**Discrete Mathematics in Computer Science**

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# Exercise Sheet 3

**Due: Thursday, October 14, 2021**

**Exercise 3.1** (2 marks)

Show that $\mathbb{Z}$ is countably infinite by specifying a bijection from $\mathbb{Z}$ to $\mathbb{N}_0$.
*You need to show that your function is a bijection.*

**Exercise 3.2** (2 marks)

Show that $\mathbb{Q}$ is countable. You may use the fact that $\mathbb{Q}_+$ is countable.

*Hint: Use the countability of* $\mathbb{Q}_+$ *to show the countability of* $\mathbb{Q}_- = \{q \mid q \in \mathbb{Q}, q < 0\}$.

**Exercise 3.3** (2 marks)

Let $S$ be the set of all possible outcome sequences of finitely many coin flips. More precisely, $S$ contains for all $n \in \mathbb{N}_1$ all possible sequences that can result from flipping a coin $n$ times.
Show that $S$ is countable.

**Exercise 3.4** (1 mark)

Explain why the following statements are wrong.

(a) $\{0,1\} \times \{2\} = \langle \{0,2\}, \{1,2\} \rangle$

(b) $|\{0,1\} \times \emptyset| = 2$

**Exercise 3.5** (3 marks)

Consider the following binary relation $R$ over $\mathbb{N}_0$:

$$R = \{(x,y) \mid y = z \cdot x \textit{ for some } z \in \mathbb{N}_0 \textit{ with } z > 1\}$$

Is $R$ reflexive, irreflexive, symmetric, asymmetric, antisymmetric, transitive? Briefly justify your answer for each property.
*Hint: Carefully check the definition of* $R$*, especially the conditions for* $z$*, and consider which corner cases might arise.*

**Submission rules:**
Upload a single PDF file (ending .pdf) generated using LaTeX. Put the names of all group members on top of the first page. Use page numbers or put your names on each page. Make sure your PDF has size A4 (fits the page size if printed on A4).