

# Discrete Mathematics in Computer Science

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## Exercise Sheet 7

**Due: Thursday, November 11, 2021**

### Exercise 7.1 (3 marks)

Consider the digraph  $G$  which corresponds to the relation  $R = \{(x, y) \mid x \leq y\}$  over  $\{0, 1, 2, 3\}$ .

- Draw a graphical representation of  $G$ .
- Specify the predecessors and successors of 1.
- Specify the formal definition of the graph  $G'$  induced by  $G$ .

### Exercise 7.2 (1 mark)

What is the maximal amount of arcs a digraph  $G = (N, A)$  can have? Specify your answer in relation to  $|N|$  and justify your answer.

### Exercise 7.3 (3 marks)

Are the following statements true? Justify your answer.

- Every graph with  $n$  edges has a path of length  $n + 1$ .
- Every graph with at least one cycle has an infinite amount of tours.
- If in a digraph  $G$  nodes  $v_i$  and  $v_j$  are mutually reachable then there is a cycle containing  $v_i$  and  $v_j$ .

### Exercise 7.4 (3 marks)

- Prove that for all  $n > 1$  there is a strongly connected digraph with  $n$  nodes and  $n$  arcs.
- Prove that every strongly connected digraph with  $n > 1$  nodes has at least  $n$  arcs.

### Submission rules:

Upload a single PDF file (ending .pdf) generated using L<sup>A</sup>T<sub>E</sub>X. Put the names of all group members on top of the first page. Use page numbers or put your names on each page. Make sure your PDF has size A4 (fits the page size if printed on A4).