Planning and Optimization

G4. Monte-Carlo Tree Search: Framework

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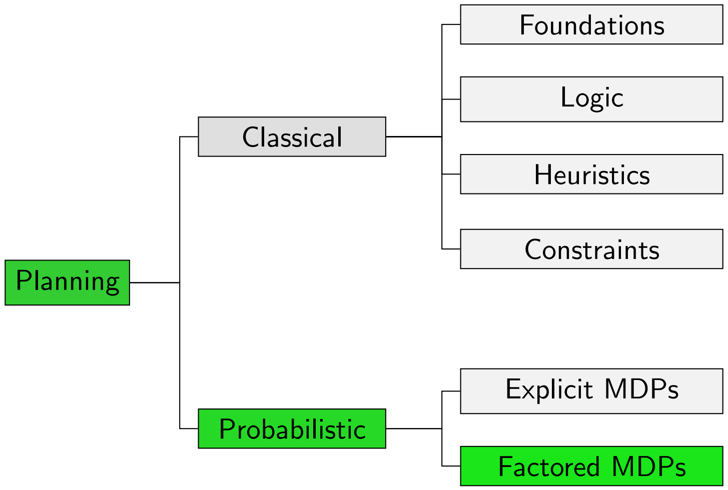
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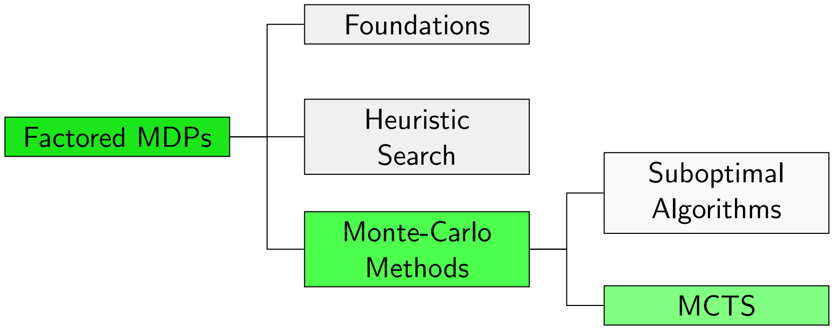
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G4.1 Motivation

Motivation

Previously discussed Monte-Carlo methods:

- Hindsight Optimization suffers from assumption of clairvoyance
- Policy Simulation overcomes assumption of clairvoyance by sampling the execution of a policy
- Policy Simulation is suboptimal due to inability of policy to improve
- Sparse Sampling achieves near-optimality without considering all outcomes
- Sparse Sampling wastes time in non-promising parts of state space

Monte-Carlo Tree Search

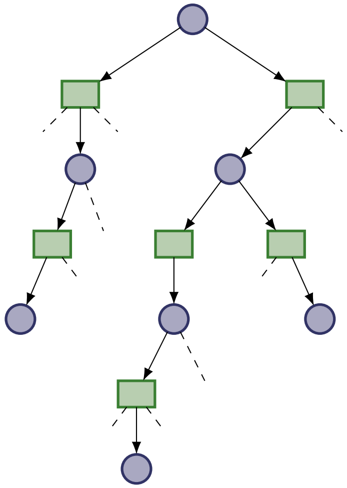
Monte-Carlo Tree Search (MCTS) has several similarities with algorithms we have already seen:

- Like (L)RTDP, MCTS performs trials (also called rollouts)
- Like Policy Simulation, trials simulate execution of a policy
- Like other Monte-Carlo methods, Monte-Carlo backups are performed
- Like Sparse Sampling, an outcome is only explicated if it is sampled in a trial

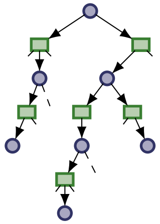
G4.2 MCTS Tree

MCTS Tree

- Unlike previous methods, the SSP is explicated as a tree
- Duplicates (also: transpositions) possible, i.e., multiple search nodes with identical associated state
- Search tree can (and often will) have unbounded depth



Tree Structure



- Differentiate between two types of search nodes:
  - Decision nodes
  - Chance nodes
- Search nodes correspond 1:1 to traces from initial state
- Decision and chance nodes alternate
- Decision nodes correspond to states in a trace
- Chance nodes correspond to actions in a trace
- Decision nodes have one child node for each applicable action (if all children are explicated)
- Chance nodes have one child node for each outcome (if all children are explicated)

MCTS Tree

Definition (MCTS Tree)

An MCTS tree is given by a tuple $\mathcal{G} = \langle d_0, D, C, E \rangle$, where

- D and C are disjoint sets of decision and chance nodes (simply search node if the type does not matter)
- $d_0 \in D$ is the root node
- $E \subseteq (D \times C) \cup (C \times D)$ is the set of edges such that the graph $\langle D \cup C, E \rangle$ is a tree

Note: can be regarded as an AND/OR tree

Search Node Annotations

Definition (Search Node Annotations)

Let $\mathcal{G} = \langle d_0, D, C, E \rangle$ be an MCTS Tree.

- Each search node $n \in D \cup C$ is annotated with
  - a visit counter $N(n)$
  - a state $s(n)$
- Each decision node $d \in D$ is annotated with
  - a state-value estimate $\hat{V}(d)$
  - a probability $p(d)$
- Each chance node $c \in C$ is annotated with
  - an action-value estimate (or Q-value estimate) $\hat{Q}(c)$
  - an action $a(c)$

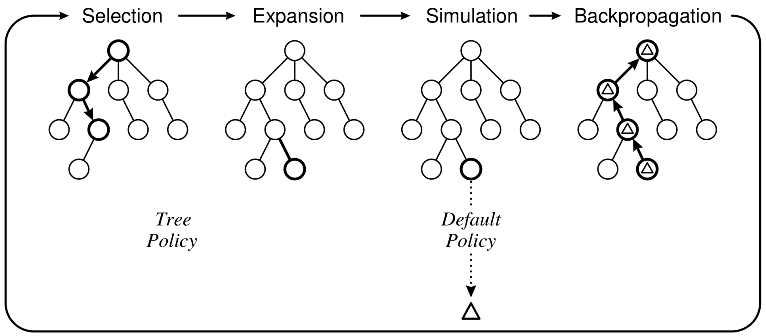
Note: some annotations can be computed on the fly to save memory

G4.3 Framework

Trials

- The MCTS tree is built in trials
- Trials are performed as long as resources (deliberation time, memory) allow
- Initially, the MCTS tree consists of only the root node for the initial state
- Trials (may) add search nodes to the tree
- MCTS tree at the end of the i-th trial is denoted with $\mathcal{G}^i$
- Use same superscript for annotations of search nodes

Trials



Taken from Browne et al., "A Survey of Monte Carlo Tree Search Methods", 2012

Phases of Trials

Each trial consists of (up to) four phases:

- Selection: traverse the tree by sampling the execution of the tree policy until
  1. an action is applicable that is not explicated, or
  2. an outcome is sampled that is not explicated, or
  3. a goal state is reached (jump to backpropagation)
- Expansion: create search nodes for the applicable action and a sampled outcome (case 1) or just the outcome (case 2)
- Simulation: simulate default policy until a goal is reached
- Backpropagation: update visited nodes in reverse order by
  - increasing visit counter by 1
  - performing Monte-Carlo backup of state-/action-value estimate

Monte-Carlo Backups in MCTS Tree

- let $d_0, c_0, \ldots, c_{n-1}, d_n$ be the decision and chance nodes that were visited in a trial of MCTS (including explicated ones),
- let h be the cost incurred by the simulation of the default policy until a goal state is reached
- each decision node d_j for $0 \leq j \leq n$ is updated by

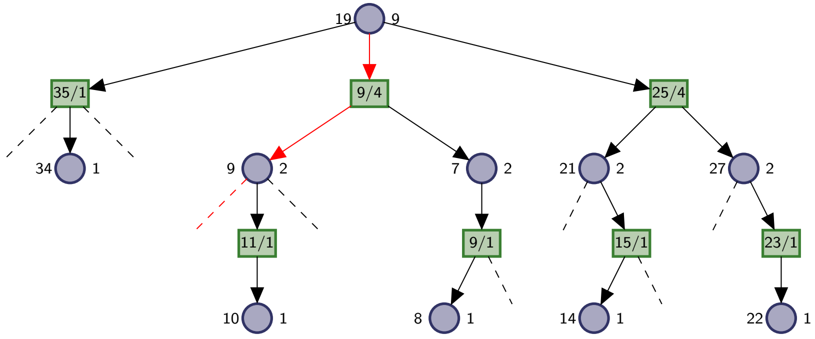
$$\hat{V}^i(d_j) := \hat{V}^{i-1}(d_j) + \frac{1}{N^i(d_j)}\left(\sum_{k=j}^{n-1} cost(a(c_k)) + h - \hat{V}^{i-1}(d_j)\right)$$

- each chance node c_j for $0 \leq j < n$ is updated by

$$\hat{Q}^i(c_j) := \hat{Q}^{i-1}(c_j) + \frac{1}{N^i(c_j)}\left(\sum_{k=j}^{n-1} cost(a(c_k)) + h - \hat{Q}^{i-1}(c_j)\right)$$

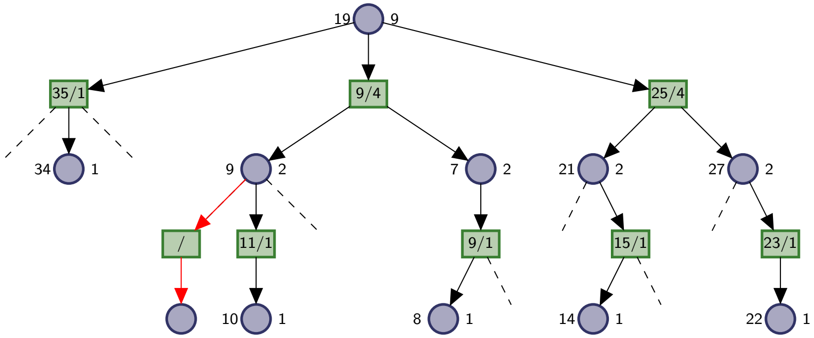
MCTS: (Unit-cost) Example

Selection phase: apply tree policy to traverse tree



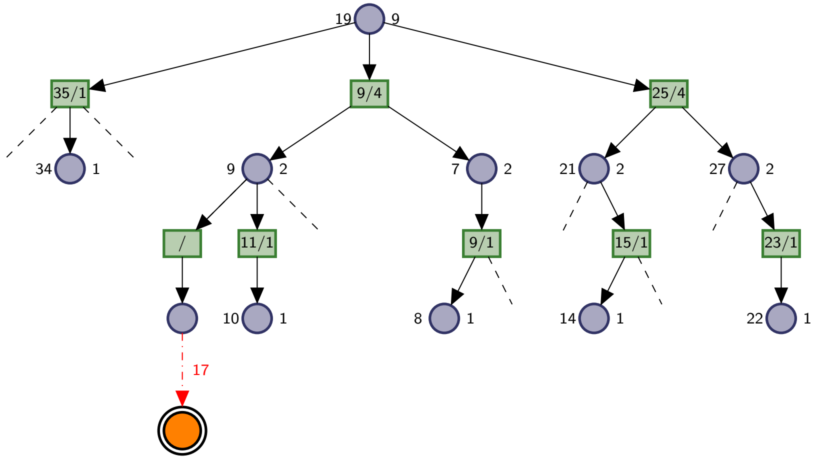
MCTS: (Unit-cost) Example

Expansion phase: create search nodes



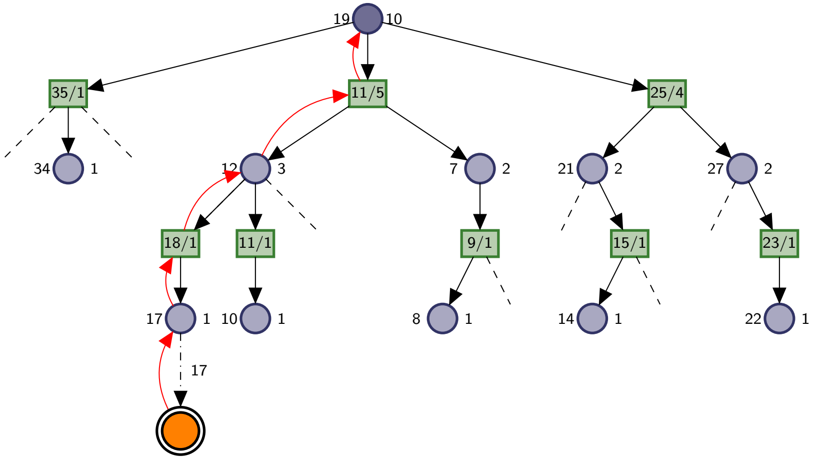
MCTS: (Unit-cost) Example

Simulation phase: apply default policy until goal



MCTS: (Unit-cost) Example

Backpropagation phase: update visited nodes



MCTS Framework

Member of MCTS framework are specified in terms of:

- Tree policy
- Default policy

MCTS Tree Policy

Definition (Tree Policy)

Let $\mathcal{T}$ be an SSP. An MCTS tree policy is a probability distribution $\pi(a \mid d)$ over all $a \in A(s(d))$ for each decision node d.

Note: The tree policy may take information annotated in the current tree into account.

MCTS Default Policy

Definition (Default Policy)

Let $\mathcal{T}$ be an SSP. An MCTS default policy is a probability distribution $\pi(a \mid s)$ over actions $a \in A(s)$ for each state s.

Note: The default policy is independent of the MCTS tree.

Monte-Carlo Tree Search

```
MCTS for SSP $\mathcal{T} = \langle S, A, c, T, s_0, S_\star \rangle$
$d_0$ = create root node associated with $s_0$
while time allows:
    visit_decision_node($d_0, \mathcal{T}$)
return $a(\arg\min_{c \in \text{children}(d_0)} \hat{Q}(c))$
```

MCTS: Visit a Decision Node

```
visit_decision_node for decision node $d$, SSP
$\mathcal{T} = \langle S, A, c, T, s_0, S_\star \rangle$
if $s(d) \in S_\star$ then return 0
if there is $a \in A(s(d))$ s.t. $a(c) \neq a$ for all $c \in$ children($d$):
    select such an $a$ and add node $c$ with $a(c) = a$ to children($d$)
else:
    c = tree_policy($d$)
cost = visit_chance_node($c, \mathcal{T}$)
$N(d) := N(d) + 1$
$\hat{V}(d) := \hat{V}(d) + \frac{1}{N(d)} \cdot (\text{cost} - \hat{V}(d))$
return cost
```

MCTS: Visit a Chance Node

```
visit_chance_node for chance node $c$, SSP $\mathcal{T} = \langle S, L, c, T, s_0, S_\star \rangle$
$s' \sim \text{succ}(s(c), a(c))$
let $d$ be the node in children($c$) with $s(d) = s'$
if there is no such node:
    add node $d$ with $s(d) = s'$ to children($c$)
    cost = sample_default_policy($s'$)
    $N(d) := 1$, $\hat{V}(d) :=$ cost
else:
    cost = visit_decision_node($d, \mathcal{T}$)
cost = cost + $cost(s(c), a(c))$
$N(c) := N(c) + 1$
$\hat{Q}(c) := \hat{Q}(c) + \frac{1}{N(c)} \cdot (\text{cost} - \hat{Q}(c))$
return cost
```

G4.4 Summary

Summary

- Monte-Carlo Tree Search is a framework for algorithms
- MCTS algorithms perform trials
- Each trial consists of (up to) 4 phases
- MCTS algorithms are specified by two policies:
  - a tree policy that describes behavior “in” tree
  - and a default policy that describes behavior “outside” of tree