Planning and Optimization

E1. Constraints: Introduction

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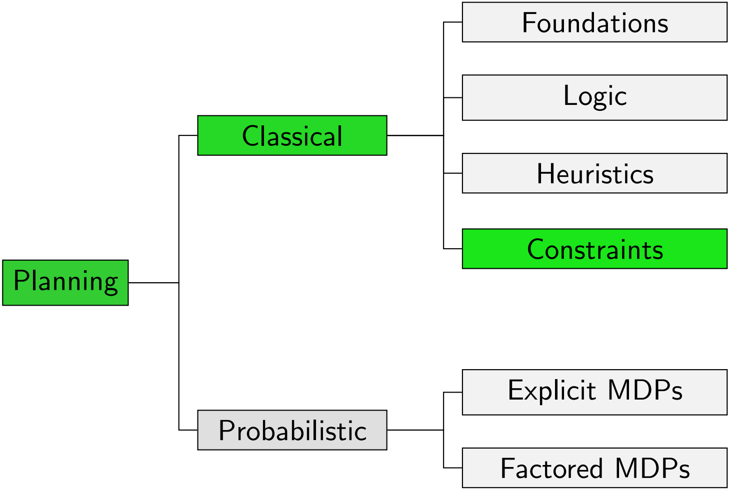
Universität Basel

Content of this Course



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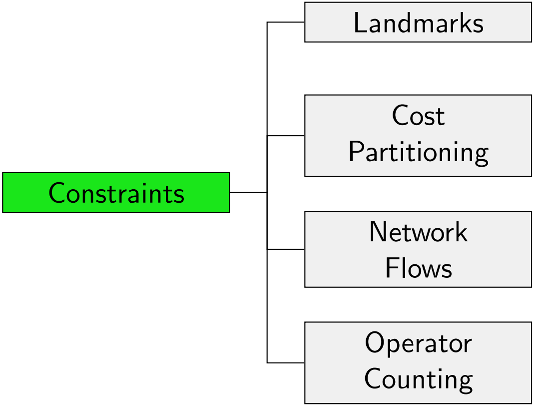


Constraint-based Heuristics

Coming Up with Heuristics in a Principled Way

General Procedure for Obtaining a Heuristic

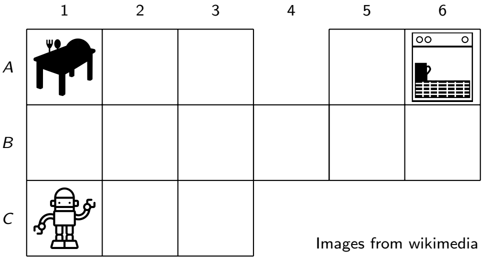
Solve a simplified version of the problem.

Major ideas for heuristics in the planning literature:

- delete relaxation
- abstraction
- landmarks
- critical paths
- network flows
- potential heuristic

Landmarks, network flows and potential heuristics are based on constraints that can be specified for a planning task.

Constraints: Example



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Constraints: Example

Example

Consider a FDR planning task $\langle V, I, O, \gamma \rangle$ with

- $V = \{\mathit{robot\text{-}at}, \mathit{dishes\text{-}at}\}$ with
  - $\mathrm{dom}(\mathit{robot\text{-}at}) = \{\mathrm{A1}, \dots, \mathrm{C3}, \mathrm{B4}, \mathrm{A5}, \dots, \mathrm{B6}\}$
  - $\mathrm{dom}(\mathit{dishes\text{-}at}) = \{\mathrm{Table}, \mathrm{Robot}, \mathrm{Dishwasher}\}$
- $I = \{\mathit{robot\text{-}at} \mapsto \mathrm{C1}, \mathit{dishes\text{-}at} \mapsto \mathrm{Table}\}$
- operators
  - move-x-y to move from cell x to adjacent cell y
  - pickup dishes, and
  - load dishes into the dishwasher.
- $\gamma = (\mathit{robot\text{-}at} = B6) \wedge (\mathit{dishes\text{-}at} = \mathrm{Dishwasher})$

Constraints

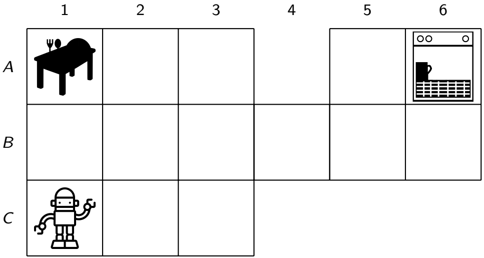
Some heuristics exploit constraints that describe something that holds in every solution of the task.

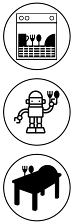
For instance, every solution is such that

- a variable takes some value in at least one visited state. (a fact landmark constraint)

Fact Landmarks: Example

Which values do *robot-at* and *dishes-at* take in every solution?





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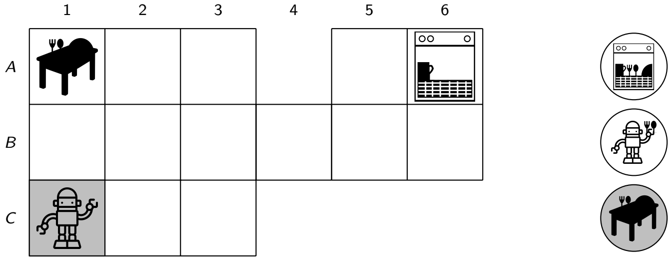

- robot-at = C1, dishes-at = Table (initial state)

Fact Landmarks: Example

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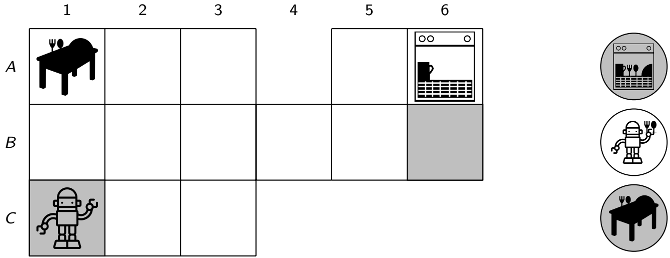

- `robot-at` = C1, `dishes-at` = Table (initial state)
- `robot-at` = B6, `dishes-at` = Dishwasher (goal state)

Fact Landmarks: Example

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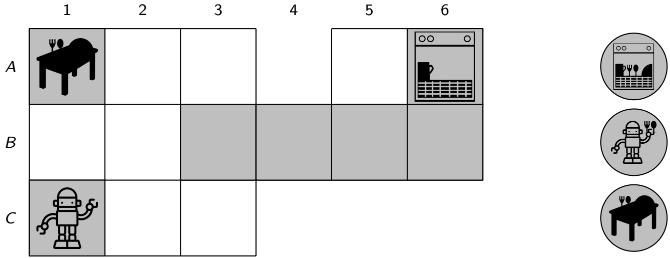

- `robot-at` = C1, `dishes-at` = Table (initial state)
- `robot-at` = B6, `dishes-at` = Dishwasher (goal state)
- `robot-at` = A1, `robot-at` = B3, `robot-at` = B4, `robot-at` = B5, `robot-at` = A6, `dishes-at` = Robot

Constraints

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For instance, every solution is such that

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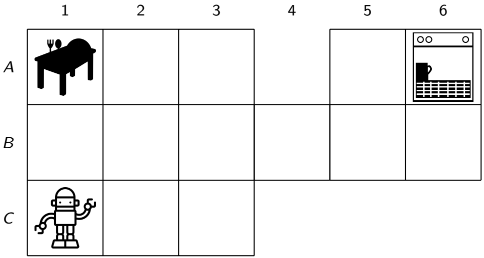
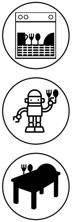
- an action must be applied. (an action landmark constraint)

Action Landmarks: Example

Which actions must be applied in every solution?





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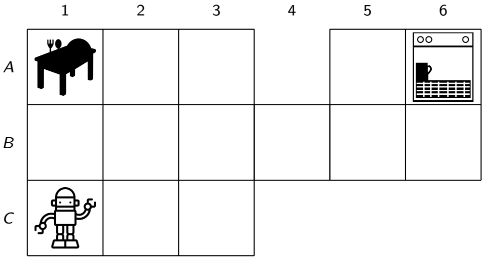

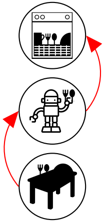

- pickup
- load

Action Landmarks: Example

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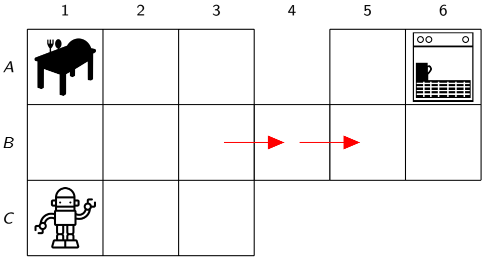

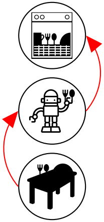

- pickup
- load
- move-B3-B4
- move-B4-B5

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Constraints

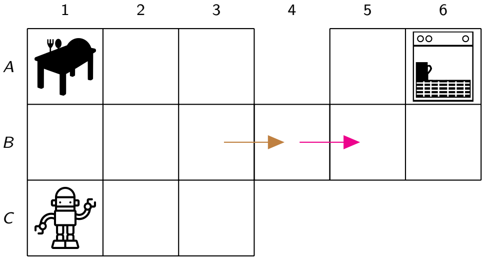
Some heuristics exploit constraints that describe something that holds in every solution of the task.

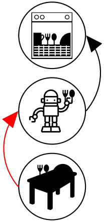
For instance, every solution is such that

- a variable takes some value in at least one visited state. (a fact landmark constraint)
- at least one action from a set of actions must be applied. (a disjunctive action landmark constraint)

Disjunctive Action Landmarks: Example

Which set of actions is such that at least one must be applied?





- {pickup}
- {load}
- {move-B3-B4}
- {move-B4-B5}

Disjunctive Action Landmarks: Example

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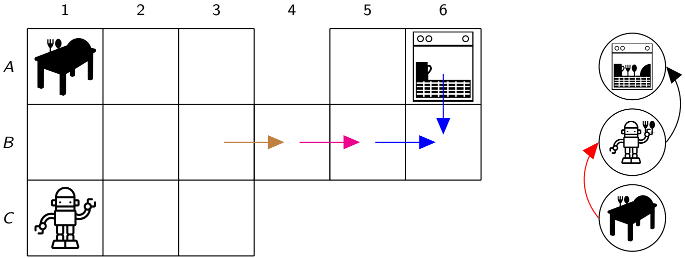
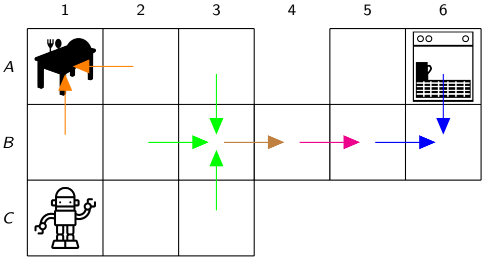
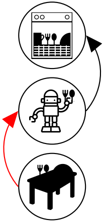


- {pickup}
- {load}
- {move-B3-B4}
- {move-B4-B5}
- {move-A6-B6, move-B5-B6}

Disjunctive Action Landmarks: Example

Which set of actions is such that at least one must be applied?





- {pickup}
- {load}
- {move-B3-B4}
- {move-B4-B5}
- {move-A6-B6, move-B5-B6}
- {move-A3-B3, move-B2-B3, move-C3-B3}
- {move-B1-A1, move-A2-A1}
- ...

Constraints

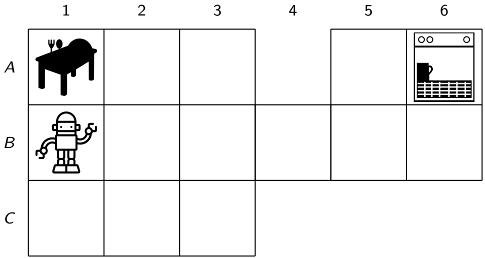
Some heuristics exploit constraints that describe something that holds in every solution of the task.

For instance, every solution is such that

- a variable takes some value in at least one visited state.
  (a fact landmark constraint)
- at least one action from a set of actions must be applied.
  (a disjunctive action landmark constraint)
- fact consumption and production is “balanced”.
  (a network flow constraint)

Network Flow: Example

Consider the fact robot-at $= B1$.
How often are actions used that enter this cell?



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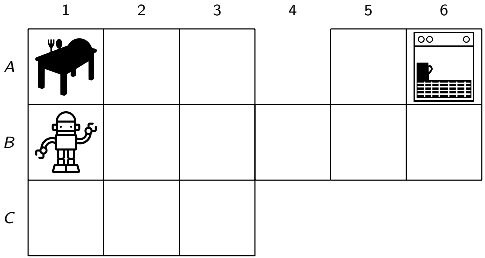

Answer: as often as actions that leave this cell

If Count_o denotes how often operator o is applied, we have:

$$\text{Count}_{\text{move-A1-B1}} + \text{Count}_{\text{move-B2-B1}} + \text{Count}_{\text{move-C1-B1}} =$$
$$\text{Count}_{\text{move-B1-A1}} + \text{Count}_{\text{move-B1-B2}} + \text{Count}_{\text{move-B1-C1}}$$

Multiple Heuristics

Combining Admissible Heuristics Admissibly

Major ideas to combine heuristics admissibly:

- maximize
- canoncial heuristic (for abstractions)
- minimum hitting set (for landmarks)
- cost partitioning
- operator counting

Often computed as solution to a (integer) linear program.

Combining Heuristics Admissibly: Example

Example

Consider an FDR planning task $\langle V, I, \{o_1, o_2, o_3, o_4\}, \gamma\rangle$ with $V = \{v_1, v_2, v_3\}$ with $dom(v_1) = \{A, B\}$ and $\text{dom}(v_2) = \text{dom}(v_3) = \{A, B, C\}$, $I = \{v_1 \mapsto A, v_2 \mapsto A, v_3 \mapsto A\}$,

$$o_1 = \langle v_1 = \text{A}, v_1 := \text{B}, 1\rangle$$
$$o_2 = \langle v_2 = \text{A} \wedge v_3 = \text{A}, v_2 := \text{B} \wedge v_3 := \text{B}, 1\rangle$$
$$o_3 = \langle v_2 = \text{B}, v_2 := \text{C}, 1\rangle$$
$$o_4 = \langle v_3 = \text{B}, v_3 := \text{C}, 1\rangle$$

and $\gamma = (v_1 = \text{B}) \wedge (v_2 = \text{C}) \wedge (v_3 = \text{C})$.

Let $\mathcal{C}$ be the pattern collection that contains all atomic projections. What is the canonical heuristic function $h^{\mathcal{C}}$?

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$$o_1 = \langle v_1 = \text{A}, v_1 := \text{B}, 1\rangle$$
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Let $\mathcal{C}$ be the pattern collection that contains all atomic projections. What is the canonical heuristic function $h^{\mathcal{C}}$?

Answer: Let $h_i := h^{v_i}$. Then $h^{\mathcal{C}} = \max\{h_1 + h_2, h_1 + h_3\}$.

Reminder: Orthogonality and Additivity

Why can we add h_1 and h_2 (h_1 and h_3) admissibly?

Theorem (Additivity for Orthogonal Abstractions)

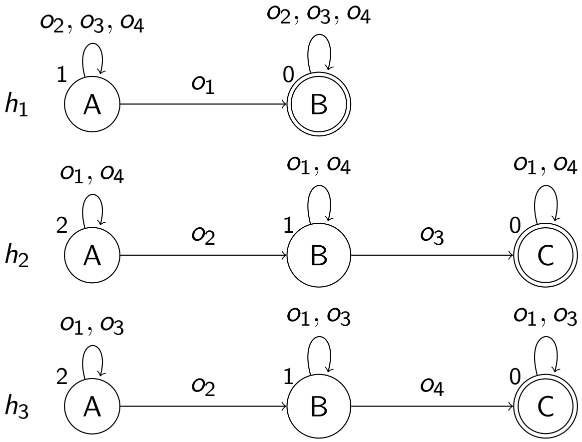
Let $h^{\alpha_1}, \dots, h^{\alpha_n}$ be abstraction heuristics of the same transition system such that α_i and α_j are orthogonal for all $i \neq j$.

Then $\sum_{i=1}^{n} h^{\alpha_i}$ is a safe, goal-aware, admissible and consistent heuristic for Π.

Consistency proof exploits that every concrete transition induces state-changing transition in at most one abstraction.

Combining Heuristics Admissibly: Example

Let $h = h_1 + h_2 + h_3$. Where is consistency violated?



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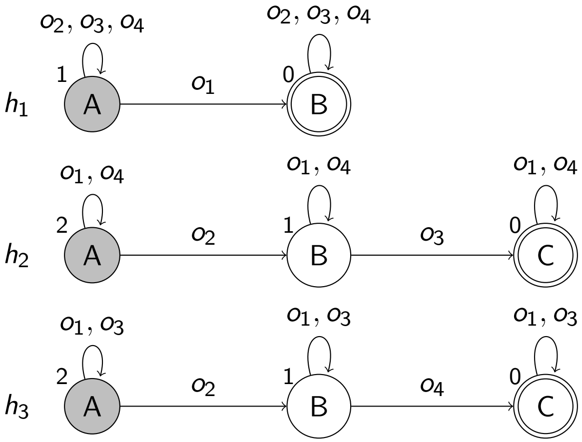

Consider solution $\langle o_1, o_2, o_3, o_4 \rangle$

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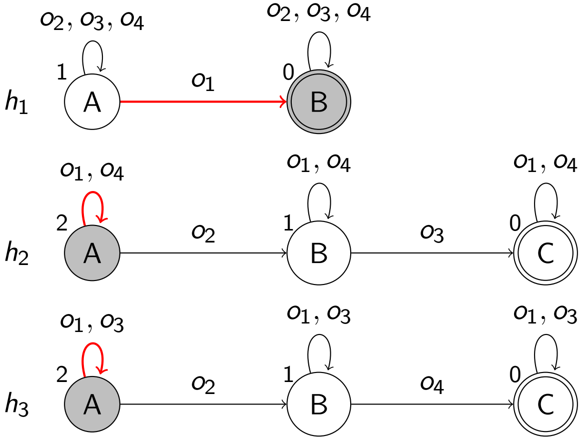

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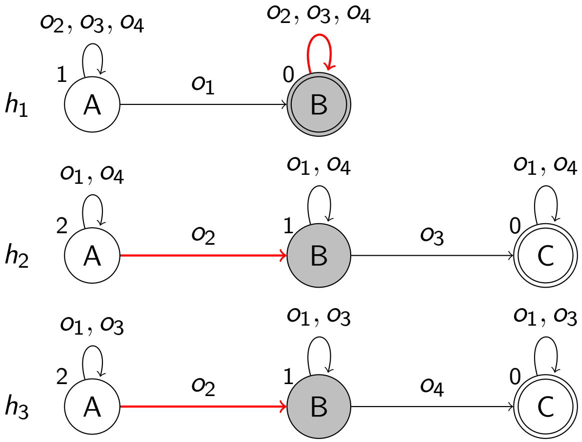

Here:
$h(BAA) = 4$
$h(BBB) = 2$

h_2 and h_3 not additive because of o_2

Consider solution $\langle o_1, o_2, o_3, o_4 \rangle$

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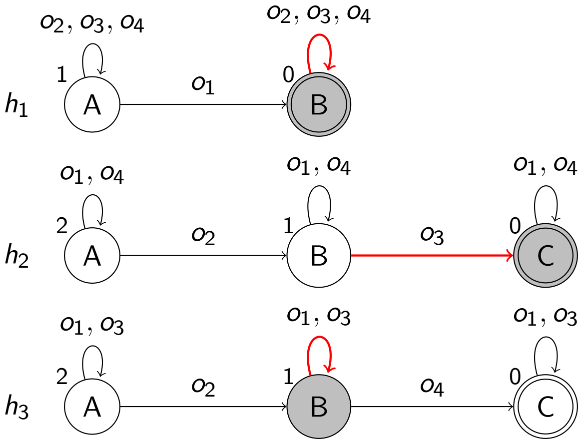

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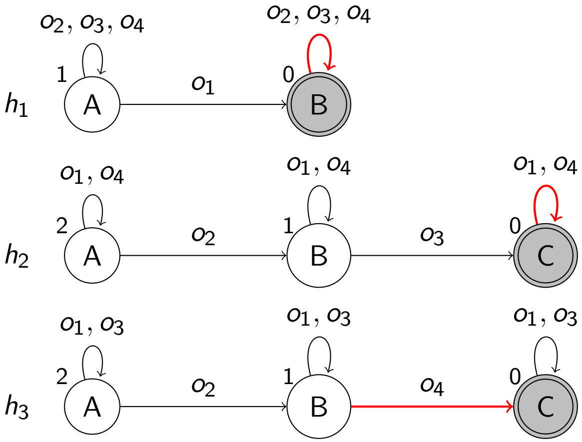

Consider solution $\langle o_1, o_2, o_3, o_4 \rangle$

Inconsistency of h_2 and h_3

The reason that h_2 and h_3 are not additive is because the cost of o_2 is considered in both.

Is there anything we can do about this?

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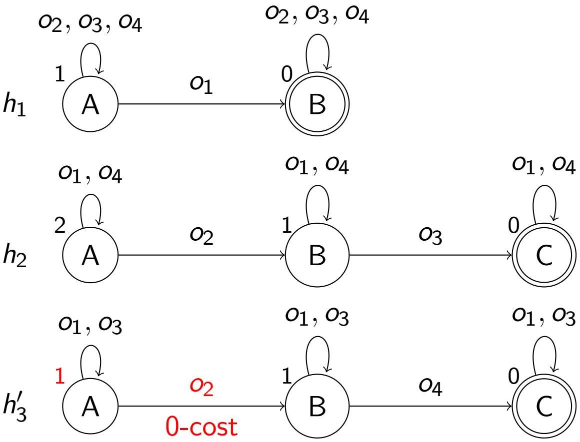
The reason that h_2 and h_3 are not additive is because the cost of o_2 is considered in both.

Is there anything we can do about this?

Solution: We can ignore the cost of o_2 in one heuristic by setting its cost to 0 (e.g., $cost_3(o_2) = 0$).

Combining Heuristics Admissibly: Example

Let $h' = h_1 + h_2 + h'_3$, where $h'_3 = h^{v_3}$ assuming $cost_3(o_2) = 0$.



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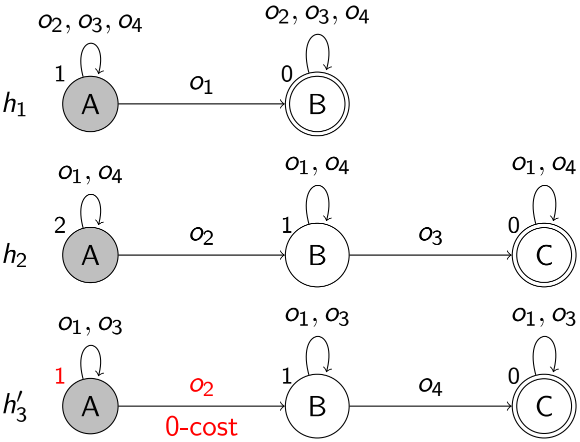

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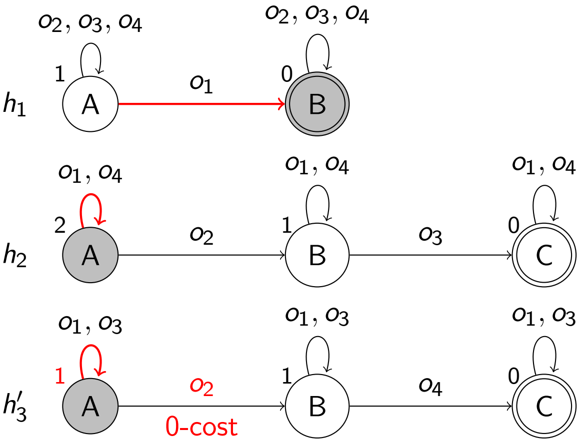

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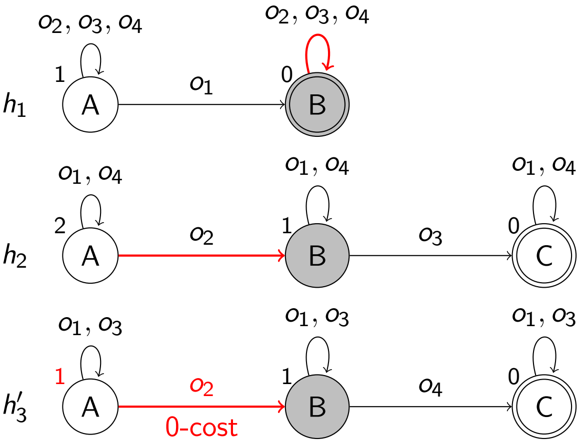

Here:
$h'(BAA) = 3$
$h'(BBB) = 2$

h_2 and h'_3 are additive

Consider solution $\langle o_1, o_2, o_3, o_4 \rangle$

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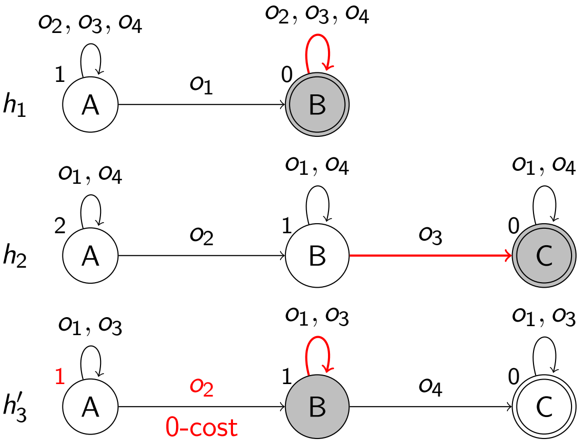

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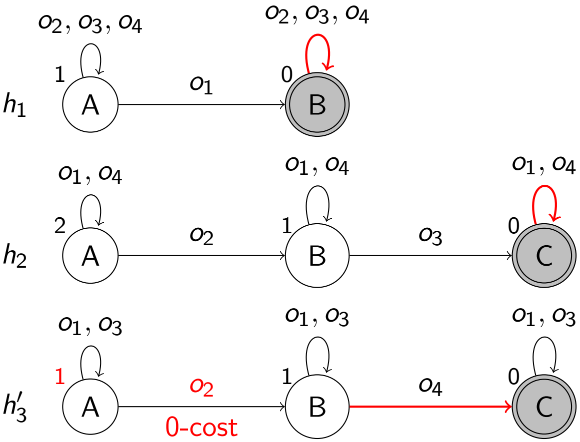

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Cost partitioning

Using the cost of every operator only in one heuristic is called a zero-one cost partitioning.

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More generally, heuristics are additive if all operator costs are distributed in a way that the sum of the individual costs is no larger than the cost of the operator.

This can also be expressed as a constraint,
the cost partitioning constraint:

$$\sum_{i=1}^{n} cost_i(o) \leq cost(o) \text{ for all } o \in O$$

(more details later)

Summary

Summary

- Landmarks and network flows are constraints that describe something that holds in every solution of the task.
- Heuristics can be summed up admissibly if the cost partitioning constraint is satisfied.