Discrete Mathematics in Computer Science

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Exercise Sheet 5
Due: Thursday, October 28, 2021

Exercise 5.1 (2 marks)

Specify a partial function $f : \{0, 1, 2, 3, 4, 5\} \nrightarrow \{u, v, w, x, y, z\}$ which satisfies *all* of the following properties:

(i) $|\text{dom}(f)| = 5$

(ii) $\text{img}(f) = \{u, v, x, z\}$

(iii) $f^{-1}[\{u, v, w\}] = \{1, 2, 4\}$

(iv) $f|_{\{2,3,4\}}$ is a total function.

You can specify your function either by listing the mapping explicitly or by drawing the graph.

Exercise 5.2 (2 marks)

Prove the following statement:
For $f : A \rightarrow B$ and $g : B \rightarrow C$, if the composition $g \circ f$ is injective, then f is injective.

Exercise 5.3 (2 marks)

Show with an example that there exists sets A, B, C and functions $f : A \rightarrow B$ and $g : B \rightarrow C$ such that neither f nor g is bijective, but $g \circ f$ is bijective.
Hint: Use the result from Exercise 5.2 to think about which properties f and g must have.

Exercise 5.4 (3 marks)

Given $n \in \mathbb{N}_1$, consider the set $S_n = \{0, \ldots, 2n - 1\}$ and the following permutation:

$$\pi_n(i) = \begin{cases} i + 2 \mod 2n & \text{if } i \text{ is even} \\ i - 4 \mod 2n & \text{otherwise} \end{cases}$$

(a) Denote π_4 in two-line and cycle notation.

(b) Specify $\sigma \pi_3$ with $\sigma = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 4 & 3 & 0 & 2 \end{pmatrix}$ in two-line notation.

(c) Specify $\pi_5^{-1} \pi_5^{-1}$ in one-line notation.

Exercise 5.5 (1 mark)

Consider a permutation π over a set S consisting of n disjoint cycles with lengths m_i for $1 \le i \le n$. How many times do you need to apply the permutation to reach the initial configuration again? In other words, for which $k > 0$ do we have $\pi^k(x) = x$ for all $x \in S$?
Briefly justify your answer.
Hint: First consider the case of a single cycle.

Submission rules:
Upload a single PDF file (ending .pdf) generated using LATEX. Put the names of all group members on top of the first page. Use page numbers or put your names on each page. Make sure your PDF has size A4 (fits the page size if printed on A4).