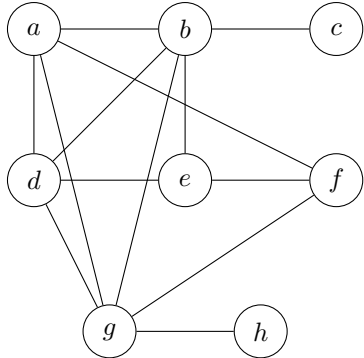


Exercise 8.5 (2 marks)

Prove that G is not planar by showing that K_5 is a minor of G . Only apply one transformation each step and provide the transformed graph as well as a description what transformation you applied to which part of the graph.

Example:

1. remove edge $\{c, f\}$:



Submission rules:

Upload a single archive containing a single PDF file (ending .pdf) generated using L^AT_EX as well as your filled out version of `Main.java`. *Do not upload any other .java files!* If you did not do the programming exercise, you may upload the single PDF directly.

Put the names of all group members on top of the first page of the PDF. Use page numbers or put your names on each page. Make sure your PDF has size A4 (fits the page size if printed on A4).