Discrete Mathematics in Computer Science

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Exercise Sheet 4

Due: Thursday, October 21, 2021

The proofs on this exercise sheet are not hard, but you have to be careful about what properties you actually need to prove. We advise you to carefully re-read the definitions from the lecture before writing your proof.

Exercise 4.1 (1 mark)
Consider the partition

$$P = \{\{1, 4\}, \{2\}, \{3, 5, 6\}\}$$

over $S = \{1, 2, 3, 4, 5, 6\}$. Specify the equivalence relation R_P induced by P.

Exercise 4.2 (2 marks)
Prove the following statement:
Let P be a partition of set S. Then every $x \in S$ is an element of *exactly one* $X \in P$.

Exercise 4.3 (2 marks)
Specify a relation over $\mathbb{N}_0$ with the required properties or explain why such a relation cannot exist.

(a) A partial order where each element is both minimal and maximal.

(b) A strict order with a least element but no minimal element.

Exercise 4.4 (2 marks)
Let R be a total order over a finite set S and let $S' = S \cup \{a\}$ for some $a \notin S$.
Prove that

$$R' = R \cup R_a \text{ with } R_a = \{(a, i) \mid i \in S'\}$$

is a total order over S'.
Hint: You need to prove four properties.

Exercise 4.5 (3 marks)
Consider the following relations over $\mathbb{N}_0$:

$$R_1 = \{(x, 2x) \mid x \in \mathbb{N}_0\}$$
$$R_2 = \{(x, y) \mid x \in \mathbb{N}_0, y \in \mathbb{N}_0, x + y \text{ is odd}\}$$

Describe each of the relations described below as a set using set-builder notation.

(a) $R_2 \circ R_1$

(b) R_1^+

(c) $R_1 \cap \overline{R_2}$

Submission rules:
Upload a single PDF file (ending .pdf) generated using LaTeX. Put the names of all group members on top of the first page. Use page numbers or put your names on each page. Make sure your PDF has size A4 (fits the page size if printed on A4).