

# Planning and Optimization

## E2. Landmarks: RTG Landmarks & MHS Heuristic

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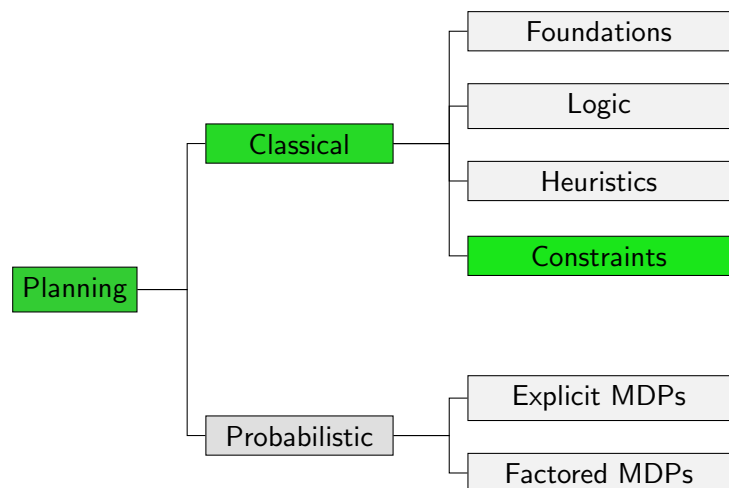
E2.1 Landmarks

E2.2 Landmarks from RTGs

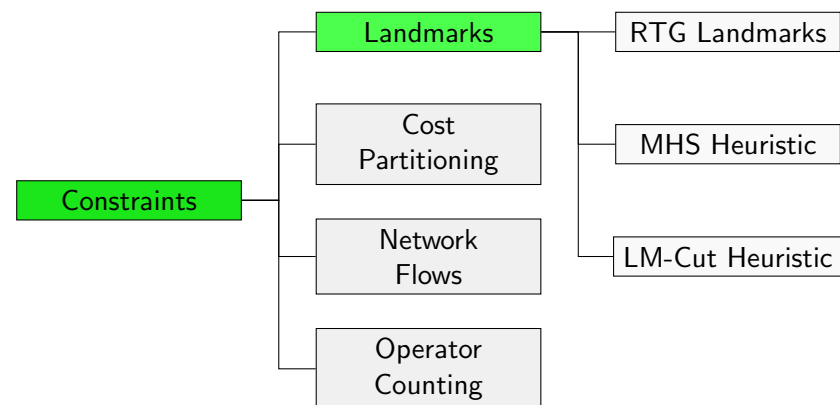
E2.3 Minimum Hitting Set Heuristic

E2.4 Summary

## Content of this Course



## Content of this Course: Constraints



## E2.1 Landmarks

## Landmarks

**Basic Idea:** Something that must happen **in every solution**

For example

- ▶ some operator must be applied (**action landmark**)
- ▶ some atomic proposition must hold (**fact landmark**)
- ▶ some formula must be true (**formula landmark**)

→ Derive heuristic estimate from this kind of information.

We only consider **fact** and **disjunctive action landmarks**.

## Definition

### Definition (Disjunctive Action Landmark)

Let  $s$  be a state of planning task  $\Pi = \langle V, I, O, \gamma \rangle$ .

A **disjunctive action landmark** for  $s$  is a set of operators  $L \subseteq O$  such that every label path from  $s$  to a goal state contains an operator from  $L$ .

The **cost** of landmark  $L$  is  $cost(L) = \min_{o \in L} cost(o)$ .

### Definition (Fact Landmark)

Let  $s$  be a state of planning task  $\Pi = \langle V, I, O, \gamma \rangle$ .

An atomic proposition  $v = d$  for  $v \in V$  and  $d \in \text{dom}(v)$  is a **fact landmark** for  $s$  if every state path from  $s$  to a goal state contains a state  $s'$  with  $s'(v) = d$ .

If we talk about landmarks for the initial state, we omit “for  $I$ ”.

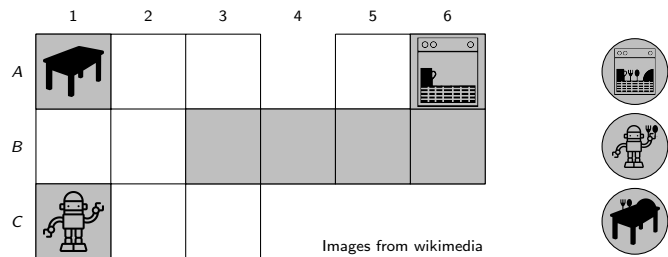
## Landmarks: Example

### Example

Consider a FDR planning task  $\langle V, I, O, \gamma \rangle$  with

- ▶  $V = \{robot-at, dishes-at\}$  with
  - ▶  $\text{dom}(robot-at) = \{A1, \dots, C3, B4, A5, \dots, B6\}$
  - ▶  $\text{dom}(dishes-at) = \{Table, Robot, Dishwasher\}$
- ▶  $I = \{robot-at \mapsto C1, dishes-at \mapsto Table\}$
- ▶ operators
  - ▶ move- $x$ - $y$  to move from cell  $x$  to adjacent cell  $y$
  - ▶ pickup dishes, and
  - ▶ load dishes into the dishwasher.
- ▶  $\gamma = (robot-at = B6) \wedge (dishes-at = Dishwasher)$

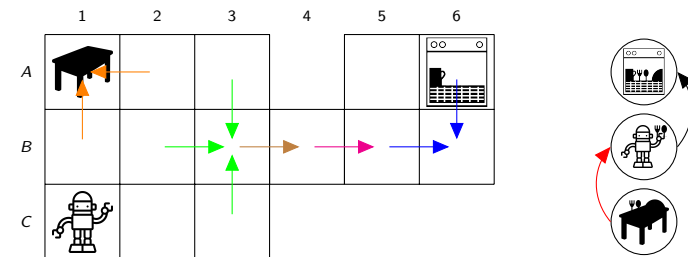
## Fact Landmarks: Example



Each fact in gray is a fact landmark:

- ▶  $\text{robot-at} = x$  for  $x \in \{A1, A6, B3, B4, B5, B6, C1\}$
- ▶  $\text{dishes-at} = x$  for  $x \in \{\text{Dishwasher, Robot, Table}\}$

## Disjunctive Action Landmarks: Example



Actions of same color form disjunctive action landmark:

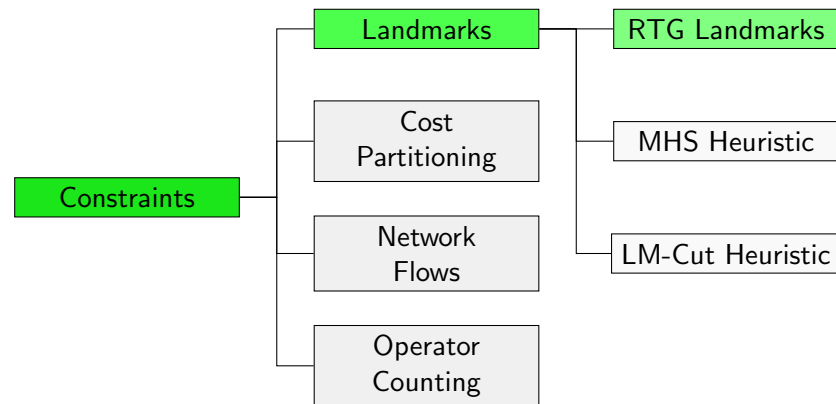
- ▶ {pickup}
- ▶ {load}
- ▶ {move-B3-B4}
- ▶ {move-B4-B5}
- ▶ {move-A6-B6, move-B5-B6}
- ▶ {move-A3-B3, move-B2-B3, move-C3-B3}
- ▶ {move-B1-A1, move-A2-A1}
- ▶ ...

## Remarks

- ▶ Not every landmark is informative. **Some examples:**
  - ▶ The set of all operators is a disjunctive action landmark unless the initial state is already a goal state.
  - ▶ Every variable that is initially true is a fact landmark.
- ▶ Deciding whether a given variable is a fact landmark is as hard as the plan existence problem.
- ▶ Deciding whether a given operator set is a disjunctive action landmark is as hard as the plan existence problem.
- ▶ Every fact landmark  $v$  that is initially false induces a disjunctive action landmark consisting of all operators that possibly make  $v$  true.

## E2.2 Landmarks from RTGs

## Content of this Course: Constraints



## Computing Landmarks

How can we come up with landmarks?

Most landmarks are derived from the **relaxed task graph**:

- ▶ RHW landmarks: Richter, Helmert & Westphal. Landmarks Revisited. (AAAI 2008)
- ▶ **LM-Cut**: Helmert & Domshlak. Landmarks, Critical Paths and Abstractions: What's the Difference Anyway? (ICAPS 2009)
- ▶  **$h^m$  landmarks**: Keyder, Richter & Helmert: Sound and Complete Landmarks for And/Or Graphs (ECAI 2010)

We discuss  **$h^m$  landmarks** restricted to  $m = 1$  and to STRIPS planning tasks.

## Incidental Landmarks: Example

### Example (Incidental Landmarks)

Consider a STRIPS planning task  $\langle V, I, \{o_1, o_2\}, \gamma \rangle$  with

$$V = \{a, b, c, d, e, f\},$$

$$I = \{a \mapsto \mathbf{T}, b \mapsto \mathbf{T}, c \mapsto \mathbf{F}, d \mapsto \mathbf{F}, e \mapsto \mathbf{T}, f \mapsto \mathbf{F}\},$$

$$o_1 = \langle \{a\}, \{c, d, e\}, \{a, b\} \rangle,$$

$$o_2 = \langle \{d, e\}, \{f\}, \{a, d\} \rangle, \text{ and}$$

$$\gamma = \{e, f\}.$$

Single solution:  $\langle o_1, o_2 \rangle$

- ▶ All variables are fact landmarks.
- ▶ Variable  $b$  is initially true but irrelevant for the plan.
- ▶ Variable  $c$  gets true as “side effect” of  $o_1$  but it is not necessary for the goal or to make an operator applicable.

## Causal Landmarks

### Definition (Causal Fact Landmark)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a STRIPS planning task.

An atomic proposition  $v = \mathbf{T}$  for  $v \in V$  is a **causal fact landmark**

- ▶ if  $v \in \gamma$
- ▶ or if for all goal paths  $\pi = \langle o_1, \dots, o_n \rangle$  there is an  $o_i$  with  $v \in \text{pre}(o_i)$ .

## Causal Landmarks: Example

### Example (Causal Landmarks)

Consider a STRIPS planning task  $\langle V, I, \{o_1, o_2\}, \gamma \rangle$  with

$$V = \{a, b, c, d, e, f\},$$

$$I = \{a \mapsto \mathbf{T}, b \mapsto \mathbf{T}, c \mapsto \mathbf{F}, d \mapsto \mathbf{F}, e \mapsto \mathbf{T}, f \mapsto \mathbf{F}\},$$

$$o_1 = \langle \{a\}, \{c, d, e\}, \{a, b\} \rangle,$$

$$o_2 = \langle \{d, e\}, \{f\}, \{a, d\} \rangle, \text{ and}$$

$$\gamma = \{e, f\}.$$

Single solution:  $\langle o_1, o_2 \rangle$

- ▶ All variables are fact landmarks for the initial state.
- ▶ Only  $a, d, e$  and  $f$  are causal landmarks.

## What We Are Doing Next

- ▶ Causal landmarks are the desirable landmarks.
- ▶ We can use a simplified version of RTGs to compute causal landmarks for STRIPS planning tasks.
- ▶ We will define landmarks of AND/OR graphs, ...
- ▶ and show how they can be computed.
- ▶ Afterwards we establish that these are landmarks of the planning task.

## Simplified Relaxed Task Graph

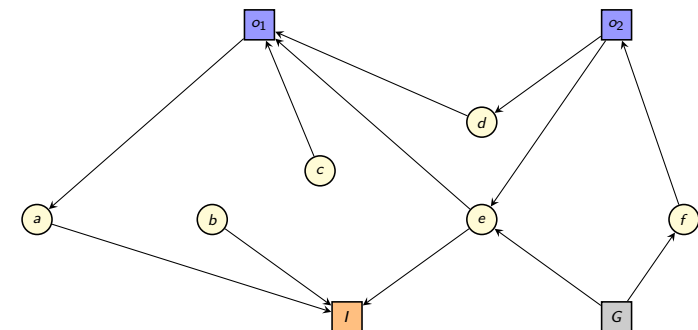
### Definition

For a STRIPS planning task  $\Pi = \langle V, I, O, \gamma \rangle$ , the **simplified relaxed task graph**  $sRTG(\Pi^+)$  is the **AND/OR graph**  $\langle N_{\text{and}} \cup N_{\text{or}}, A, \text{type} \rangle$  with

- ▶  $N_{\text{and}} = \{n_o \mid o \in O\} \cup \{v_I, v_G\}$   
with  $\text{type}(n) = \wedge$  for all  $n \in N_{\text{and}}$ .
- ▶  $N_{\text{or}} = \{n_v \mid v \in V\}$   
with  $\text{type}(n) = \vee$  for all  $n \in N_{\text{or}}$ , and
- ▶  $A = \{ \langle n_a, n_o \rangle \mid o \in O, a \in \text{add}(o) \} \cup$   
 $\{ \langle n_o, n_p \rangle \mid o \in O, p \in \text{pre}(o) \} \cup$   
 $\{ \langle n_v, n_I \rangle \mid v \in I \} \cup$   
 $\{ \langle n_G, n_v \rangle \mid v \in \gamma \}$

## Simplified RTG: Example

The simplified RTG for our example task is:



## Characterizing Equation System

### Theorem

Let  $G = \langle N, A, \text{type} \rangle$  be an AND/OR graph. Consider the following system of equations:

$$LM(n) = \{n\} \cup \bigcap_{\langle n, n' \rangle \in A} LM(n') \quad \text{type}(n) = \vee$$

$$LM(n) = \{n\} \cup \bigcup_{\langle n, n' \rangle \in A} LM(n') \quad \text{type}(n) = \wedge$$

The equation system has a unique maximal solution (maximal with regard to set inclusion), and for this solution it holds that

$n' \in LM(n)$  iff  $n'$  is a landmark for reaching  $n$  in  $G$ .

## Computation of Maximal Solution

### Theorem

Let  $G = \langle N, A, \text{type} \rangle$  be an AND/OR graph. Consider the following system of equations:

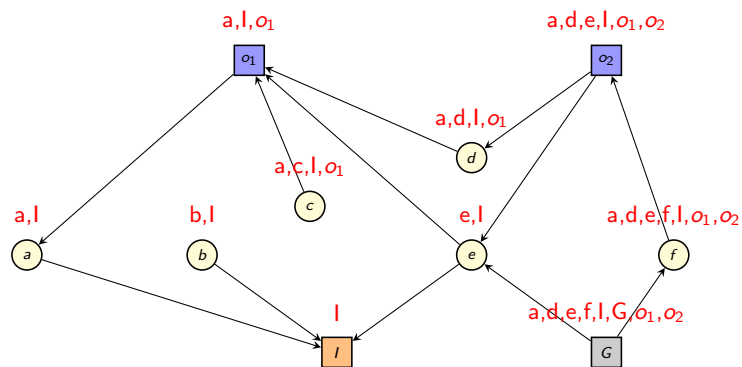
$$LM(n) = \{n\} \cup \bigcap_{\langle n, n' \rangle \in A} LM(n') \quad \text{type}(n) = \vee$$

$$LM(n) = \{n\} \cup \bigcup_{\langle n, n' \rangle \in A} LM(n') \quad \text{type}(n) = \wedge$$

The equation system has a unique maximal solution (maximal with regard to set inclusion).

**Computation:** Initialize landmark sets as  $LM(n) = N_{\text{and}} \cup N_{\text{or}}$  and apply equations as update rules until fixpoint.

## Computation: Example



(cf. screen version of slides for step-wise computation)

## Relation to Planning Task Landmarks

### Theorem

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a STRIPS planning task and let  $\mathcal{L}$  be the set of landmarks for reaching  $n_G$  in  $sRTG(\Pi^+)$ .

The set  $\{v = \mathbf{T} \mid v \in V \text{ and } n_v \in \mathcal{L}\}$  is exactly the set of causal fact landmarks in  $\Pi^+$ .

For operators  $o \in O$ , if  $n_o \in \mathcal{L}$  then  $\{o\}$  is a disjunctive action landmark in  $\Pi^+$ .

There are no other disjunctive action landmarks of size 1.

(Proofs omitted.)

## Computed RTG Landmarks: Example

### Example (Computed RTG Landmarks)

Consider a STRIPS planning task  $\langle V, I, \{o_1, o_2\}, \gamma \rangle$  with

$$V = \{a, b, c, d, e, f\},$$

$$I = \{a \mapsto \mathbf{T}, b \mapsto \mathbf{T}, c \mapsto \mathbf{F}, d \mapsto \mathbf{F}, e \mapsto \mathbf{T}, f \mapsto \mathbf{F}\},$$

$$o_1 = \langle \{a\}, \{c, d, e\}, \{a, b\} \rangle,$$

$$o_2 = \langle \{d, e\}, \{f\}, \{a, d\} \rangle, \text{ and}$$

$$\gamma = \{e, f\}.$$

- ▶  $LM(n_G) = \{a, d, e, f, I, G, o_1, o_2\}$
- ▶  $a, d, e$ , and  $f$  are causal fact landmarks of  $\Pi^+$ .
- ▶  $\{o_1\}$  and  $\{o_2\}$  are disjunctive action landmarks of  $\Pi^+$ .

## Landmarks of $\Pi^+$ Are Landmarks of $\Pi$

### Theorem

Let  $\Pi$  be a STRIPS planning task.

All fact landmarks of  $\Pi^+$  are fact landmarks of  $\Pi$  and all disjunctive action landmarks of  $\Pi^+$  are disjunctive action landmarks of  $\Pi$ .

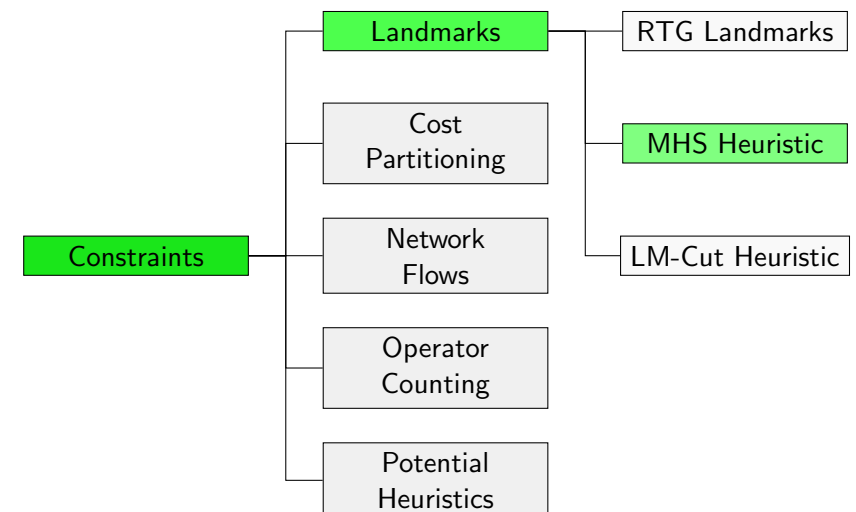
### Proof.

Let  $L$  be a disjunctive action landmark of  $\Pi^+$  and  $\pi$  be a plan for  $\Pi$ . Then  $\pi$  is also a plan for  $\Pi^+$  and, thus,  $\pi$  contains an operator from  $L$ .

Let  $f$  be a fact landmark of  $\Pi^+$ . If  $f$  is already true in the initial state, then it is also a landmark of  $\Pi$ . Otherwise, every plan for  $\Pi^+$  contains an operator that adds  $f$  and the set of all these operators is a disjunctive action landmark of  $\Pi^+$ . Therefore, also each plan of  $\Pi$  contains such an operator, making  $f$  a fact landmark of  $\Pi$ .  $\square$

## E2.3 Minimum Hitting Set Heuristic

## Content of this Course: Constraints



## Exploiting Disjunctive Action Landmarks

- ▶ The cost  $cost(L)$  of a disjunctive action landmark  $L$  is an admissible heuristic, but it is usually not very informative.
- ▶ Landmark heuristics typically aim to combine multiple disjunctive action landmarks.

How can we exploit a given set  $\mathcal{L}$  of disjunctive action landmarks?

- ▶ Sum of costs  $\sum_{L \in \mathcal{L}} cost(L)$ ?  
 $\rightsquigarrow$  **not admissible!**
- ▶ Maximize costs  $\max_{L \in \mathcal{L}} cost(L)$ ?  
 $\rightsquigarrow$  **usually very weak heuristic**
- ▶ **better:** Hitting sets

## Hitting Sets

### Definition (Hitting Set)

Let  $X$  be a set,  $\mathcal{F} = \{F_1, \dots, F_n\} \subseteq 2^X$  be a family of subsets of  $X$  and  $c : X \rightarrow \mathbb{R}_0^+$  be a cost function for  $X$ .

A **hitting set** is a subset  $H \subseteq X$  that “hits” all subsets in  $\mathcal{F}$ , i.e.,  $H \cap F \neq \emptyset$  for all  $F \in \mathcal{F}$ . The **cost** of  $H$  is  $\sum_{x \in H} c(x)$ .

A **minimum hitting set (MHS)** is a hitting set with minimal cost.

MHS is a “classical” NP-complete problem (Karp, 1972)

## Example: Hitting Sets

### Example

$$X = \{o_1, o_2, o_3, o_4\}$$

$$\mathcal{F} = \{\{o_4\}, \{o_1, o_2\}, \{o_1, o_3\}, \{o_2, o_3\}\}$$

$$c(o_1) = 3, \quad c(o_2) = 4, \quad c(o_3) = 5, \quad c(o_4) = 0$$

What is a minimum hitting set?

**Solution:**  $\{o_1, o_2, o_4\}$  with cost  $3 + 4 + 0 = 7$

## Hitting Sets for Disjunctive Action Landmarks

**Idea:** **disjunctive action landmarks** are interpreted as instance of **minimum hitting set**

### Definition (Hitting Set Heuristic)

Let  $\mathcal{L}$  be a set of disjunctive action landmarks. The **hitting set heuristic**  $h^{MHS}(\mathcal{L})$  is defined as the cost of a minimum hitting set for  $\mathcal{L}$  with  $c(o) = cost(o)$ .

### Proposition (Hitting Set Heuristic is Admissible)

Let  $\mathcal{L}$  be a set of disjunctive action landmarks for state  $s$ . Then  $h^{MHS}(\mathcal{L})$  is an admissible estimate for  $s$ .



## Hitting Set Heuristic: Discussion

- ▶ The hitting set heuristic is the **best possible** heuristic that only uses the given information. . .
- ▶ . . . but is NP-hard to compute.
- ▶  $\rightsquigarrow$  Use approximations that can be efficiently computed.  
⇒ **LP-relaxation**, **cost partitioning** (both discussed later)

## E2.4 Summary

## Summary

- ▶ **Fact landmark**: atomic proposition that is true in each state path to a goal
- ▶ **Disjunctive action landmark**: set  $L$  of operators such that every plan uses some operator from  $L$
- ▶ **Relaxed task graphs** allows efficient computation of landmarks
- ▶ **Hitting sets** yield the most accurate heuristic for a given set of disjunctive action landmarks
- ▶ Computation of **minimal hitting set** is NP-hard